COLORING CONJECTURES WITH SOUND, SILENCE, SYNTAX AND GESTURE: A MULTIMODAL POETIC ANALYSIS

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Three fields to a tree
Three trees to a hound
Three hounds to a horse
Three horses to a human being
Three human beings to a deer
Three deer to an eagle
Three eagles to a salmon
Three salmon to a yew
Three yew trees to an age

(Guss, 1985, p. 245; Hull, 1932. Layout modified) [1]

The Legend of the Oldest Animals is a folklorist's term for genres of stories, proverbs and poems that have been recorded widely throughout Europe, from Hesiod's poetry in the 7th century B.C., with many medieval and early Renaissance examples, to relatively recent oral tellings (Bath, 1981; Hull, 1932). Sometimes, the stories represent early scientific beliefs regarding the astounding lifespan of certain species. They can provide a method for calculating dates, including the time period between destructive millennial events (Hull, 1932). In an Irish tale, also reported by Hull, the character Fintan transforms himself through various animal forms, capitalizing on their longevity, in order to preserve traditional knowledge across the ages. Despite this variability in purpose, the Oldest Animals stories share a foundation in oral traditions and a reliance on a poetic structure to portray a mathematically-informed cosmology.

A poetic structure occurs when a speaker repeats a phrase or sentence that was spoken previously while retaining some of the form of the prior statement. The repeated form could involve any linguistic, auditory or expressive feature, including rhyme, silence, alliteration, consonance, assonance, gesture or syntax. I emphasize the presence of syntactic repetition because these structures encompass big enough stretches of discourse—words in relation to other words—to express fairly complex or precise mathematical ideas. However, poetic analysis of any repeated linguistic or bodily action can reveal mathematical thinking (see, for example, Smythe, Hill, MacDonald, Dagenais, Sinclair & Toohey, 2017, pp. 126–7). The repetition can involve statements by the same speaker or by multiple speakers and can serve many interactional and logical functions, such as demonstrating involvement with another person, editing a previous comment, or legitimizing an argumentation structure (Staats, 2016a, 2016b, 2017; Tannen, 1989). This article will discuss ways in which the interlaid rhythms of poetic structures, repetitions of syntax, sound and gesture, arise in spontaneous speech as a means to express mathematical insight.

In the example of an Oldest Animals poem above, each sentence has the form *Three As to a B.* This establishes both syntax and words that are repeated in subsequent lines. The sentence as a unit of analysis, however, is not sufficient to capture the exponential character of the poem. If we read only a single sentence, without relating it to the broader structure, we will have a proportional understanding of animal lifespans, but not an exponential one. A more complete unit could be *Three As to a B / Three Bs to a C.* This captures both the sentence-level syntactical repetition and the "reversal" in the position of *B* (Staats, 2016a, 2016b) that establishes the exponentiation. One way to draw attention to the repetition is to use indentation as above or multiple types of underlining, as I do in examples below, to indicate the repetitions that seem significant for a particular discourse selection.

Understanding poetic structure is important for scholars who seek a close analysis of how mathematical ideas develop through talk. Repetition helps students talk about math because the syntax establishes relationships among mathematical ideas or images. Repeating the phrase lets students conserve or modify these relationships as they experiment with different conjectures. Spontaneous mathematical talk occasions the interplay of academic and informal discourses in ways that become central resources for learning (Barwell, 2015; Planas & Setati-Phakeng, 2014). This article will illustrate how poetic structures use layers of syntax within and across sentences, along with words, sound dimensions of speech, and gesture to amplify the speakers' mathematical concepts.

Multimodality and grammar in mathematics education

Poetic structure analysis depends on noticing similarities in syntax across different mathematical comments. This concern with syntax coordinated with sound and gesture is developed within an approach to discursive multimodality that is drawn from conversation analysis (Bergmann, Brenning, Pfeffer & Reber, 2012). Multimodal interactional grammar holds that grammars are not stable, pre-existing forms of knowledge that format speech, but rather, that patterns of spoken interaction and grammatical knowledge constantly inform each other. Some linguists argue that repeated phrases like *Three As to a B / Three Bs to a C* that emerge through dialogue become temporary grammatical units (Du Bois, 2014). Multimodal interactional grammar

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takes prosody—patterns of sound in language such as pitch, intonation, pause or loudness—combined with gesture and syntactic structures, as interactional modes that allow people to create meaning. Syntactic units—a phrase, clause, or a sentence—are often marked multimodally through prosody and gesture, though the structure of these units and their correspondence with turn-taking and other social purposes are under debate (Bergmann, Brenning, Pfeffer & Reber, 2012; Szczepek Reed, 2012).

In mathematics education, multimodal analysis has shown that prosody, gesture, sounds created through gesture and rhythm can be coordinated to express mathematical understanding (Bautista & Roth, 2012; Radford, Bardini & Sabena, 2007). Radford has developed a comprehensive view of multimodality that he terms *sensuous cognition*, that "all our relations to the world (hearing, perceiving, smelling, sensing, *etc.*), are an entanglement of our body and material and ideational culture" (2014, p. 350). Through sensuous cognition, students can synchronize multiple semiotic modes, including the rhythms of gesture, speech intensity, and writing and words, to express mathematical generalizations (Radford, Bardini & Sabena, 2007). Bautista and Roth (2012) demonstrate that the sounds students make—vocally and through performances with apparatus—bring their mathematical consciousness into the social world by highlighting similar and different attributes of objects and ideas. Alibali, Nathan *et al.* (2014) show that the coordination of speech and gesture links mathematical ideas, particularly in discussions of new material.

Within mathematics education, then, there is growing awareness of the mathematical expressivity of non-lexical modes. Interactional grammar suggests that spontaneous speech often involves syntactic units that are emphasized through prosody and gesture. Repeated syntax and its mathematical affordances, however, have been studied very little. The following discussion suggests that mathematical structure conferred through repeated syntax is indeed sensuous because it is marked, amplified and embellished through the more clearly embodied actions of gesture, sound and silence. In this article, I introduce poetic structure as an additional discursive modality—alongside prosody, gesture and writing—for creating mathematical meaning.

Property noticing in the hexagon task

This article reports on a task in which two students, Sheila and Joseph, were asked to find a formula for the perimeter of a string of n adjacent hexagons, as pictured in Figure 1. The task was based very closely on a proposed measure of readiness for undergraduate study (Wilmot *et al.*, 2011, p. 287). Sheila and Joseph were undergraduate students who had recently completed a procedurally oriented university class on polynomial, exponential and logarithmic functions. They participated in a paid, one hour video and audio recorded problem-solving session. They worked for about 40 minutes without any researcher intervention. A correct answer for the task is $p = 4n + 2$. Sheila and Joseph developed an answer of the form $\#H(6) - 2(N - 1) =$ in which $\#H$ and N both stand for the number of hexagons (Staats, 2016a; 2016b). Both students were highly engaged throughout the conversation. In the earlier part of the conversation, Sheila voiced many of the significant results, but in the stage reported below, Joseph was more mathematically active.

The key issue in the task is that the interior sides of a hexagon string do not contribute to the perimeter. One of the students, Joseph, noticed this property and over the course of about four minutes stated the property five times before his partner agreed to it. Property noticing is a moment in a student's mathematical understanding that can be expressed either individually or through collective mathematical activity (Pirie & Kieren, 1994; Towers & Martin, 2014). Joseph was in an individual stage of property noticing in the following passages. He attempted to bring the observation into his collaborative work with Sheila, with only some success.

Joseph's five restatements of a property create an unusual condition for thinking about language as a resource for learning. We can expect that with each repetition, Joseph might deploy a slightly different set of linguistic resources designed broadly to accomplish the same thing: to secure his partner's

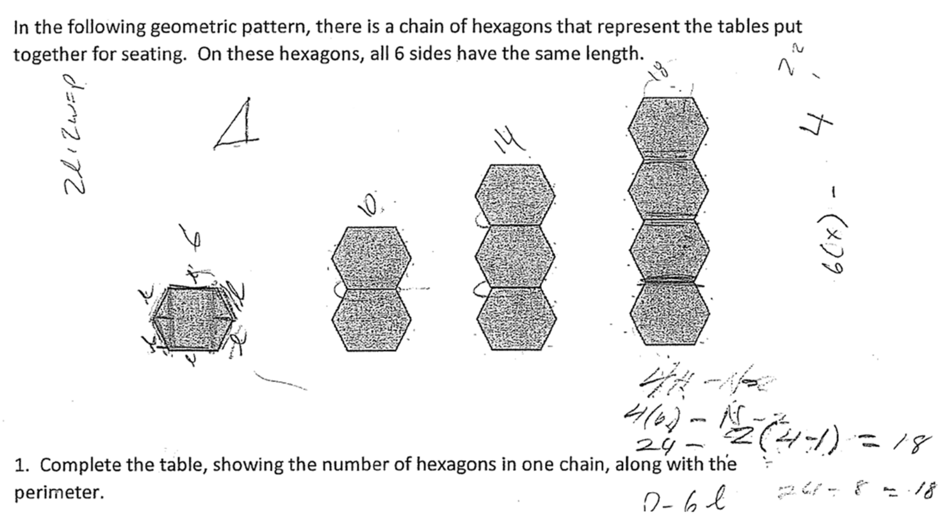

Figure 1. Task diagram for n = 1 to n = 4 hexagons.

acknowledgement of the property. We can consider which discursive strategies Joseph conserved and which he changed as he tried to make his point. I highlight five expressive modes that a speaker could combine in various ways to create a new strategy for explanation: introducing academic vocabulary; elements of prosody; gesture; writing mathematical formulas; and poetic structures. In the selections below, I transcribe Joseph's property noticing statements closely using a transcription key [2], and I present other statements in ordinary prose with ordinary punctuation.

I have suggested that for studies of repetition in mathematics discourse, it is helpful to identify phrases that include repeated grammar and at least one word in common, to ensure that we focus on comments that have some topical connection to each other. If there seem to be multiple ways to identify poetic structure repetitions, I suggest choosing the longest possible stretch of discourse that involves repeated syntax (Staats, 2016a, 2016b). These two guiding principles for identifying poetic structure repetitions help us select among different possible ways to parse statements.

Five attempts to explain a property

In this part of the conversation, the students had already developed a numerical method for solving the hexagon problem corresponding to their written formula $\#H(6) - 2(N - 1) =$. The task then asked them to explain their method in terms of the diagram. After a brief discussion of whether this refers to the table of values or the images of hexagons on the handout, Joseph stated a property of the hexagon diagram for the first time in the conversation.

First attempt, turn 171

171.1 *Joseph* Then this must be the diagram (1.0) so ↑how can we use ↓this (0.2) to prove our point (1.4) *[Pencil taps the diagram at each "this".]*

171.2 so the **hex**agon has ↑six sides (1.4)

171.3 but when you put a hexagon in a **chain**↑ they share **two** sides (0.6) so you're taking away **two** sides (0.8) from the chain↓ (1.6) *[During 171.2 to 174, Joseph's hand obscures the paper. From his hand position, it is likely that he used some small gestures, but they did not result in Sheila's acknowledgement of the property.]*

171.4 so each time you **add** another hexagon in a chain↑

172 *Sheila* But it's not asking, this one

173 *Joseph* you're losing two sides↑

174 *Sheila* This one is not asking about the diagram. It says, how do we know this is true? Why does it work? Because the formula matches the diagram and table.

The major poetic structure in this exchange is the sentence at 171.3, repeated with small shifts at 171.4 and 173. Single underlining is intended to draw attention to the similar syntax in two opening phrases of when you put a hexagon in a **chain** and each time you **add** another hexagon in a chain↑. Double underlining draws attention to syntactic similarities in the two concluding phrases you're taking away **two** sides and you're losing two sides↑.

The beginning and ending of each of these four underlined phrases is fairly strongly marked through prosody, either through pause, intonation or emphasis. For example, *so you're taking away two sides* is bounded on either side by a distinct pause. The poetic structure *when you put a hexagon in a chain* is marked as a bounded unit as well, but perhaps less distinctly, with an opening pause and a small level of sound emphasis on the word *chain*. The co-occurrence of semiotic modes is important because it suggests that poetic structure repetitions are more than just a trick of transcription, but rather, that they are an authentic component of Joseph's attempt to externalize his mathematical insight. As James Gee points out, validity in discourse analysis improves when multiple forms of analysis reinforce an interpretation and when multiple linguistic features support a significant social function, such as sharing mathematical insight (Gee, 2014). Pauses, intonation changes, and emphasis all coincide with Joseph's manner of chunking information into brief mathematical observations.

Furthermore, the mathematical ideas that are defined through repetition have the effect of establishing a mathematical system of comparison and contrast. For example, the poetic structure in Joseph's first explanation introduces a strategy that he uses in some of his subsequent explanations, too: a cause/effect, action/result, or if/then form of argumentation. His comment at 171.3 proposes that the action of attaching a new hexagon to a chain results in the loss of two interior sides. The repetition at 171.4 and 173 continues this cause/effect relation by conserving some of the grammatical structure. This very repetition or similarity, however, allows focus on something new: the shift from *when you put a hexagon…* (171.3) *to each time you add another hexagon…* (171.4), which seems to shift towards a more generalized proposition. While the pronoun *you* in both phrases carries a sense of generalized action (Rowland, 2000), the comment *when you put a hexagon* fixes the action at a certain moment in time, *when*, with a singular hexagon. By contrast, in 171.4, we shift from a moment in time to *each time*, and a single hexagon shifts towards the more generalized sense that there could always be *another hexagon*. The subtle interplay of similarity and difference established by the poetic structure is exactly the linguistic resource that expresses Joseph's mathematically precise idea.

Sheila was not convinced, however, that Joseph's statement fulfilled the task. In the next few turns, the students discussed what it means to explain a method in terms of a diagram. Sheila wondered if they needed to draw a new diagram which led to Joseph's second and third statements of the property.

Second attempt, turn 179

179.1 *Joseph* ↑I don't think so I ↑think you just have to explain so (1.8) a ↑hexagon has six sides↓

	(0.4) *[Just before "a hexagon", he makes a heavy tap on the base of the n = 1 diagram.]*
179.2	and as you **add** (1.2) an additional hexagon (1.2) *[At "add," there is a light touch on the base; at "additional," a light touch on top.]*
179.3	you add six↑ (0.8) *[He touches base before "six" and touches top after "six".]*
179.4	because they ↑share two sides↑ (0.6) *[He is circling the top of the n = 1 diagram.]*
179.5	you subtract↓ two sides↓ *[His pencil is above the page. At "subtract," there's a beat in the air near the top of the hexagon and at "sides," there's a beat near the base.]*
179.6	from their total↓ (0.8) number of (0.8) sides↓
180 *Sheila*	Um. I think this is, I think this is
181 *Joseph*	Which is essentially this.

In his second attempt to explain the property, Joseph used two poetic structures. One is the repetition at 179.2 and 179.3 of *you add / (an additional hexagon; six)*. The other is the repetition at 179.4 and 179.5 of *(because they share; you subtract) / two sides.* Both of these pairs advance an "action/result" argument, just as in Joseph's first attempt. Again, each of these four lines was strongly bounded by prosody, particularly pause, but also with some intonation, emphasis, and at 179.5, with a beat gesture. In his second explanation, Joseph continued to use poetic structures, but he overlaid them with a new combination of discursive resources. Joseph introduced the words *add* and *subtract* as precise computational terms, in contrast to the activity of *adding* or *putting* a new hexagon into a chain in his first explanation.

Poetic structure repetitions establish a kind of rhythm in speech that opens up new expressive possibilities. As we noticed in the first explanation, once a speaker establishes bounded units of speech through a poetic structure, certain words and phrases appear in parallel positions. Sometimes these positions are conserved as in *you add* (179.2 and 179.3) and *two sides* (179.4 and 179.5). Sometimes the phrases in the parallel positions shift as in *they share …you subtract* (179.4 and 179.5). This rhythm is most noticeable in the words and syntax, but prosody and gesture can form a sub-rhythm that controls focus across different mathematical ideas.

In 179.2, for example, two light gestures highlight the assonance of sounds in *add an additional hexagon* and with a pause, focus attention on the activity of appending new hexagons. In 179.3, the emphasis shifts from this activity to the new computational sense of *adding six*. This shift of focus towards the *six* is controlled with intonation, pause and a two part pointing gesture. The gestures in the second explanation come in pairs and seem to presage the role of the interior sides in this task. This sub-rhythm of prosody and gesture shifts the hypothetical action on the diagram of *add/additional hexagon* and towards the computational technique of *add/six*.

Another sub-rhythm at 179.4 and 179.5 supports this interpretation. The rising intonation on *share/sides* is contrasted with a slight falling intonation on *subtract/sides*, which, like 179.2 and 179.3, emphasizes a shift from action to computation but also realizes the cause/effect argumentation structure. The sense of cause/effect was heightened as the circling gesture at 179.4 shifted to a two part beat gesture on *subtract/sides*. With elaborately marked and bounded poetic structures, Joseph's second explanation coordinated prosody and gesture with poetic structures to animate the relationship between an action and a computation.

Sheila began to engage Joseph's idea, but Joseph sensed a question of whether one side or two sides are lost when hexagons join together, leading to his third explanation of the property.

Third attempt, turn 181

182 *Sheila*	The, the number of tables has six sides. And with that multiplied you minus, as they join together they lose one side. So for every-
181 *Joseph*	One side for (0.2) **one** (0.2) hexagon. *[Overlapping with Sheila's 182 to 183. No gestures; Joseph's hand is away from the paper.]*
183 *Sheila*	-For every, for every two tables, one side is lost. So for the, for the, well, uh, the formula just states it, right? How do you state the formula?

With each attempt to explain the property, Joseph deployed a different combination of discursive resources. Here, his statement, *One side for one hexagon*, relied on a prominent poetic structure with no gestures. A subtle sub-rhythm used pauses and a small bit of emphasis to highlight the clarification of ***one** hexagon*. His strategy in this third attempt was minimalism. This case highlights the way in which poetic structures can focus attention on a key pattern using very few words.

Fourth attempt, turn 185

185 *Joseph*	Well they want you to use the diagram. So using this we have to explain how that formula works. So there are six sides, which is, which corresponds to –
186 *Sheila*	Let's see. Let's do 4. For every, for all these tables, four tables, there's only three intersections *[Joseph takes an extra piece of paper and writes "4".]*
187.1 *Joseph*	But they **all** lose two sides at an intersection (1.6) so (0.4) a **hex**agon has six sides, right↑ (0.6) so 4 times 6 is 24↑ (1.2) *[Joseph has now written 4 ·6 = 24.]*
187.2	um but each (1.0) table when each table touches the other one (1.2) *[Joseph has taken a second piece of paper, and at "when," he holds his hand up with palm down.]*

187.3 they lose two sides↓ *[At "lose," he closes his palm into a fist.]*

188 *Sheila* Oh, that doesn't work out mathematically.

Joseph's fourth explanation used poetic structures less prominently, though there are short repetitions at 187.1 and 187.3 and again within 187.2. Less reliance on poetic structures could be associated with the new modality of writing mathematical formulas that also introduced new verbal syntax. The small level of repetition that exists still expresses the cause/effect argumentation structure, and this is supported through gesture. The open-handed gesture at 187.2 seems to be a gesture of asserting, and the closed fist at 187.3 seems to grasp a conclusion.

Joseph's final explanation focused on the $n = 4$ hexagon case in the task diagram of Figure 1. He merged poetic structures, prosody, gesture, and a written formula to gain Sheila's agreement.

Fifth attempt, turn 189

189.1 Joseph Because ↑here's one and ↑here's one (0.8) **here's** one **and here's** one (0.4) **here's** one **here's** one (0.4) so you lose **six** sides (0.8) *[Joseph's hand has obscured the view again, but we can hear him scratching over the interior pairs of sides, and we can see him moving from the bottom pair to the middle pair to the top pair, making the marks that are visible on the interior sides of the n = 4 case in Figure 1. His hand comes away after the last "here's one".]*

189.2 so **24** (0.8) *[On the extra sheet of paper, Joseph writes a minus sign in 4 ·6 = 24 –]*

189.3 would be the number of ↑sides if they weren't touching (0.4) but because they're ↑touching (0.4) you lose six↓ *[He writes 6 after the minus sign.]*

189.4 and that comes down to 18. *[Joseph has now written 4 ·6 = 24 – 6 = 18.]*

190 *Sheila* Yeah. Okay, go ahead and write it down. You're better at that than I am.

191 *Joseph* I'm just more visual.

The first part of turn 189 has a strong poetic structure based on *here's one and here's one* in which the beginning and ending is marked and bounded off with prosody and a combination of gesture and drawing. Alibabi, Nathan *et al.* (2014) consider this action as a "writing gesture" that is distinguished from writing linguistic or mathematical text. The various gestures that Joseph made near the base and top of a hexagon in several previous statements were here made lasting and tangible through a writing gesture. The last part of turn 189.1, *you lose six sides*, and the subsequent poetic structures in 189.3 are all bounded with pause and intonation.

The cause/effect argumentation structure is in play in both, 189.1 and 189.2 to 189.4. In earlier phases of the conversation, Sheila had been highly involved in checking emerging formulas with direct computation leading up to the formula $\#H(6) - 2(N - 1) =$. Because Sheila has raised the computational approach again, Joseph is concerned throughout the fifth explanation to validate the calculation by demonstrating the basis of each number. In 189.1, he uses a poetic structure to establish compellingly that there are six interior sides in the $n = 4$ case. Near 189.3, there is a slight dramatic feel to Joseph's explanation. The rising tone was accompanied by a musical lilt in Joseph's voice at *because they're* ↑*touching* (189.3). One can sense this effect while reading the transcript by noticing the unfinished subtraction at 189.2, and the impression of incompletion that it creates. By tracing the multimodal interaction of prosody, writing and poetic structure near 189.3, we can better understand Joseph's final, successful expression of the hexagon property.

At 189.2, Joseph begins with *24*, probably because this was previously established and written information. In terms of intonation, there are two rising pitches in 189.3 that are part of the supposition of the argument. This intonation shift is also coordinated with the poetic structure in the *number of* ↑*sides if they weren't touching* (0.4) but because *they're* ↑*touching* (0.4). These two rising pitches near the two components of a poetic structure are a quiet sub-rhythm that helps shift attention from the 24 sides perspective to the issue that arises when sides touch. The falling tone in the conclusion of the argument co-occurs with the poetic structure *you lose six* and the written completion of the subtraction statement. The poetic structure, writing mathematical text, pause and intonation are all coordinated to reproduce the cause/effect argument that Joseph used throughout his property noticing statements. At 190, Sheila responded with a long, thoughtful sounding *Yeah*. It sounded as if she decided that she agreed with Joseph.

The fifth explanation recapitulates and extends the second explanation through an animated and elaborated transition from action towards the more standardized, academic argumentation of calculation. In a broader sense, Joseph's final, successful explanation responds to much of Sheila's approach throughout the conversation.

The moment when Joseph writes the 6, performs the subtraction, and brings the 18 into being, has some feeling of being a performance of the validation and not merely an explanation or logical assertion of facts. Performances can emerge from merely literal ways of speaking through discursive features that include taking responsibility to an audience for competent expression, poetic structures (in this source, the term is parallelism), and specialized speech forms—perhaps here, including mathematical ones (Bauman, 1977). The multimodal coordination of poetic structures, prosody and writing in Joseph's fifth explanation suggests that its performative quality may have helped Sheila engage Joseph's property explanation more directly than she had before. In selection 5, Joseph found a way to align his attention to properties of the task diagram with Sheila's attention to computations, as she put it, making things *work out mathematically.* Joseph was finally successful when he created a performance that merged Sheila's more computational understanding into his own.

Discussion and conclusion

Joseph used poetic structures in all five of his property explanations, prominently in four of them. The beginnings and endings of poetic structures were marked multimodally through pause, intonation change, vocal emphasis and gesture. Poetic structures put ideas or phrases into parallel positions that provide a foundation for elaborate multimodal expressions that foreshadowed conclusions, shifted attention from known to less-known concepts and emphasized the logical structure of arguments. This suggests that poetic structures are not merely a transcription tool but an experiential linguistic resource for expressing and animating mathematical insights.

The repetitive character of grammar and other modalities in mathematical conversation demonstrates how every expression has a social content, that it is both re-forming an idea received in the past and pre-forming the idea for others' use in the future. Every repetition recasts an idea, but it also repositions a person and her/his contributions, her/his stance and her/his focus of attention. This positioning acts within the more explicit logic of the lexical and grammatical flow of talk, simultaneously with the foreshadowing of intonation and pause that helps other speakers participate in a discovery.

Multimodal poetic structure analysis could lead to several directions for new research in mathematics education. Joseph's fifth explanation suggests that attention to multimodal poetic structures could highlight why some types of explanation are more successful than others, in particular, by enhancing the ability of interlocutors to participate in mathematical understanding. Joseph's third explanation highlights the potential for using poetic structures as a way to explain a lot about a mathematical system using very few words. This kind of research could lead to new, intentional questioning and discussion techniques for teachers.

In a broader sense, multimodal poetic analysis can contribute to the need for discourse research that attends closely to students' mathematical thinking (Ryve, 2011). In my view, both the grammatical modality and the more-clearly-embodied modalities of sound, silence and gesture are important to this goal. The parallel grammatical phrases represent the conceptual richness and specificity of students' ideas and become temporary, shared units of analysis in a dynamic chain of expression. Repeated gesture, pause and sound dimensions signal to other participants how these units are related, or that they are about to be acted upon. Multimodal poetic structure analysis, then, lives at the intersection of students' content knowledge and intersubjective experience of creating mathematics.

Poetic analysis is not restricted to any particular language because poetic structures are, in principle, possible in any language. Nearly all grammars involve morphemes or units of meaning within words, phrases or other levels of expression that can be switched out for other units, so that speakers can create a series of syntactically similar but changing utterances. In this way, poetic structure analysis could contribute to current research on identifying grammatically-based linguistic resources in multiple languages (Edmonds-Wathen, Trinick & Durand-Guerrier, 2016). It would make this emerging field of research more dynamic, so that we can listen to students' interactive development of grammatically-based mathematics ideas in any language.

The Legend of the Oldest Animals unifies multiple timescales and observations into a broader theory of embodied knowledge. Poetic structure analysis could similarly connect different levels of mathematics education discourse research. Mathematics classroom talk always references past learning or social experiences, but it also always involves students using language towards a mathematical goal. Moments in which speakers repeat and recast previous speech may prove to be a crucible for understanding linkages among received mathematical knowledge, moments of embodied argumentation and intersubjective awareness.

Notes

[1] Guss' version is derived from several examples in Hull, 1932.

[2] Transcription is based on the following key:

Symbol	Definition
↑or↓	High pitch or low pitch, respectively, compared to nearby words. Could occur at beginning, middle or end of word.
Bold text	Emphasis on a word through loudness, vowel lengthening, or stress
(x.y)	Pause, estimated, in seconds.
Underlining styles	Single, double or bold underlining represent the different poetic structures in one turn at talk
[italics]	Descriptions of gestures

This transcription approach is a simplified version of notations used in conversation analysis (Hepburn & Boden, 2013). Conversation analysis transcription seeks to represent many features of the sound of utterances; standard symbols of written language such as commas, periods and question marks are replaced with notations that represent sound features such as pause or changes in intonation. My transcription choices involved trying to capture the variety of sound and gesture modalities that mark boundaries of syntactic units. I estimated the length of pauses using Rymes' method of speaking a multi-syllabic word or phrase while listening to the recording (2009). I calibrated my speaking speed for the phrase *nu-one-thou-sand-one* to one second several times just before listening to the moment in the recording. Though this method does not give an absolute measure of pause length, it can demonstrate that a pause exists relative to surrounding words.

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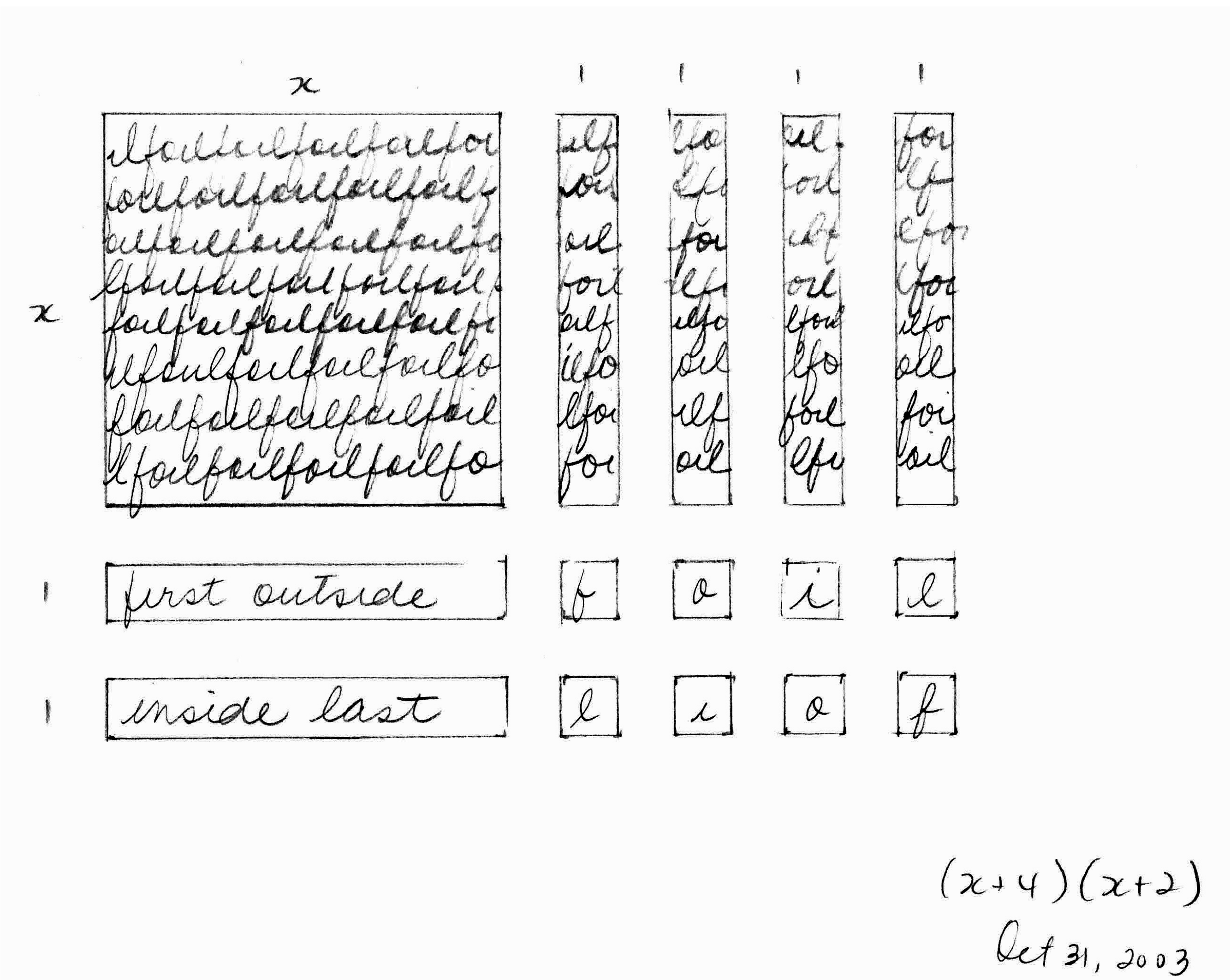

Polynomial Study #16, by Elaine Simmt of the University of Alberta.