MATHEMATICAL PRECISENESS AND EPISTEMOLOGICAL SANCTIONS

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This article is about Samir, an eleven-year-old, multilingual emergent speaker of Swedish, the language of learning and teaching in his classroom. Samir and three of his peers were working on a group activity on angles. Samir entered the group activity confidently saying, "How good I am" when solving a task with his peer Darko. However, at the end of the activity Samir's talk about himself had completely changed from self-confidence to insecurity. Submissively he begged his peer Darko to rely on his mathematical knowledge saying, "Forgive me, please trust me."

In this article, I examine how a focus on preciseness in the mathematics classroom could affect group activity. The preciseness I am interested in relates to the ways mathematical concepts, in this case angles, are described in discourse between students. In this context, I consider how 'micro-invalidations', seemingly minor but potentially damaging criticisms, can limit students' opportunities to learn, particularly in a social and language diverse classroom.

During the group activity, Samir was exposed to micro-invalidations. At the end of the activity, Samir's peer Darko did not trust Samir's ability to make reliable mathematical claims any longer. Samir lost the authority to make such claims as they had become invalid in the eyes of his peers. I explicate the idea that Samir's loss is linked to discourses on (Western) mathematics that embrace mathematical rigidity and preciseness.

Mathematical concepts used in formal mathematics are rigid, based on absolutist, axiomatic conceptions. Their use elicits discourses that comprise preciseness and certainty, acknowledged aspects of the Romance of (Western) mathematics (Lakoff & Núñez, 2000). Such discourses influence school mathematics. Preciseness impedes flexible conceptions of mathematics and evokes ideas about mathematical rigidity. This resembles an absolutist view of mathematics that rejects mathematics as a cultural product. Although the absolutist approach has been challenged, for instance by Lakatos, it is not always easy to see the cultural roots of mathematics. Prediger (2002) argues that it is (paradoxically) the human desire for certainty that dehumanises mathematics. In brief, according to Prediger, mathematicians use a strong practice of coherence and consensus to be certain about mathematical propositions. That is, mathematicians agree on what is appropriate and what is not appropriate mathematical propositions. By the time that their agreements reach the layman, this process is hidden. The mathematicians' high coherence and the wide consensus has obscured the human dimension and cultural origin of mathematical propositions, hence making them appear impartial.

In the mathematics classroom aspects of preciseness, rigidity, coherence and consensus play out as a matter of being right or wrong. Although, as pointed out by Wagner (2009), it is when the act of forbidding the 'wrongs' is challenged that significant mathematics might emerge, classroom mathematics is more often about fostering consensus on what is easily assumed as 'rights' (deFreitas & Sinclair, 2014). Therefore, putting forward mathematical claims of knowledge can be a risky business because the chance of being 'precisely wrong' is at stake. Being wrong may expose the claimer to micro-invalidations. Students whose claims repeatedly are invalidated are at the risk of ceasing to perceive themselves as potential participants in school mathematics practices (Andersson, Valero & Meany, 2015). Judging claims based on whether or not they are to be acknowledged as claims of knowledge is an epistemological matter. Therefore, I consider the micro-invalidations that Samir was exposed to, small epistemological sanctions.

Multilingual students' participation in verbal-reasoning group activities may provide both mathematics and language learning opportunities (*e.g.,* Moschkovich, 2015; Planas, 2011, 2014). While such participation is crucial for emergent speakers of the language of learning and teaching, it is far from unproblematic. In Samir's case, it means potential exposure to epistemological sanctions, which affects the ways he speaks and feels about himself when doing and thinking about mathematics and perhaps, in the long run, his mathematics learning. One way of grappling with this issue is to provide language- and content-integrated learning opportunities that take into account the epistemic role of school language discourse (Prediger & Krägeloh, 2016). Another way is learning to meet the Other in the classroom. That is, learning to meet marginal and minority ways of engaging with mathematics in open and respectful ways (Guillemette & Nicol, 2016) or creating dialogical spaces (Chronaki, 2011). Yet another way is re-thinking what kinds of discursive spaces absolutist mathematics shapes and how students (and teachers) navigate those spaces (McGarvey & Sternberg, 2009). None of these ways excludes the others.

In this article, which draws on a previous research report (Ryan, 2018), my concern is to understand how discursive spaces based on an absolutist mathematics shape the interactions of Samir and his peers. I argue that discourses of preciseness in mathematics may elicit micro-invalidations directed towards students who do not justify their claims in a (school) mathematically precise way, possibly because they are not participating in the epistemic school language discourse.

I draw on Wittgenstein's ideas on language games (from his *Philosophical Investigations*) to grapple with Samir and his peers' reasoning about angles and Samir's talk about himself during the activity. I use here the notion of I-language games and the game of giving and asking for reasons.

I-language games and the game of giving and asking for reasons

I-language games do not raise questions about what 'I' am nor what it is to be 'me'. They start from the question 'How do I talk about me?', presupposing that I do not merely talk in one way about myself. I change my I-language games constantly depending on where I am, whom I talk to, how I feel, what I am doing, *etc.*

All of the language games in which I use the word 'I' or when I talk about myself are interwoven. Together they form a coherent and ever-changing structure. By the use of the word 'I' we draw attention to ourselves. We need not explicitly be talking about ourselves, who we are or how we feel. We draw attention to ourselves also when we do and talk about other things than ourselves. For instance, when we do mathematics, we use the word 'I' explicitly and/or implicitly. By doing so we share our "mental/psychological states, experiences, feelings, thoughts" (Beristain, 2011, p. 108) while we are busy doing mathematics. Hence, studying students' I-language games gives access to their ways of sharing experiences and feelings about themselves when they, for example, participate in a group activity of reasoning about angles. This means that studying Samir's I-language games has the potential to illuminate his ways of sharing feelings and thoughts about himself in response to epistemological sanctions such as micro-invalidations. These feelings and thoughts might lead to stigmatisation and eventually perceived marginalised membership in mathematical communities (Gutiérrez, 2017).

When students engage in reasoning in mathematics class their I-language games are interwoven with other language games such as the *game of giving and asking for reasons* (GoGAR). GoGAR is at the heart of inferentialism (Brandom, 1994, 2000)—a neo-pragmatic philosophical theory of language use and meaning. GoGAR concerns people's practical face-to-face reasoning, that is the kind of reasoning that may occur when students deal with group activities in mathematics. GoGAR is not about how to build good arguments based on data, warrants and conclusions. Rather, it attends to how we deal with the uncertainty and anxiety of making ourselves understood and of understanding the Other when we reason. To grapple with this uncertainty, we keep track of, or score, our interlocutors' doings and sayings. This score-keeping is built around two normative status levels that we assign to claims: commitments and entitlements. The idea is that, when we claim that things are such and such, we undertake a commitment to the claim. This does not mean that we cannot change our mind and commit to another kind of claim in the next instance. It merely means that while I am committed to a particular claim I can be held responsible for it in the sense that I can be asked if I have good reasons for it.

Keeping track of whether our interlocutors have good reasons for their claims involves assessment. Hence "there must be in play also a notion of entitlement to one's commitments: the sort of entitlement that is in question when we ask whether someone has good reasons for her commitments" (Brandom, 2000, p. 43). To think that an interlocutor has provided good reasons for a claim means that we find the claim normatively appropriate in the realm of the social practice that the claim is caught up in. If we are satisfied with the reasons given and hence find our interlocutor reliable, we acknowledge the claim and undertake it ourselves. An entitlement is a social status that a commitment has received within a community, *e.g.,* a mathematical community and/or a classroom community or a practice.

There are epistemological aspects to GoGAR. It is concerned with how interlocutors keep score of commitments and entitlements and judge what they and others take to be claims of knowledge. If we acknowledge claims, we take them to be knowledge-claims that we are prepared to use in our own reasoning. Thus, in practical face-to-face reasoning, before we acknowledge a claim of knowledge and use it as a piece of knowledge ourselves, we assess the reasons that support it or we simply rely on the claimer to have good reasons.

The assessment of claims is normative. It is shaped by the norms present in the situation. This means that there are sayings and doings that are normatively appropriate as well as normatively inappropriate from the perspective of the meaning of the claim. For instance, claiming that a 100° angle is acute is usually considered normatively inappropriate in a mathematics classroom. Claiming that the same angle is obtuse is normatively appropriate. If I commit myself to that claim, my claim has the social status of entitlement due to the norms of the mathematical community.

From an absolutist perspective of mathematics, norms comprising preciseness shape the assessment of mathematical claims. Imprecise use of mathematical concepts violates the norms of mathematics as precise and certain. Such violation (*i.e.,* lack of entitlement to a claim) calls for some kind of sanction, which can be internal, external or both (Brandom, 1994). External sanctions like exclusion, nullification or disregarding of a person's beliefs or statements are micro-invalidations which, when assigning status in GoGAR, shape epistemological sanctions. Epistemological sanctions might lead to disqualification from eligibility to undertake commitments, a kind of authority loss. Hence, a student who fails to give reasons for a specific claim involving a mathematical concept, which her/his interlocutors assess as inappropriate, risks being exposed to external epistemological sanctions in the form of micro-invalidation due to the failure. Being repeatedly exposed to external epistemological sanctions could affect a student's I-language games. Changed feelings about oneself could, as in Samir's case, evoke I-language games to change from positive to negative ones. In other words, "the 'crime' is cognitive but the penalty affective" (Rowland, 2000, p. 123). To avoid exposure to epistemological sanctions interlocutors can attach vagueness and uncertainty to their claims. That is, they can use shielding hedges. To use a shielding hedge is for instance to say, '*I think* that the angle is 90°' instead of saying, 'The angle is 90°'. Using 'I think' means a weakened commitment to the claim and thus a protection from being

held responsible for it. This reduces the risky business of making mathematical claims. Or, by using 'I think' the claimant shows uncertainty and thereby risks undermining his/her authority as a producer of entitled knowledge claims for interlocutors to use in their own reasoning. Moreover, 'I think' talk is part of I-language games and hence affects the way a person draws attention to her/himself.

From "How good I am" to "Forgive me, please trust me"—context and three snapshots from a group activity on angles

"How good I am [at solving mathematical problems]" was one of the first turns recorded at Samir's and his peers Eva, Greta and Darko's desk when I was a participant observer during a regular math class in their social- and language-diverse grade 5 classroom (students aged 11). In the classroom, the language of teaching and learning was Swedish. Although multiple languages were represented in the classroom, none but Swedish was heard. According to Barwell (2005) this is a common thing when the teacher, as in this case, does not share the languages of the students. In addition, in this particular class no students shared a language with another student, other than Swedish. Among the four students, Darko was born in Sweden to immigrant parents and spoke Serbian and Swedish at home. Samir, who had arrived from Palestine 2.5 years before, was an emergent Swedish speaker who spoke Arabic and Hebrew at home. Greta and Eva spoke only Swedish at home. During the lesson, the four students were working on two tasks that entailed drawing angles which the other pair of students were to measure and label (Task 1) and to decide which angles were right, acute and obtuse and give reasons for their judgement (Task 2). Samir and Darko formed one pair and Greta and Eva the other. Below are three snapshots [1] from the interaction that illustrate how Samir's I-language games changed from positive to negative as he was exposed to epistemological sanctions when the students engages in GoGARs.

Prior to the lesson, I interviewed Samir. He told me that he liked mathematics and was good at it. This attitude was occasionally shared in the classroom. In other words, Samir's I-language games, when interwoven with mathematics, were usually positive.

Snapshot 1

Samir and Darko measured the angles drawn by Greta and Eva. They wrote the magnitude of the angles on a piece of paper which they handed over to the girls. They denoted one of the angles as 80/100°, the two numbers placed above each other as on a protractor scale. Greta asked them to give reasons for claiming that the angle is 80/100°.

Greta	Yes, but no. It cannot be both *[80° and 100°]*
Samir	Yes, because they are above each other
Greta	Yes, but they *[pause]* it does not mean that it is the same *[pause]* it *[the angle]* is not 100 slash 80. It must be one of them.
Samir	Yes, yes *[pause]* I get it, I made a mistake *[pause]* where is the protractor

Samir then used the protractor to re-measure the angle. He suggested that it is 100° and Darko that it is 80°. Taking up Darko's suggestion, Samir wrote 80° on the piece of paper. The boys' subsequent discussion of a 40°/140° angle caused Greta to give reasons, hence engage in GoGAR, for claiming that angles are labeled using only one number and to explain how the scales of the protractor work. Though both Samir and Darko were at first committed to the double labelling, when they could not give normatively appropriate reasons for this claim and thus realised that they were not entitled to it, Darko told Samir "Why didn't you say so?", making him accountable for the loss of entitlement. Samir was held responsible for the loss of the social status as a reliable claimer of mathematical knowledge.

Snapshot 2

The four students were occupied with Task 2; drawing one right, one obtuse and one acute angle. The other pair of students were to judge which was which. Samir decided that he should draw the right angle. Directly after he had finished drawing, Greta urged Eva to use the protractor to check that their right angle was *exactly* 90°. Such preciseness was not necessary as Task 2 was about looking at the angles and judging which was which. Probably inspired by Greta's urge for their right angle to be precisely 90° the following turns were uttered by the boys.

Darko	Well done *[pause]* what degrees is it *[the right angle that Samir drew]*
Samir	It is *[pause]* eh eh I am good at forgetting.
Darko	Mmm *[pause]* yes you are good at forgetting.
Samir	Yes, that is why my name is Forgetty.

Darko wanted Samir to measure their right angle, *i.e.,* he wanted Samir to commit himself to a claim of the angle being exactly 90°. Samir was just about to do so when he started to say "It is" but appeared to change his mind and claimed instead to be "good at forgetting". Samir seems to avoid being responsible for claiming the angle to be precisely right which might (as in Snapshot 1) cause him entitlement loss and exposure to epistemological sanctions.

Snapshot 3

Although Task 2 was not about measuring angles that is what the students did. Greta questioned the boys' claim about the magnitude of an angle they had measured. The boys reasoned with Greta about whether to denote it 145° or 155°. Samir claimed it is 155°, a claim that Darko initially supported.

Darko	Wait *[to Greta][pause]* look it is 155.
Greta	Not *[1]*55. *[1]*55 is there *[showing on the protractor][pause]* this is *[1]*45.
Samir	I thought *['trodde' in Swedish]* it was *[pause]*
Darko	I am not going to trust you anymore.
Samir	I thought *['trodde' in Swedish]* it was so *[pause]*

Darko You cannot just think *['tro' in Swedish]* so.

Samir May I *[pause]*

Darko I asked you specifically and you just said yes.

Samir Forgive me *[pause]* please trust me.

In this snapshot Samir and Darko use the Swedish word *tro* (think) and its past tense *trodde* (thought). *Tro* is one of the three possible Swedish translations of the English word 'think'. The other Swedish words are *tänka* and *tycka*. *Tycka* refers to having a personal opinion. *Tänka* refers to mental activity. *Tro* refers to holding a belief that one is not sure about.

Greta justified her claim by showing Samir and Darko where, on the scale of the protractor, the two angles 145° and 155° are located. Samir apparently undertook Greta's claim and simultaneously stated that his initial claim was based on that he "thought it was" an appropriate one. To avoid being exposed to sanctions due to a possible lack of entitlement for claiming the angle to be 155° Darko dismissed Samir's claim and seemed to argue that "think so" is not enough to justify a claim, hence challenging Samir's reliability. In the last turn Samir appears to think that his reliability and thus authority to make claims that will be acknowledged and thus given the social status of an entitlement is lost and he begs Darko to forgive him and to reassign him reliability.

Throughout the three snapshots the normative appreciation of preciseness in mathematics opens up a space which makes it possible for Greta to claim that an angle cannot be both 80° and 100°. That a right angle must be measured and be precisely 90°, and that an angle measures 145° not 155° and not simply (as the task required) 'obtuse'. This very space, inherent in Western mathematical ideas of angles and the artefacts used to measure and denote them, licences the entitlement losses Samir is held responsible for as the students play GoGAR. From the entitlement losses follow epistemological sanctions which makes Samir change his I-language games from positive to negative talk, being a forgetter, a wrong-doer and a person without reliability. It causes him to play negative I-language games to avoid further exposure to potential epistemological sanctions. Samir does not use shielding hedges (Rowland, 2000, 2014) in any of the snapshots. Instead, in Snapshot 2 he claims to be "forgetty". In Snapshot 3 he adds uncertainty to his previous claims by saying that "he thought it was so". This does not work out though while Darko does not seem to accept shielding hedges being attached to claims afterwards. Attaching shielding hedges is a kind of epistemic school discourse (Prediger & Krägeloh, 2016) that students use as ways of dealing with the risky business of making mathematical claims when preciseness is dictating the epistemic discourse. Either Samir did not want to undermine his authority as a producer of knowledge claims or he was not aware of the epistemic school discourse that incorporates use of shielding hedges.

Moreover, the normative appreciation of preciseness appears to prevent the four students from reasoning about alternative ways of denoting angles or from exploring each other's conceptions of angles. The concealment of mathematicians' consensus (Prediger, 2002) is efficient in that it appears to have excluded any appreciation of dissensus (deFreitas & Sinclair, 2014). The concealment of consensus, the dehumanisation of mathematics, moves the students into a space in which their pursuit of preciseness and universal certainty is possible at all.

To engage in mathematical reasoning guided by preciseness appears to be risky business, particularly for emergent speakers of the language of learning and teaching. The mono-lingual, mono-cultural and mono-mathematical normativity diminishes her/his resources for giving reasons for claims in GoGARs. This suggests that when inviting students to group activities involving reasoning, educators need to be sensitive to the exposed situation of emergent speakers and provide student awareness of potential epistemological sanctions evoked by enthroned preciseness. Although it is Samir who is exposed to epistemological sanctions, it could in fact happen to any student whose ways of expressing mathematical knowledge differs from the 'pursuit-of-preciseness-and-certainty' normativity. I claim that emergent speakers and students whose social languages differ from the school language are at greater risk of exposure to epistemological sanctions than other students are. I elaborate on the theoretical reason for this below.

It is common to think of language and epistemological issues as separate, hierarchically ordered, hence presupposing that when the epistemological is considered the semantic meaning is already settled. That is to think that we already know the meaning of the claims we consider and that the question is merely under what circumstances they could be justified. However, according to inferentialism one cannot treat the *meaning* of a claim as already settled before the knowledge content is judged (Brandom, 2013). We consider the meaning of a claim at the same time as we consider its knowledge value. Hence, the semantic and the epistemological are inseparable elements that constantly are in flux (Derry, 2013). Consequently, the person who is in least command of the language at play in face-to-face reasoning, is potentially the one whose claims are most likely to be questioned, while that person's claims are at the greatest risk of being the least accurate or precise according to the norms that guide the situation. At the same time, the emergent speaker of a language (be it a social or national language) is the one who might be the least equipped in terms of language to give reasons for her/his claim, in order for it to potentially be acknowledged by the interlocutors and thus get the status of entitlement. Hence, the claims of an emergent speaker of the language of learning and teaching are at greater risk of being nullified or disregarded than claims of students in command of the language at play. Therefore, learning to meet the Other, to respect his/her knowledge and ways of expressing it (Guillemette & Nicol, 2016) is crucial in social and language diverse contexts. It is particularly important when preciseness guides the norms that shape face-to-face reasoning. Learning to meet the Other and her/his knowledge when engaging in GoGARs may have the potential of saving Samir and others at risk from playing negative I-language games about themselves when reasoning about mathematics.

To close, as stated by Gutiérrez and Dixon-Roman,

> We need to be constantly considering the forms of mathematics and what they seek to deal with. As society presents new demands, new technologies, new possibilities, we must ask ourselves whether our current version of mathematics is adequate for dealing with the ignorance that we have. (2011, p. 32)

And, I would like to add, how our current version of mathematics shapes discourse in social and language diverse contemporary mathematics classrooms. This is imperative at present as global migration is reaching new stages, diversifying local contexts such as mathematics classrooms. This paper suggests that in social and language diverse classrooms, students and teachers do not merely navigate language diversity as an outcome of migration; they also navigate epistemological aspects of mathematics. Learning to meet the knowledge of the Other, learning to avoid epistemological sanctions calls for meta-understanding of language diversity which embrace epistemological aspects of mathematics.

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Note

[1] Each snapshot comprises a transcript that I translated from Swedish to English. Although I experienced doing the translations as a relatively straightforward matter, there is a translation challenge in Snapshot 3 discussed below. Moreover, I am aware that there are nuances of the languages that may influence my own as well as the readers' focus and interpretation of the excerpts.

References

Andersson, A., Valero, P. & Meaney, T. (2015) 'I am [not always] a maths hater': shifting students' identity narratives in context. *Educational Studies in Mathematics* **90**(2), 143–161.

Barwell, R. (2005) Working on arithmetic word problems when English is an additional language. *British Educational Research Journal* **31**(3), 329–348.

Beristain, C.B. (2011) Language games and the latticed discourses of myself. In Gálvez, J.P. & Gaffal, M. (Eds.) *Forms of Life and Language Games,* pp. 107–119. Berlin: Walter de Gruyter.

Brandom, R. (1994) *Making It Explicit: Reasoning, Representing, and Discursive Commitment.* Cambridge, MA: Harvard University Press.

Brandom, R. (2000) *Articulating Reasons.* Cambridge, MA: Harvard University Press.

Brandom, R. (2013) Reply to "Brandom on knowledge and entitlement": Schmoranzer and Seide. In Prien, B. & Schweikard, D.P. (Eds.) (2013) *Robert Brandom: analytic pragmatist,* pp.163–171. Berlin: Walter de Gruyter.

Chronaki, A. (2011) "Troubling" essentialist identities: performative mathematics and the politics of possibility. In Kontopodis M., Wulf C. & Fichtner B. (Eds.) *Children, development and education,* pp. 207–226. Dordrecht: Springer.

Derry, J. (2013) Can inferentialism contribute to social epistemology? *Journal of Philosophy of Education* **47**(2), 222–235.

de Freitas, E. & Sinclair, N. (2014) *Mathematics and the Body: Material Entanglements in the Classroom.* New York: Cambridge University Press.

Guillemette, D. & Nicol, C. (2016) Mathematics education and social justice or learning to meet others in the classroom: a collective reflection. *For the Learning of Mathematics* **36**(3), 38–39.

Gutiérrez, R. (2017) Political conocimiento for teaching mathematics. In Kastberg, S., Tyminski, A.M., Lischka, A. & Sanchez, W. (Eds.) *Building Support for Scholarly Practices in Mathematics Methods,* pp. 11–37. Charlotte, NC: Information Age Publishing.

Gutiérrez, R. & Dixon-Román, E. (2011) Beyond gap gazing: how can thinking about education comprehensively help us (re)envision mathematics education?. In Atweh, B., Graven, M., Secada, W. & Valero, P. (Eds.) *Mapping Equity and Quality in Mathematics Education,* pp. 21–34. New York: Springer.

Lakoff, G. & Núñez, R. (2000) *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being.* New York: Basic Books.

McGarvey, L. & Sterenberg, G. (2009) Ethical concerns: Negotiating truth and trust. *For the Learning of Mathematics* **29**(1), 35–39.

Moschkovich, J.N. (2015) Academic literacy in mathematics for English Learners. *Journal of Mathematical Behavior* **40**, 43–62.

Planas, N. (2011) Language identities in students' writings about group work in their mathematics classroom. *Language and Education* **25**(2), 129–146.

Planas, N. (2014) One speaker, two languages: learning opportunities in the mathematics classroom. *Educational Studies in Mathematics* **87**(1), 51–66.

Prediger, S. (2002) Consensus and Coherence in Mathematics. *Philosophy of Mathematics Education Journal* **16**, 1–10.

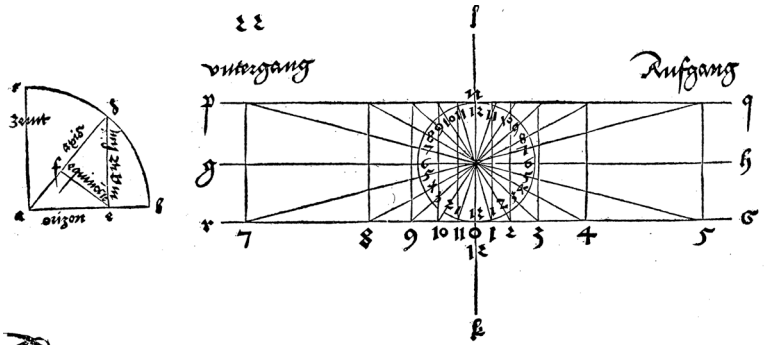
Prediger, S. & Krägeloh, N. (2016) "x-arbitrary means any number, but you do not know which one": The epistemic role of languages while constructing meaning for the variable as generalizers. In Halai, A. & Clarkson, P. (Eds.) *Teaching and Learning Mathematics in Multilingual Classrooms: Issues for Policy, Practice and Teacher Education,* pp. 89–108. Rotterdam: Sense Publishers.

Rowland, T. (2000) *The Pragmatics of Mathematics Education: Vagueness in Mathematical Discourse.* London: Falmer Press.

Rowland, T. (2014) Hedged maxims. *For the Learning of Mathematics* **34**(2), 45–46.

Ryan, U. (2018) "From how good I am" to "Forgive me…please trust me"—microaggressions and angles. In Bergqvist, E., Österholm, M., Granberg, C. & Sumpter, L. (Eds.) *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education,* Vol. 4, pp. 75–82. Umeå, Sweden: PME.

Wagner, D. (2009) If mathematics is a language, how do you swear in it? *Montana Mathematics Enthusiast* **6**(3), 449–458.



Albrecht Dürer's diagram showing how to correctly construct and orient a sundial, from his 1525 *Unterweisung der Messung mit dem Zirkel und Richtscheit [Instructions for Measuring with Compass and Ruler].*