Communications

What is a concept?

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I have been re-reading a chapter by Rosch (1999) entitled *Reclaiming concepts,* as well as a new book entitled *Enaction* (Stewart, Gapenne & Di Paolo, 2011). Focusing on learning, I am challenged by:

> what *can* be "learned" is both *enabled* and *constrained* by the epigenetic landscape. Development, and therefore learning, is essentially an endogenously self-generating process; it is therefore unnecessary - and impossible - to "instruct" it from the outside. [ ] at any particular stage in the dynamical process of development, only a very restricted set of "next steps" are possible. (Stewart, 2011, p. 9)

Rosch, meanwhile, discusses the way that concepts are central to cognitivism and then, in sharing her own ideas on concepts, shows that they are inaccessible to cognitivism For Rosch, linked to categorization:

> Concepts are the natural bridge between mind and world [...] [and] occur only in actual situations in which they function as participating parts of the situation rather than either as representations or as mechanisms for identifying objects; concepts are open systems by which creatures can learn new things and can invent (Rosch, 1999, p. 61)

I would like to comment on these two quotations through sharing what I noticed in reading FLM 31(1), which arrived whilst I was re-reading Rosch. I was, naturally, being "triggered by what I COULD be triggered by" (to paraphrase Proulx, as cited by Samson and Shäfer, 2011, p. 38), and so I noticed the word "concept" while continuing to reflect on learning and on what teaching environments might look like without instruction.

I was particularly struck by the von Glasersfeld (1995) quotation, "concepts [ ] have to be individually built up [ ] operating mentally in a way that happens to be compatible with the perceptual material at hand" (p. 184), sandwiched between Tall's (2011) "crystalline concepts" and Samson and Schäfer's (2011) article which refers to enactivism in the title Given von Glasersfeld's idea of building, I wondered where the building is? I can, however, read this statement as talking about Rosch's bridge between mind and the "perceptual material at hand".

In Tall's article, I was drawn to his use of the term crystalline "to apply to a mathematical context that constrains a particular knowledge structure to have necessary properties that can be determined from the situation" (p. 5) This idea feels more strongly in the world and uses the ideas of enabling and constraining in a seemingly similar way to Stewart (2011). Tall also considers invention and discovery to be part of the landscape, similarly to Rosch.

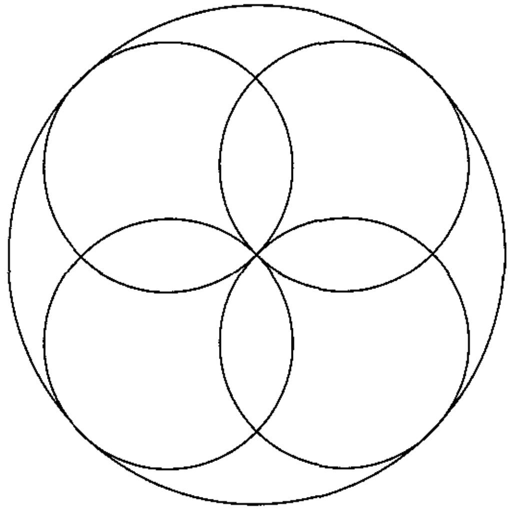

Figure 1. Seeing circles.

In Samson and Shäfer's article, there is a focus on the role of the teacher, with pupils "encouraged to arrive at plausible explanations" (p. 37). A dissonant chord led to me reading the article in some detail. Varela's "preliminary formulation of an enactive approach" to cognition is in two parts. In Samson and Shäfer's article, the first part, "perception consists of perceptually guided action," is quoted without the second, "cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided" (Varela, 1999, p. 12) Over time, each individual comes to see more, or learns, linked to their actions As Tall discusses, words such as reification or condensation have been used to try to describe this immediate perception of complex ideas

During a session focused by John Mason at the recent Association of Teachers of Mathematics (ATM) Easter Conference, I became aware of this aspect of seeing. We were asked what we saw in the diagram shown in Figure 1 The original diagram was coloured but the colours were not important to me. What I "saw", directly, was that the four smaller circles had the same area as the large circle and, so, when we were asked for the relationship between the black (coloured yellow in the original) and grey (coloured red in the original) areas (see Figure 2), I simply had to check the colours, the petals, the black "overlaps" and the left-over pieces between the small circles and the large circles, the grey "underlaps", to know that those areas must be the same. I was also recognizing this seeing as an example of my having learnt in the past and that that learning was directly available to me in this situation.

I cannot get away from noticing the word "concept" at the moment: the following example is from the opening lecture at ATM, given by Alf Coles, quoting Dick Tahta:

> ... metaphors of ownership and control: obtaining the meaning, having the understanding, getting the concept. And, consequently, of course, there will be the

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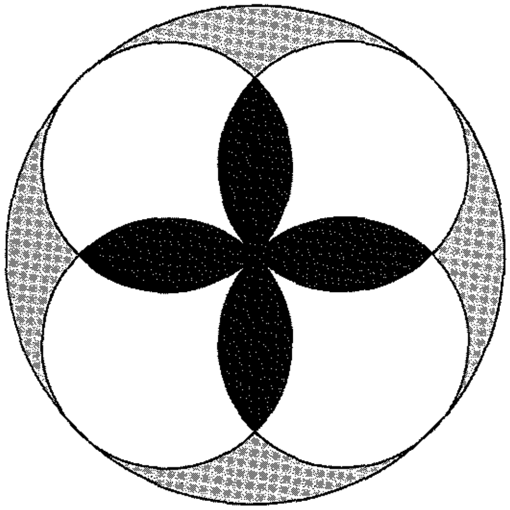

Figure 2. Overlaps and underlaps

mathematical *descaminados*, the shirtless who have not understood, who never get the concept?

I always have some concept of what we may both be considering. I will certainly never have yours [1]

So, what would a classroom look like in which students were learning? And were not having the same concepts! In Samson and Schäfer's (2011) article, the teacher would not be introducing different equivalent expressions for the generality, they would work contingently with what the students could themselves see. Here is an example from Alf Coles's classroom (Coles & Brown, 2006), working with students on a similar problem to Samson and Shäfer's within a project exploring students' "need for algebra". The students were 12 years old and used to sharing their ideas on what we call "common boards". The board records work in progress, often written by the students themselves. Students choose where to focus their attention, being opened up to more possibilities as and when they are able to see them. The concept of equivalence emerges as a process of noticing what looks different and what must be the same. How? Why? Following Rosch, meaning develops over time in response to what can be seen within a group, although no one individual is likely to see everything written down, nor, indeed, do all individuals see the same. Teaching is not "instruction". It becomes a process of facilitation of ways of working mathematically: the students do the mathematics, creating crystalline concepts, learning naturally as they categorise and make distinctions, inventing and discovering

Note

[1] The quotation is from "Take care of the symbols" (undated) from the private papers of Dick Tahta

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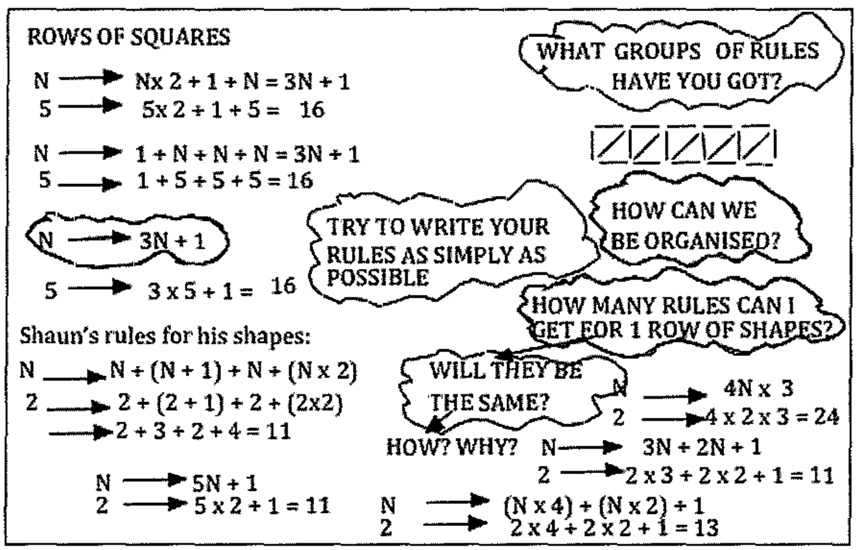

Figure 3. Example of a common board (Coles & Brown, 2006, p. 69).