CECI N'EST PAS UN "CIRCLE"

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There is a famous painting by René Magritte. It depicts a smoker's pipe with the caption *Ceci n'est pas une pipe* (this is not a pipe). The joke, still appreciated at the beginning of the twentieth century, was that it was not a pipe; it was a painting of a pipe. The painting has fuelled many discussions about the attachment of signifiers to signifieds: how exactly do symbols and words represent an object?

As soon as we enter the domain of language we inevitably move to a host of filters that condition our understanding of the material we are examining. This article is concerned with the perception of mathematical concepts [1] and seeks to explore some of the linguistic filters and socio-cultural factors that influence human understanding of such concepts. The core of our enquiry is a consideration of how learners in different cultural settings construct mathematical knowledge as a result of their socially determined needs, raising the question of whether or not mathematical concepts are consistent across the cultures.

Background

The context is a teacher-training programme in Uganda where Krista, formerly a primary teacher in the UK, was training teachers for primary education in Uganda and helping them to develop their skills in mathematics. She had not worked on mathematics at this level since towards the end of her own schooling.

Krista carried out research within a practitioner enquiry frame as part of a distance education course initiated by Tony. She experienced a steep learning curve in mathematics, made steeper as she became more aware of how mathematics was constructed in Ugandan schools and how its derivation from curricula in the West compounded difficulties for the students she was teaching. The ontological conception of mathematics itself was brought into question.

Krista's raised awareness of the cultural issues also brought into question her own agency within this development context as she faced the challenge of mediating between the externally defined demands of the western-inspired curriculum and the more immediate educational needs of her students.

This article comprises recent shared writing sandwiching some personal reflective diary work by Krista created three years ago (see the next section). Our enquiry attends to the cross-cultural perception of mathematical concepts and is based on discussion of the ontological basis of mathematics in the light of this reflective diary.

Krista: Encircling teacher practice

The following writing, setting the context for the journal extracts, has been taken from my Master's dissertation:

The Runyankore language allows for very accurate description of precise localities. For example, a child can be sent into a large banana plantation with instructions on which bunch to collect. The trees are not in rows or in any "organized" arrangement and the plantations can be vast. However, the instructions as to the location are exact and the child is able to follow them precisely. The language used is descriptive, involving the slope of the ground, the angle and the way the trees are "facing", a description of a particular group of trees, and in relation to the sun. The translation into English can hardly be done and causes quite a lot of frustration to the person asked to try. An equivalent description in an English culture would probably be based around approximate distances using standard units or counting rows, with the trees being planned in rows in order to overcome the problem of locating a particular spot.

Being an equatorial country, the sun consistently rises and sets at the same time and in exactly the same place each day. This stable consistency in position (and appearance) has allowed for the development of sun-based location and its associated language in a much more profound way than has been possible in a European language. Perhaps this would have made the development of alternative systems unnecessary. The words for East and West are derived from the rising and the setting of the sun. There are no specific words for North and South. They are not necessary. These directions can be described in relation to the sunrise and the sunset.

Runyankore does not include words for any geometrical shapes. This might indicate that the Banyankore have either not needed these words, not used the shapes or have not felt the need to explain and classify them as such. There are not words for triangle, rectangle or even for square. There is a word, *oriziga*, meaning circular or curved but it does not refer to a circle specifically (as I shall illustrate shortly). Once again, the language has developed descriptively rather than finding ways to categorise as the English language favours. Where the English language can use the word sphere, a description involving the word for curved and comparison to similarly shaped objects will be needed in Runyankore.

Extract from reflective journal: April 2001

Me: So if I give you an orange, a football, this small ball, and this stone, what shapes are they? How would you describe the shape in vernacular?

Him: They are all a circular and shaped like the small ball.

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Me: But in English you would say sphere?

Him: Sphere, yes. But that is a *muzunga* [white] word.

The absence of these words surely makes a profound statement about the direction of the development of the indigenous ethno-mathematics. At present the western curriculum is being taught in schools using the English language in order to improve the children's English but also to overcome the problems of awkward translations.

However, the concepts of simple regular shapes such as squares and triangles are so basic to the imposed curriculum that they are included in the syllabus for nursery and Primary 1. These children have little or no English so the words *sikwera* (square) and *turyango* (triangle) have been incorporated into the language. But the concept is a taught one and it is an unfamiliar one. These basic shapes that surround a western child in their everyday life are only theoretical, abstract and purely academic shapes on the chalkboard to the Banyankore children. There is no allowance for this in the school syllabus. These children progress on to Pythagoras' Theory just as quickly.

Extract from reflective journal: February 2001

In order to promote as much discussion as I could, I chose to start the lesson with an open question. I reworded the question many times in order to choose one that was obviously as open and as unoppressive as I could. Using a (handmade) chart with four diagrams on: a sphere, a spiral, an oval and a regular polygon with 16 sides, the question that I gave was "Why are these not circles?"

At first, many students were not involved so I quickly told them that they should discuss for a few moments in small groups and then they could put their ideas out to the class. The discussions went well and there was a lot of describing and fairly argumentative discussion.

During the second and third lessons I was much less intrusive in the class discussion and allowed the students to debate across the class more. I was surprised at the level of discussion that the question had caused. I had assumed that we would be describing the properties of the sphere in comparison to a circle, but I realised that the students were not very confident with the words sphere or oval, and even less so with spiral.

A majority of students were arguing that the sphere was a circle and it was when they were convincing the non-believers that I came in and steered the discussion by explaining 2D and 3D. The confusion may have been because of my representing 3D on a 2D learning aid. The oval was more straightforward and seemed to be a language issue only – as did the spiral. I became aware of my use of the words circular and circle and tried to explain the difference. But I am surprised at the confusion over the use of the word circle.

When I taught this lesson my focus was on student interaction. At that time I was measuring the success of my teaching by the quality of the interaction. I was also anxious to promote as much discussion as possible in order to find out as much as I could about the students' thinking. I hoped that by doing this I could become "closer" to them and teach more effectively.

Extract from reflective journal: August 2001

This second extract is a recording of a session using the same activity six months later. However, by this time I am more aware that the possible confusions may be attributed to the different cultural background and uses of language and classification systems:

I presented a small group of students with a diagram of a circle, a sphere, a spiral and an oval. After a few moments, I asked them what they could see and if they knew what the shapes are.

Student 1: They are circles!

Student 2: Well this one is more of a circle. This is a pure circle [pointing to the circle].

Me: What about these other ones then? Are they the same?

Student 2: This one is bent and too pointed [referring to the oval].

[Pause.]

Me: Mmmm? [prompt to continue].

Student 3: This one is more of a coil [referring to the spiral].

Me: Why is it not a circle?

Student 3: Because it is not closed and it is not as perfect as this one [the circle]. It is continuing.

Student 1: But it is a circle sort of.

Student 2: It is a coil.

[Pause.]

Me: What about this one?

Student 4: This is a circle [general agreement over the sphere]. But why have you shaded it?

I decide that it is a fault on my part to try and represent a three-dimensional sphere as a diagram and use a nearby globe as the sphere.

Me: How is this [the globe] different from this [the circle]?

Student 2: This one [the globe] it has this line – the equator – around it.

[I am really regretting not bringing a plain ball!]

Me: How is the shape of this [globe] or say the volleyball or the football, different from this circle? or are they the same?

Student 3: They are both similar.

The students then begin to argue in Runyankore and from the gestures they seem to be discussing the three dimensions of the globe.

When I ask them for a conclusion in English they have decided that they are both circles.

So, I explain about the three dimensions and that the globe is a circle from all sides, which they are fully aware of and have been discussing. I give them the word sphere, which they all eagerly write down.

One student now points out that he has heard the word in Geography when learning about hemispheres and they now see the connection. A connection that is obviously emphasised and perhaps limited to my use of the globe as a learning aid.

The above dialogue shows that although there is no discrepancy in describing the differences and the similarities of the various circular shapes, I think there is a mismatch in the use of the word circle between myself and my students. There is also a mismatch in the language structure for categorising these shapes. The English word circle is being used in place of the Runyankore word *oriziga*. But I think that a more accurate translation for *oriziga* must be the English circular. The students then are correct when they say that "they are all circles" because some students mean that they are all circular. In Runyankore, then, all these circular shapes are separated by a description incorporating the word for circular. For example, the oval is usually described as meaning "circular like a stone" and the spiral is *engata* meaning circular like a basket. As mentioned before, the sphere is circular like a ball.

In English, specific words have evolved in order to denote each category exactly. Each category has precise requirements. Much of the academic mathematics is based around the requirements of each category and the consequential properties of the individual shapes. Mathematical values and attitudes in a western society emphasise this need to categorise according to precise properties and generally to leave nothing uncategorized or explained. These are characteristics of a western technological society and not of a practical one (Bishop, 1988 and Kline, 1962). Thus, I feel that these students have not only to work in a second culture, but also become aware of a different structure for categorising. If adjusting from a language with direct translations of all these words, I feel that the adjustment would be comparatively simple.

I had originally assumed that the students would use statements such as "this one is a sphere because". But if I am to align my teaching with the students thinking and to make the subject more accessible, then I should shift my own thinking and provide an activity that begins with discussing the differences and similarities rather than focusing on each category and its label.

Circular arguments

As the preceding journal extracts from Krista demonstrate, her teaching developed into an on-going displacement of a term (circle) and an examination of how this displacement impacted on intersubjective understandings of mathematics itself. The mathematical term was necessarily, in this instance, a function of how these local and broader social relations were understood.

In some of the transcripts above, it is as if Krista was saying "this is not a circle" – Ceci n'est pas un "circle" – as she pointed to a representation of something seen by her as "circular", but seen by her students as a "circle". What, then, is a real circle, or even, when is a circle, and who decides? What form of authority would one invoke to adjudicate alternative claims? Or rather, how might this process of adjudication proceed? And what sorts of things would be offered in evidence? Who would be called as expert defense witnesses? What status would "circle" have at each stage of the proceedings?

Clearly, this short article is not going to sort out the ramifications of attempting to impose linguistic frames upon mathematical conceptualizations understood in a more, might we say, "mathematical" way (Barwell, 2004). Nevertheless, it seems inevitable that we would touch on this territory. In his classic, *The origins of geometry*, Husserl referred to this issue explicitly and sought to get at some account of how mathematical primitives came into being:

> It is easy to see that even in [ordinary] human life, and first of all in every individual life from childhood up to maturity, the originally intuitive life which creates its originally self-evident structures through activities on the basis of sense experience very quickly and in increasing measure falls victim to the *seduction of language*. Greater and greater segments of life lapse into a kind of talking and reading that is dominated purely by association; and often enough, in respect to the validities arrived at in this way, it is disappointed by subsequent experience. (quoted in Derrida, 1989, p. 165, *original emphasis*)

How might the association between the derivation of such language from social practices and the mathematical objects this produces be better understood? Alternatively, through which social processes might we establish some agreement that we are talking about mathematics here and such social concerns could be treated as *ceteris paribus*.

The very notion of "signified" here is problematic, potentially an illusion, insofar as two linguistic domains, which themselves evolve, cannot have an intersection within which a word has a positive and unconditional meaning. We simply cannot constrain the self-generating aspects of language towards settling, once and for all, the linguistic status of mathematical objects, that is, whether they are products of discourse or not (Radford, 2003). The Lacanian writer Zizek (*e.g.*, 1989) would take the notion of illusion seriously and speak positively of a fantasy layer structuring the "reality" beyond. [2]

Our concern here, however, is not just an issue of mathematics' ontological status. In the concluding section, we shall briefly consider how alternative understandings of the teacher, her students and mathematics are predicated in the research and teaching processes themselves. The journal extracts, however, also more specifically demonstrate the linguistic filters that are in operation and the socio-cultural factors that influence both the teacher's and the students' understandings of the mathematical concept of a circle. We seem to have a hermeneutic circle of understandings passing through various explanatory domains, English/Runyankore (perhaps also mathematics/geography, novice/expert, descriptive/abstract). In one, "circle" is seen as a noun, in the other, a type of adjective. And quite apart from that the term resides in two alternative worlds where people move around in different ways and understand or experience their movements quite differently.

There are words such as circle, sphere, *oriziga*, *engata*, spiral, ball or coil that appear in sentences that get spoken in the presence of certain objects or diagrams and the task of education here seems to be at least partly about how to distinguish them. Yet these sentences are being visited from a range of experiences in a range of linguistic home-bases whilst creating an illusion of signifiers being isomorphic to signifieds, or even that the signifieds exist in a sort of clear cut way (Brown, 2001).

The extracts show that the phenomenological experience of "circle" is different for each of the participants yet, as authors writing this, we feel haunted by those who might see "circle" as a mathematical notion, potentially untainted by such personal constructs. The hermeneutic circle of "circle" takes the word through a broad range of scenery waving to passers by, as they wave back in partial recognition – "we thought we knew you but you seem different now". But, as Krista holds on to her circular calling card, she herself changes in her understanding of what she is trying to do (perhaps of how her teaching relates to her students' learning, or of how she understands the students learning), in her way of sharing words, in her ways of getting words to do things, or of letting them do things to her.

As the calling card is passed around, it bounces or rotates or swirls or rolls or gyrates or orbits or radiates, and "arrives at its destination" (Lacan, 1988, p. 53). The card (the word "circle") stays the same, but people respond to it differently, shape themselves around it differently, as do Krista's written analyses. This evokes Lacan's emphasis on the signifier standing in for, even producing, the person it seeks to locate. He suggests that

> the displacement of the signifier determines the subjects in their acts, in their destiny, in their refusal, in their blindnesses, in their end and in their fate, their innate gifts and social acquisitions notwithstanding, without regard for character or sex, and that, willingly or not, everything that might be considered the stuff of psychology, kit and caboodle, will follow the path of the signifier. (Lacan, 1988, pp. 43-44)

Rounding up reflexivity – the cyclical research process

Krista's analyses took place at different times within successive modes of immersion in linguistic domains, which she sought to observe, understand, participate within (or resist) and transform through her participation. She recorded successive perspectives on successive actions. Yet, in the research process, it was the writing generated by her that provided anchorage, but only in the limited sort of way in which the word "circle" served as an anchor for more mathematically oriented discourse. The word itself was more stable than the way it held meaning.

Similarly, the writings simultaneously sought to explain the past and shape the future, but in the meantime provided orientation and a conceptual space for examining how the term "circle" was being used. And yet each component of this writing was constantly in the process of having its status amongst its neighbours unsettled. Krista was involved in the production of stories that had a limited shelf life as "stories in their own right".

The process of research entailed generating perspectives and framings to enable solutions to professional difficulties or perhaps to recast the difficulties into a more manageable form. [3] As such, the process provided a "trace technology" comprising the generation of successive reflective writings that produced an archive seen through a "perpetual present" (Luke, 2003, p. 336). Citing Derrida, Luke emphasises that the notion of "archive" transcends mere associations with the past.

In our analysis, the reflexively defined researcher followed the path of the signifier that had been set in motion through previous actions. The on-going collation of writings was about formatting the future, setting a trajectory for an on-going journey, in which the term "circle" would be met over and over again. Yet the consistency of such terms is necessarily tainted by their derivation from, and evolution through their seduction by, language. They are historically and ideologically defined entities. The passage of time, however, can provide the distance necessary to see the previous frame as being outside of oneself. And of how it had encapsulated the teacher, the learner and the mathematical objects that they had sought to share.

Notes

[1] The ontological basis of mathematics remains a subject for debate. Nevertheless, pedagogical strategies and demonstrable skills of learners are often seen as subordinate to the mathematical conceptions they seek to depict (Brown, 2001). In some countries, teachers of mathematics are increasingly externally defined within the formal education system, which reifies mathematics as a set of concepts and procedures and specifies the teacher as deliverer of these. Moreover, the teacher's capacity to contribute to the construction of mathematical ideas is limited and both teacher and learner are respondents to externally imposed demands (Brown and McNamara, 2004). On the other hand, constructivist theory, which has dominated mathematics education for the last 15 years, has generally been predicated on people interacting with knowledge. The way a learner constructs mathematical knowledge is framed by his or her own social existence.

[2] This issue has been discussed in detail in relation to mathematics education in Brown and McNamara, 2004.

[3] Atkinson, D., Brown, T. and England, J. (forthcoming) *Regulative discourses in education: a Lacanian perspective.* Bern, Peter Lang.

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