

Exploring Difficulties in Teaching Mathematics through Investigations in the Primary Classroom

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It is interesting for those of us working with student teachers and teachers in the UK at the moment because we appear to have “right” on our side when we introduce them to the merits of investigational work. A substantial proportion of the statutory requirements for mathematics laid down in the National Curriculum documents seems to support to spirit of this type of work. However I am not sure this is always helpful.

Many teachers remain frightened of the more open spirit of this approach and at times it seems that they would rather make a case for fitting what they already do to the words used in National Curriculum documents than explore this spirit themselves. Of course this is unfair in some ways—rapid changes in the legal requirements on teachers in UK schools have led to unprecedented pressure, and it is not necessarily easy to engage with the fairly complex philosophy that underpins investigational work.

In some respects, though, it was simpler in a pre-National Curriculum era to try to give teachers experience of this kind of work and offer some words in its support, knowing that some would reject it and feel free to do so while others would take it on with the more open enthusiasm of somebody impressed by its potential and not coerced into complying with it. Subversion, whether conscious or not, is always a possibility when a curriculum is laid down. In secondary schools, for example, we have the phenomenon of children being taught investigational processes in an almost algorithmic way.

Evidence from official reports supports a general feeling amongst mathematics educators that in primary schools the quality of children’s investigational work can be quite limited. Since the spirit that informs investigational work is a significant part of mathematics it is worth considering some of the difficulties faced by teachers who try to work in this way.

I have chosen to explore these issues by looking at the work of Karen, a student in the final year of her BEd degree, a 4-year primary teacher training course that enables students to study a main subject at degree level for 40% of the time, in addition to achieving qualified teacher status. I have selected her because her main subject is mathematics and she does not have the lack of subject confidence that some teachers have; she is also familiar with and enjoys the investigational process herself. Partly as a result of her experiences she needs no persuading about the mathematical or personal benefits of working investigational. Although already a qualified teacher by virtue of having successfully completed her final teaching practice she clearly lacks wide experience. This I see as an advantage for my purposes, however, because she has not

reached a stage where the easy flow of “control techniques” available to experienced teachers might lead her to turn a blind eye to some of the important teaching issues that arise in the complex areas of teaching investigational.

Since she starts with a relative confidence stemming from a good mathematics background and a belief that investigational approaches are worthwhile, it seems easier to separate the issues that arise for her in the area of teaching rather than in her background knowledge base or attitude. She also manages to write with an appealing honesty that makes these issues more transparent to me as a reader.

The present structure of our degree course enables final year students to spend a good deal of time carrying out two practical studies in the classroom. In the study I am considering there is generous tutor support (six hours of one-to-one discussions per student). Karen, like all the students who studied mathematics at degree level for part of their teaching degree, was automatically part of the Specialist Mathematics Teaching Studies group that I worked with. For her final assessment she had to produce a 6,000 word dissertation based on a study involving her own mathematics teaching and carried out in a local primary school. Each year a substantial proportion of the students elect to take a look at investigational work or at problem solving. Many of their dissertations open with words referring to their lack of teaching confidence in an area they feel they should be familiar with as mathematics specialists and in which they would like to put their ideas and ideals into action. In addition they also often say that, although they are convinced of the case for this kind of work and have enjoyed the approach with the tutors who take them for main subject studies, they have not usually seen any comparable practices in the schools they have been placed in.

In a session with me very shortly before the start of her study into investigational mathematics, Karen told the group that she had done little other than follow the prescribed mathematics scheme on her final teaching practice. Final teaching placements bring their own constraints, of course, and as I subsequently discovered when I read her final dissertation she *had* done some investigational work but felt that what she had done was inadequate, or at least left too many questions unanswered. This supplied the impetus for her choice of study. Karen was very quiet and thoughtful in my teaching group and not especially voluble in tutorials, so it was a surprise as well as a pleasure to read such an articulate account of her investigations into investigational mathematics teaching.

In the following sections I summarise parts of Karen’s dissertation [Bell, 1994]. In summarising her comments I have inevitably paraphrased and condensed the original

text, but I have tried hard to retain the sense, and where possible the actual words used. When I report on the actions that Karen took in the classroom I base this on her written words. Karen herself is satisfied that I have rendered the correct sense of what she wrote. Where it seems important that Karen's actual words should be recorded, I have indicated this by the use of italics. I also intersperse my own comments and reactions to the events that Karen describes and the issues she raises. Where my interpretations of her actions are tentative I indicate this. Since I was not present during her teaching sessions with children, the interpretations are based on what she has written and on my recollections or notes of my discussions with her prior to her writing the dissertation. Karen and I have discussed the draft of this article and she is happy with the interpretations I have made.

Karen introduces her study by confirming the extent of her lack of confidence in teaching investigatively, mentioning in passing that the children in the class of 9 and 10 year olds she chose to work with (her final teaching practice class) also seemed to be very lacking in confidence. This lack in both the children and herself led her *to tend to give the children all the answers*. It is a common theme in students' dissertations that it is initially difficult to involve children in investigational work because they have picked up messages from successive teachers that progressing at a good pace through the prescribed content of the lessons is important. Entering into the more "leisurely" spirit of investigations, even when these appear in the textbook, may seem to the child to be opposed to this message. In such cases the children's worries about inconsistent demands need to be addressed so that they can feel relaxed enough to engage with the investigational tasks. (For some students this may be only a short term "agreement".)

Karen writes that she wanted to look at the part that discussion played in the children's investigational work, at their interactions with each other, and at the way they were able to develop strategies over the six sessions that she anticipated working with them. She was also interested in gaining more insight into her own role, in particular with regard to her questioning techniques; her choice of activities; her ability to engage these four 9-10 year old children in adopting an investigative manner; and a range of other issues connected with own interventions. She felt herself, prior to the study, to be *very dominant in (her) mathematical work with children* and too prone to giving answers rather than inviting children to reflect. After some deliberation and discussions with me, she decided to introduce the idea of investigational work to the children gradually by adapting a problem from the textbook used in the class.

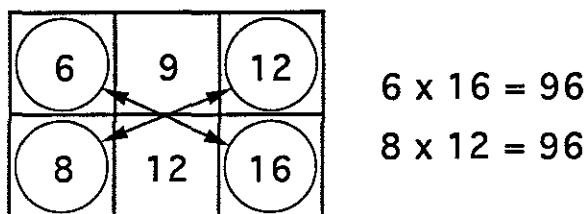


Figure 1

This involved looking at the relationship apparent in a rectangle of numbers selected from a "multiplication square" and investigating whether this relationship held for other rectangles contained in the square.

Karen noticed that the children became very engaged with this problem. She was very happy that the ensuing discussion was in contrast to previous sessions during teaching practice where she felt that she had *spoken too much, given too many answers and not given the children time to think*. However, as soon as she suggested the children work in pairs using the textbook for reference, the lively discussion stopped and was replaced by silent individual working. The children also showed concern about others copying their work, a response that Karen attributed to the negative effects of always working from a scheme. She suggests that the use of the books, which was mainly associated with working alone, had triggered established responses about copying and cheating, though she had intended there to be sharing. In this silent atmosphere she found herself forced into a different role and realised in listening to a tape of the session afterwards, that she had responded to the silence by asking inappropriate questions eliciting one word answers:

In a desperate attempt to change this I began to push the children to talk with each other, asking questions such as, "Does this rectangle work?" and "Have you tried a square one?"

On reflection she found herself agreeing with Jaworski [1986] that:

the way a question is posed can limit your vision considerably, or provide too wide a range of possibilities to make choices easy. [p 29]

She wished that she had rephrased these questions as "Which rectangles have you looked at and what have you found?" and "What happens if you use a square?" As a result she writes of her intention to look at the questions she would pose in the next session.

The next activity she proposed involved "arithmogons" as shown below. She saw this as being a more open activity. It involves finding missing numbers by using the rule that the numbers in the squares at any two vertices must add up to the number in the circle between them. The children worked on one large arithmogon and wrote potential solutions on blank cards which were then placed on this diagram. Although the example in Figure 2(a) proved successful the one in Figure 2(b) soon began to cause problems.

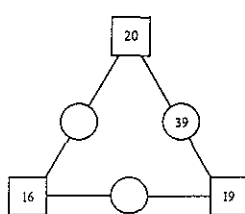


Figure 2(a)

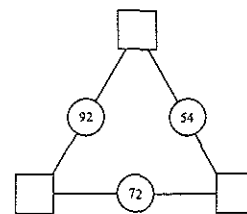


Figure 2(b)

All four children wanted to try out their ideas at once, and this led to noise and confusion. Karen later realised that there was insufficient structure in the situation and that she had mistakenly thought it inappropriate to intervene since she was committed to providing open-ended activities. In trying to *limit her dominance* as she puts it, it seems to me that she had in fact reduced it to such an extent that the children were unlikely to take any control of the situation because they lacked the necessary "outsideness", or perhaps "authority", to sort out the tangle they were in. She also realised that the children lacked some important skills, e.g., the ability to tackle a problem in stages. Retrospectively she recognised that she might have changed the problems by simplifying them and altered the physical situation in order to allow more involvement by each of the children. Importantly, she realised the need to provide *a balance between prompting children and giving them answers* and realised that the skills involved in knowing *when or when not to intervene would not occur overnight*. Turning again to Jaworski she discovered (too late, she felt) the remark that a complete upheaval of normal practices could be very disruptive, so for the next session she decided to follow Jaworski's advice by adopting a games approach to *get them used to asking questions and working together*.

Although experienced teachers might say that problems arising from four children working on one board could be anticipated, I had an intuition, later confirmed in a conversation with Karen, that the decision to use this arrangement was closely related to the appeal of it as an "open" activity. In her dissertation Karen characterised this activity as *at the opposite end of the investigation spectrum* to the previous one, and although not everybody would agree, in terms of the broadening of her own confidence it was. In my experience the appeal of "openness" is high for some students, perhaps partly in reaction to the closed nature of much of the teaching they see. After a setback of the kind Karen had, some students decide that it simply doesn't work to try to be more open, and content themselves with situations that are less challenging for the children and to themselves. However, instead of rejecting her ideal of more open working, Karen decided that the structure she had chosen was inappropriate.

Curiously, despite her intention to look at her own questioning style in this session, she then proceeds to ask virtually no questions at all—or at least she does not report this. I am wondering now whether she was in fact still concerned about the fact that she had previously asked too many questions and if, in this sense, she *has* attended to looking at how she asks questions by tacitly deciding to ask none at all. Questioning is connected to another of the important issues she sets out to explore, concerning how much a teacher should intervene. Has she perhaps chosen to see this in terms of whether she asks questions or not, and whether she supplies information or not, rather than considering the types of questions that can be asked if one decides to ask them? In a recent conversation, Karen was unable to illuminate my speculations here, apart from commenting that there was a big difference for her between her views about the nature of intervention at the beginning of

her study and at the end. What is certainly impressive is the sharp contrast between the relative lack of "success" of the session and the insightful analysis of it she provides.

Karen next follows Jaworski's advice to use mathematics games as a way of encouraging children to ask questions and to work together. She introduces the children to three games and decides to look now at the interactions between children, the questions asked by her and by the children, and to consider her role in the session more carefully. She writes that she sees the games as useful in breaking down their stereotype of what mathematics is, providing a framework for collaboration to get the children used to working with each other, and providing some *time and space for the children to develop understanding*, as well as giving her a more fluid framework within which to *practice (her) role and build (her) confidence up*. She also felt that the games would *give rise to opportunities to intervene and present the children with issues to consider*.

In one board and dice game, within the time and space she has provided for herself to *practice intervening*, she makes use of a problem that arises about different ways of making 12, directing the group to take a break from the game in order to investigate this. She is delighted to find that one child explores this in a systematic way, one operation at a time, and seeing this proves useful for the other children in the group. When it seems that this investigation threatens to overawe them, and answers are arriving randomly, she again intervenes and suggests that they limit themselves to the numbers from 0 to 20. Later, she capitalises on the fact that the children appear to be using her questions to direct themselves into more productive areas of exploration by involving them in a game called "Draw that Shape", where the children are given shapes drawn on cards and their partner has to ask questions that can be answered "yes" or "no" in order to identify the shape. While recognising that it is often difficult to be sure that a new direction is not a diversion, or an intervention is not an intrusion, Karen felt that she had learnt a great deal about the art of intervening in a productive way and decided to move on to trying to apply these skills in the context of an investigational rather than a game setting.

She decided that the investigation should be relevant to them and to their *stage on the investigative scale*, and eventually chose to work with them on a version of the 4-cube problem from the Wiskobas Project in Holland [IOWO Team, 1976]. She introduced a gnome called Gnorman who writes to the children asking them to help him make as many houses as possible from four cubes, which are only allowed to touch (whole) face to (whole) face. She chose this for several reasons, one being that, in this form, the investigation is quite closed, and although a certain kind of richness is lost as a result, more control was available to her.

There is an enduring attraction in this fantasy-based investigation for many of my students and it seems to me that it derives in part from the way that it is possible to use the cantankerous gnome as a kind of alter ego whom the student can blame for the more demanding aspects of the exploration. Where the teacher might be accused of changing her mind and being awkward there is a sense in which

one expects nothing else of a gnome, and the awkwardness becomes all part of the fun. In addition, children of primary school age enjoy receiving letters from the fictitious Gnorman. Even though they don't really believe that he exists, they are prepared to suspend their disbelief. I know some mathematics educators who are quite negative about this approach to setting up an investigation, believing that the mathematical problem should have sufficient interest in itself, but my experience is that it has inspired many students to take their first step into investigative mathematics.

With her more relaxed attitude towards intervention now, Karen seemed to find herself in a position to "simply notice" what the children were doing and to act on it as necessary. Many interesting discussions about similarities and differences arose which the children resolved for themselves. For example, the following two houses are eventually accepted by the group as different after one child argues that the number of floors is a significant factor in making houses different even though the shapes are identical.

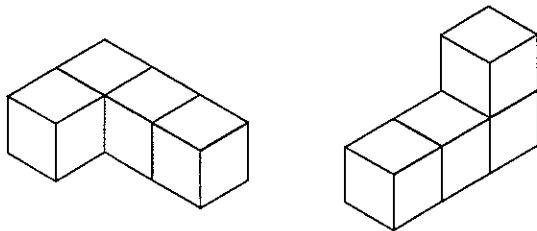


Figure 3(a)

Figure 3(b)

Considering her role, Karen observed that setting up the investigation using a letter apparently written by somebody else had helped to shift her from the role of teacher into the more comfortable role of guide since the children were prepared to accept that they were working on somebody else's problem, whose solution might be unknown to Karen:

The whole nature of the activities changed, as in the children's eyes, I had not set up the activity, I was just passing information on. Because of this the children tended not to look to me for answers. My input guided them rather than telling them what to do. With the pressure of the children looking to me for answers lifted, I had time to consider where to intervene and what kind of questions to ask.

I have noticed this technique working successfully in other circumstances and it seems to me that it shows how paying attention to contexts of this kind with primary school children can radically alter the success or otherwise of work arising out of investigative mathematics.

The children were very self-sufficient in this situation but Karen found places where she could move into action to help children. For example, she decides at one stage to introduce a child to a strategy for generating new houses from existing ones. She asks him to consider the shape shown in Figure 4(b) and move just one block to a different place rather than continue making a total reconstruction

each time, as he had been doing. The question itself was enough to enable him to *take on the task for himself* eventually, but Karen comments that she had to restrain herself from giving additional help by *moving away and giving him time and space* to resolve the issue for himself when this took what seemed to her like a very long five minutes.

Having built up her confidence Karen decides to tackle a more open problem. She (or rather Gnorman) asks the children to find out how many different designs they can find for a 3×3 tile using only the following type of tile:



Figure 4

The enormity of this task swamps the children and Karen comments that

the children randomly chose patterns. The activity had no structure. The progress the children had made in the previous sessions was not evident.

Disheartened, but with new skills in place since the last time a session had disintegrated, Karen decides to intervene and does so with considerable skill. She initially asks how many 3×3 designs there are and is met with blank faces. When told that there are too many to find, she refers them back to a previous experience when they had been stuck. She asks how they overcame this, and one child says that they changed the problem to make it easier. Challenged to find a way of making this easier, another child, Grant, suggests looking at smaller patterns.

Grant: We could use less tiles so the patterns are smaller (indicating a 2×2 arrangement of unit tiles)

Karen: How's that going to make it easier?

Grant: Well, there's only four different tiles so it's bound to have less patterns

As a result she moves them to the related problem of finding the number of designs for a 2×2 tile and, perhaps because the smaller scale enables him to focus more clearly, Grant suggests the strategy of moving one square (top left) at a time to produce the following designs:

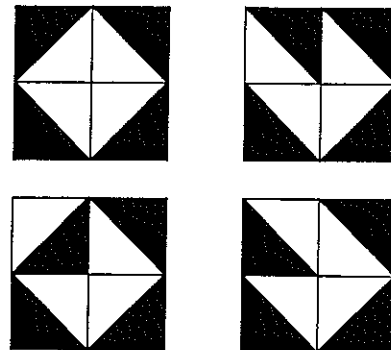


Figure 5

Although he believes at this stage that he has all the solutions, an intervention by a member of the group alerts to the fact that there are more, and supplies an example. Grant then applies the same strategy to this example to produce another three solutions. Later, in a section considering her own role in the session, Karen used this incident to demonstrate the real advantages of working in groups where anyone confident enough can intervene and enable other individuals to move forward. Another child in the group later took Grant's idea in an different direction, and Karen used this as a further example of the value of working in groups.

Reflecting on this episode later, Karen considered that the group members were unable to make the decision to change the problem on their own since they lacked confidence and for this reason it was up to her to intervene to move the investigation on.

The task faced by the children was too great. They'd never really experienced such freedom. Therefore in this case it was the role of the teacher to establish that because the activity is open the children can make their own rules up.

With the activity back within the capability of the children, Karen felt able to resume her role of asking questions and making interventions where she considered these would be helpful:

There were a few instances where the children asked "Can I have that one?" and my reply was with another question like "What do you think?", "Which patterns have you already got?", "Can your partner help you?" This technique reversed the question and encouraged children to find out for themselves whether or not they could have that particular pattern. By reversing the question back to the children I felt eventually that the children themselves would adopt this questioning approach and wouldn't need to ask. This however would be a long way into the future.

This reference on how long it takes for skills to develop within children reminds me of her earlier reference to how long she feels it will take her to develop the skills of intervention and reveals the depth of her understanding about the time dependence of such issues even when in an ideal sense she is already aware of what is needed.

In conclusion Karen reflected on a number of issues about the whole project. Her first comment was that:

My original definition of an investigation was too cut and dried. After completing this study I can see it as a much greater issue.

This comment of Karen's reminded me of the security that many find in the definition of a particular word. The establishment of a National Curriculum in the UK after so many years without an "official" curriculum in primary mathematics intensified the call for a definition of a mathematical investigation—which, until that time, had been only a minority interest in primary schools. Some teachers seemed to be saying, "If you want me to do this type of work tell me exactly what it is you want me to do".

In a process-dominated area like investigative mathematics a definition is essentially unhelpful until a teacher has had some experiences on which he or she can draw. In any case I have always felt uneasy about providing a definition if I sense that the person asking for one is attempting to close down discussion rather than open it up. Imagining a definition to be the highest peak in the landscape shown below seems to imply one has arrived at a goal and there is nowhere else to go: everything is somehow settled.

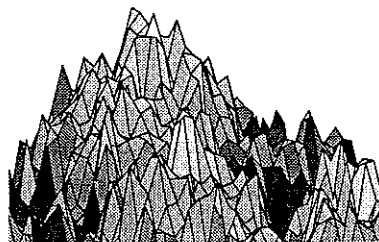


Figure 6

Arriving too soon can be a mistake. It bypasses the interest and satisfaction of exploring the rich range of possibilities in the valleys or slopes that lead to a "definition"—which is only a temporary resting place despite the momentary exhilaration of achieving it. I much prefer the idea of trying to characterise investigational work by considering observations of children at work that offer insights into the processes and the learning that is taking place. In fact, after tentatively suggesting that investigative mathematics is a process of asking the correct questions so as to begin to answer them, in the introduction to her dissertation, Karen effectively shelves her attempts to define it. Instead she builds up a store of observations of her experiences that enable her to characterise investigational work in a way that suits the particular context within which she finds herself.

Descriptions of such experiences seem to offer a richness that can be deconstructed in a variety of ways and from a variety of viewpoints rather than reduced and matched against a definition. The existence of different names for "investigations" seems to reflect a tendency for innovative teachers to prefer to climb the slopes, or even the difficult faces leading up to the peaks, and plays an essential part in keeping the spirit of mathematical investigations alive. Karen is contributing to a living tradition of innovative teaching by people who want to offer valuable learning experiences to children and to explore the hints offered by other people's words about rich and exciting areas.

The spirit of investigational work is maintained and regenerated by the debate that ensues when people compare and contrast their experiences in areas that offer sufficient richness to sustain such a debate. Such exchanges can take place in print but are more likely to occur in conversation between "enthusiasts". They seem to offer an organic way of counteracting the tendency for valuable experiences to become first labelled and then debased into fashionable but flaccid activities lacking the life-force of the original conceptions.

In summarising her dissertation Karen recognises a number of significant aspects to the work that she had done with the children. For example:

- She had enabled children to work co-operatively; the children had helped each other significantly by their interventions and suggestions to each other.
- The children had transferred some skills from one session to another.
- Discussion had helped individuals clarify concepts by attempting to articulate them.
- She became aware that in order to produce worthwhile mathematics children needed to be more in control of what they were doing, learning to make their own rules, to devise systematic approaches and to consider and test hypotheses.
- The teacher's role is more appropriately that of facilitator than instructor.
- Children need to be given time and space to develop their own methods of questioning and tackling problems.
- Activities need to be relevant to the children's interests.
- Investigational processes are not specific to mathematics but have cross-cultural implications.
- The meaning of the word investigation is difficult to define and its meaning can shift from activity to activity.
- Working with a small group is different to working with the whole class, but encouraging independence is a spin-off of working investigational and has implications for whole class work.

Several things strike me as I consider the experiences that Karen underwent and her analysis of them. I am impressed by the level of honesty throughout her writing. Learning can be an uncomfortable and frustrating experience and it requires a particular kind of openness to explore mistakes in the way that she did, even though counter-balanced by examples of teaching that very much "went right". I am also struck, more forcibly than ever before, by the fact that the resourceful use of investigational approaches is an excellent indicator of skilful teaching in a much more general sense—one which I suspect transcends artificial subject boundaries.

One of the difficulties we face in asking for better investigational work in schools is that we are, in a disguised way, asking for more skilful teachers. I have often felt that a primary school teacher who produces thoughtful work in any particular area is also capable of producing interesting mathematics with children, provided she has the will to do so and no insurmountable emotional problems deriving from her own history with mathematics. It seems to me that a good facilitator of mathematical investigational work can be characterised in the following ways:

- They are likely to have a view of themselves as facilitators of learning activity rather than as instructors.
- They are skilled at knowing whether, when, and how to intervene.
- They try to make things relevant to the children they work with at the level of content and/or context.
- They understand that it is necessary to work with the

existing skills and attitudes of children and to take account of their past experiences.

- They seek to make the children increasingly independent and more able to cooperate with others in appropriate ways.

It seems likely that effective teaching in spheres other than mathematics also includes these same qualities. Not surprisingly, the simple words conceal such complex skills and insights, concerning the nature of the relationships between children and teachers in learning and teaching contexts, that even the most accomplished practitioners continue to hone them on a daily basis in their classrooms.

My overall impression of Karen's dissertation is that it shows her ability to analyse what she considered to be chaos, to see the important aspects of it which needed working at, and to try again. This quality augurs well for her future teaching as she continues to develop her skills and insights in making mathematics with this investigational spirit available to the children she works with.

I am naturally drawn to consider my own part as a tutor in Karen's study. Since I was not aware at the time that I would use her dissertation as a vehicle to explore issues concerning the difficulties of investigational teaching, my scribbled notes during our tutorials reveal little of significance concerning my "role". I can say (with the easiness with which one tends to say such things) that I did what most tutors do in these cases—I listened, made some suggestions, and pointed her towards certain books and starting points for her work with the children. I tried to empathise with her situation and offer suggestions on where she might go next, given the difficulties of knowing precisely the contexts within which she worked. I offered the same "service" to each of the other students in the group. Each one demonstrated different levels of skill in taking up or rejecting my offerings but this may partly reflect my misunderstanding of their situation. There were also very different levels of skill apparent in the writing up of their experience, making it difficult in some cases to judge the relationship of the insights gained during the project to the written account of it. Some extremely interesting pieces of written work were produced, together with one quite depressing piece.

This may only be an elaborate way of saying that I have little idea what skills I do and don't have in enabling students in this area. Just as Karen is exploring her skills in helping her children to investigate, so I am exploring the skills I need to help students develop *their* awareness and skills. Fortunately, unlike her, I do not yet have to write a 6,000 word dissertation on it—although perhaps sometime I should dare myself to try!

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Afterthoughts

Having read through the above words I discover that I can't resist taking up my own challenge and at least laying down some preliminary thoughts connected to the previous paragraph. My work as a primary school teacher provided a fund of experiences on which I have drawn to build some personal theories of "how a teacher can encourage investigational work in primary school". The quotation marks are intended to indicate my awareness of the difficulties of attaching global claims to a set of personal experiences and contexts (I explore this in more depth in Delaney, [1989]). This awareness doesn't stop me offering thoughts as if they *might* be true in a larger number of contexts than those in which I have been personally involved, but it does make me more cautious about how and when I offer them. Increasingly I am having new experiences as I supervise students and look into how they can initiate investigational work in schools, and these add in subtle ways to the more direct insights I had from working with children.

Students like Karen are involved in a number of interrelated experiences when they undertake a school-based study. They act, observe, and reflect on the activities they carry out with children; they read what other people have said about the process of investigating and about possible starting points, and they discuss with a tutor what they will do and what they have done. Confident students seem to recognise, if only unconsciously, that in a local sense they are the "experts"—they know the children and the wider

context and were present when events took place. However they also recognise that others who have had a more sustained experience with children in this area can offer help coming from a different space, even though it will rarely be a perfect fit with the singular nature of their own experiences. The problem for less confident students is keeping a sense of perspective about how these different levels of "expertise" can help or hinder them when they engage in their actions locally. It is my job to be aware of these issues and to draw attention to them in helpful ways if I can.

It seems to me that, at its best, supervising a student involves a kind of collaboration. (I note in passing that this probably also applies to the best kind of teaching in schools with children.) Not surprisingly, some of the skills that I can identify as useful for a supervising tutor are those that I listed earlier as typical of skilled teachers. I feel that I have these to a greater or lesser extent in different situations. The more challenging the tutorial situation the more aware I am of the need for such skills. In the smooth flow of collaboration that seems to characterise a "good supervision experience" I tend to notice my contribution much less. My guess is that in these circumstances both supervisor and supervised are to some extent building on each other's skills. One consequence of this is that one is much more likely to focus on the stimulation generated by the topic under discussion than by the analysis of one's part in what makes it productive.

In mathematics, just as in music (and nowhere else), doing is inseparable from thinking; more than that, in both doing is identical with thinking. What is true of tones is also true of numbers: to think them is to create them.

Victor Zuckerkandl
