GIVING REASON TO PROSPECTIVE MATHEMATICS TEACHERS

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In this article, we describe the “living contradiction” (Whitehead, 1989) that became apparent to us as we confronted our “teacher selves” (Day, Kingston, Stobart, & Sammons, 2006) and our practices in mathematics teacher education. The consciousness of the living contradictions of our practice emerged as the result of our professional collaboration.

Beginning in the late 1990s, we were colleagues in a teacher education program in which all the courses were aligned and based on a constructivist theoretical perspective. With similar training as mathematics educators we collaborated to build models of student understanding in order to make decisions about our practice of teaching mathematics. As our professional collaboration grew, we became aware of the uniqueness of our work together. Our discussions are characterized by a willingness to try to represent ideas, thoughts, and feelings about our teaching as our understandings evolve. On numerous occasions one of us taught the mathematics content course to prospective elementary teachers and the other taught the same students in a subsequent semester in the course on methods of teaching mathematics. Our models of our students’ mathematics guided the design of coherent experiences for them. Our work together was fuelled by intellectually challenging and honest deliberations regarding our practice and our students’ construction of mathematical ideas. When we shifted from deliberations regarding our mathematics students’ learning in our content courses, to deliberations regarding our prospective teachers’ construction of their teacher selves in our mathematics methods courses, our greatest challenges began to emerge. In espousing our values as they relate to our goals for our students and our practice, we were taken aback by our inability to attend to the perspectives and ideas of the student who was learning to become a teacher of mathematics.

Giving reason
Duckworth (1987) describes “giving children reason” as the teachers’ disposition to consider the sense in what a child did or said, even when the “meaning was not immediately obvious” (p. 86). Giving reason, as described by Duckworth, entails the ability to honor and respect the reasons given by a student. It should not be confused with “giving kids reasons,” meaning to explain things to a learner in the hope that they will absorb the teacher’s explanations. In mathematics teaching, giving reason to the learner would mean considering the mathematics learner as a mathematical thinker with a system of knowledge that is internally consistent. With this consideration, teaching would mean gaining insight into the learner’s system of knowledge and then challenging and supporting the development of elements of that system. In mathematics education, there are many different pedagogical strategies that are linked to giving reason, including listening (D’Ambrosio, 2004; Davis, 1997; Weissglass, 1990), trusting (Kastberg, Norton, & Klerlein, 2009), caring (Hackenberg, 2005), and suspending doubt (Harkness, 2009). Using these strategies in light of a disposition to give reason to the learner, teachers gather insight into the system of concepts and processes available to learners.

Our interpretation of giving reason is drawn from our perspective of ourselves as constructivist teachers. Steffe and D’Ambrosio (1995) coined the term “constructivist teacher” to refer to teachers who “study the mathematical constructions of students and who interact with students in a learning space whose design is based, at least in part, on a working knowledge of students’ mathematics” (p. 148). In our work as teachers of mathematics, we listen, and suspend doubt, trusting that the students will share ideas that we have never heard or seen. We have a deep appreciation for the “unintended consequences” of our teaching of mathematics and care about its impact on our students (Schön, 1992). While we design our practice to enable the emergence and exploration of ideas, it is never quite possible to anticipate exactly what ideas might emerge from a particular activity. This makes our teaching exciting and intellectually stimulating. Our students’ activity is a source of inspiration, often leading us to the reconstruction and elaboration of our mathematics.

One example of the inspiration students have provided us is drawn from work with them in the area of place value. What appeared to be simple questions on the surface, such as how many hundredths there are in ten ones, became investigations that led them to make generalizations. Our readings about units and mental activity associated with units (Steffe, 1994) came to life as a result of the activities of our students. The living model (Steffe & D’Ambrosio, 1995) of mathematics we constructed from our students’ activity, provoked more complexity in our understanding of place value and in our views of how students come to reason with and about place value. From our students’ mathematical work we began to understand, in a deeper way, how students make sense of different place value units and their relationships, how and when they use the additive nature of decimal representations to support their reasoning, and the ways in which students connect different representations and contexts when dealing with place value. These understandings allowed us to develop a framework that enabled us to untangle the complexity of understanding place value, articulated...
Insights from our students

Dan and Sheila were two students in our teacher education program. These two cases elucidate instances in which we were unable to give reason to prospective teachers as they described their interactions with children, reflecting their views of effective teaching. Dan and Sheila’s words should have allowed us to infer their teacher selves. Instead of suspending doubt and understanding their teacher selves as developing and changing through the experiences in our courses (i.e., as work in progress), we were quick to evaluate their work as indicative of inadequate and ineffective teaching practices. The disappointment we felt suggested that we had been ineffective in helping them build teacher selves that resembled our teacher selves. We were striving to see practices aligned with a constructivist view of learning, yet our disposition to see things from the learners’ perspective, that is, to give reason to the learner, was lacking.

Dan

Our analysis of Dan’s learning about and enacting of good teaching underwent several iterations. It was only recently that we realized the inadequacy of our expectation that Dan accept the benefits of being and performing as a constructivist teacher. Initially our conversations were based on an analysis of Dan’s artifacts produced in class and for the program. His mathematical work revealed high levels of performance in rather standard ways. He had been a successful mathematics student and remembered the procedures for solving most types of elementary school problems. We believed that his ease with procedures and reluctance to use anything other than school-learned procedures seemed to be a hindrance to his opportunities to make sense of the mathematics of children. When Dan interviewed a child to make sense of the child’s understanding of place value, our assessment was highly critical. Dan created a worksheet for the interview (see Figure 1).

With the response to the first few questions it seemed clear to us that the child’s responses needed probing if anything was to be uncovered about his knowledge. Dan’s strategy was to encourage the child to answer all the questions on the worksheet. In his description of the child’s understanding, Dan garners information from right versus wrong answers and writes: “The child struggles with this

![Figure 1. Dan’s worksheet to assess a child’s understanding of place value.](image-url)
exercise, getting none of the ten correct.” To Dan, wrong or unexpected answers were indicative of lack of understanding on the part of the child. Our stance was critical of Dan’s views of teaching mathematics and Dan’s ways of accessing student understanding. We were unable to empathize with Dan and give reason to his actions. We were unable to enact our own views and theories of constructivist teaching. Through Dan we were living our contradictions. We were trying to teach our students to be hermeneutic listeners (Davis, 1997; D’Ambrosio, 2004) of children’s mathematics, but we were unable to be that listener with our own students as they struggled to communicate their “teacher selves”.

Dan’s views of teaching and learning were shaped by years of personal success with a traditional model of learning, yet this understanding was elusive to us. Like many prospective teachers, Dan’s experiences in the teacher education program presented many contradictions to his beliefs about good teaching and the images that he had created about his future role as a teacher. These views constituted Dan’s emerging identity as a teacher. His inner voice was louder than the voices of his instructors or authors of the readings he encountered.

Our models of our prospective teachers, and of Dan in particular, failed to include our understanding of each learner’s views of good teaching. The opportunities that we created for learning about the teaching of mathematics were based on the goal of growing prospective teachers who embraced constructivist teaching as the practice they would use as teachers. This goal, in and of itself, was a contradiction with our constructivist teacher selves. The opportunities for prospective teachers to consider and understand the views they held of effective practice would have required experiences that we had been unable to create. Prospective teachers willing to embrace a view of learning as construction of knowledge were successful in our classes. Those who resisted tended not to do so well. Several prospective teachers learned the game of school and were able to create a “teacher self” worthy of our praise, even if that “teacher self” was only an imaginary performance used by the actors in our classes. The alignment of the acting and reality was never in question for us.

Although we were aware of students “masking” their lack of understanding of mathematics, we were less aware of their “masking” their true inner “teacher selves.” The tasks we had created that exposed students’ mathematical selves did not evolve into parallel tasks that could expose prospective teachers’ “teacher selves.” Our deeper understanding of our students’ “teacher selves” would have allowed us to create much more complex models of prospective teachers as potential teachers of mathematics. Only with such models is one able to create the learning space that will generate opportunities for prospective teachers to confront and understand their ways of operating, thus creating new and more robust understandings of the complexities of teaching.

Initially we discussed how Dan had failed to learn; now we discuss how we failed to challenge Dan to consider different approaches to teaching than those that were already part of his identity as a teacher. Our evaluative stance has taken a turn away from Dan and towards ourselves and our practices in preparing teachers.

Sheila

We were very excited to meet Sheila. Her work in one of our mathematics content courses reflected a commitment to reason about the problems we posed. She was one of the students who always went above and beyond the course expectations in order to gain insight into the material. For example, when we began problem-solving she decided to read Polya’s (1945) account and integrated her understanding of the reading into her analysis of her own problem-solving. She found and read papers mentioned in class and did the suggested exercises.

Following the conclusion of the mathematics course, we invited Sheila to join us in several activities. Sheila agreed to be interviewed about her understanding of place value during a graduate class that we taught. The interview provided an opportunity for doctoral students to observe the clinical interview method and gain insight into the challenge of understanding someone else’s mathematics. Sheila also joined our work at a local school. She worked directly with several students as they tried to partition a pan of brownies.

After the episode, Sheila prepared notes of her observations of one child’s mathematics. Her notes were a narrative of her work with the child. She thoroughly described what the child did and what she asked the child to do. Sheila also shared notes from her tutoring sessions with children. Again, these focused on capturing images and actions of the child she was working with and her own associated actions.

In our view, our aims in the mathematics content course were fulfilled. Sheila was not only thinking conceptually about mathematics, she was also thinking about and working toward engaging children in thinking mathematically. For us, Sheila’s work signified progress toward becoming a constructivist teacher. Sheila gave reason to each of the children with whom she worked. She attended to the child and could recall the child’s physical movements as well as her own. She had the disposition to save students’ work. For us this meant that Sheila was honoring the children she was working with. We recognized ourselves in Sheila’s work and, as a result, felt quite proud that the activities in our mathematics classes were useful and effective. Sheila “got it” and we were proud of her and proud of ourselves!

Later, we began to see the unintended consequences of our actions. As Sheila moved toward the end of the program, she engaged in several performance assessments as part of her program of study. One of these assessments involved place value. Sheila planned to interview a child to explore their thinking about the standard algorithm for addition. She asked for help to build her interview protocol. She had piloted the interview protocol and made lists of different levels of understanding of place value that might be demonstrated using her tasks. Later, Sheila shared her final report with us, in which she shifted to asking the eleven-year-old student questions involving packages of tens and hundreds. To explore ideas about decimals, Sheila asked the child to read numbers and identify different place values associated with different digits. Reading Sheila’s report, we began to worry. Sheila seemed paralyzed by the curriculum associated with her interviewee’s grade in school, rather than driven by the goal of modeling the mathematics of the
child. It occurred to us that Sheila was an excellent student, but that perhaps we had not been such good teachers. Sheila’s document included her reflection on what she had learned about place value. The interview, and the child’s mathematics, had not added any noticings or wonders for Sheila regarding her understanding of place value. Her statement “I have a strong understanding of place value, so my ideas about place value have not changed,” led us to wonder whether we had ever really considered a model of Sheila as a developing teacher.

During student teaching, Sheila shared more evidence of her ideas. She struggled to implement a lesson from the district-adopted curriculum. In one lesson, children in front of the camera had their heads down on the desks and very few responded to Sheila’s questions. Generally, Sheila shared that she struggled to implement the curriculum that seemed to be well above the level of the children in the classroom. We suggested that she try a lesson that felt more comfortable to her. Sheila drew from another curricular resource in the area of fractions. During the lesson, the children worked to find how much of a submarine sandwich each of four children would get if they were sharing three sandwiches. Sheila instructed a child sharing his answer of $\frac{3}{4}$ to write “what is the whole” on his paper. When he became confused, she took his pencil and wrote the question on his paper. Throughout the lesson, Sheila struggled to keep the discussion focused on her question “what is the whole?” The lesson concluded with one group of children sharing their solution and process. Sheila asked who agreed or disagreed with the finding of the group. No students responded. Our observations of Sheila enacting her views of teaching revealed that she was teaching a curriculum rather than teaching the children. Seeing Sheila struggle to get through the lesson shamed us.

We began to see ourselves as failing Sheila. How did we think of Sheila and her efforts to become a teacher? How did we use a model of Sheila to construct challenges for her? In retrospect, our model of Sheila was an image of ourselves. We were happy with Sheila’s work and her dispositions and included her in our conversations and social activities. Yet, now we wondered what Sheila had given up to be with us. Had Sheila always worried about engaging the children and fitting into a school culture, but left those worries unvoiced? We share this question not to identify Sheila as a victim and ourselves as villains (Walshaw, 2010). Rather, it highlights our awareness of dimensions of giving reason that we initially did not understand in the context of mathematics teacher education.

Our professional dilemmas
Lampert (1985) argues that describing and viewing “the teacher as dilemma manager accepts conflict as a continuing condition with which persons learn to cope” (p. 192). The dilemmas described are complex collections of tensions, not solvable problems; situations in which there is an imperative to act and the teacher assumes the responsibility for action. Actions taken by the teacher cannot then solve the problems posed, as these problems have many dimensions, with each action having potential risks. Managing in such situations is the work of the teacher. Berry (2007) describes dilemmas, and more specifically tensions, drawn from her analysis of learning to teach about teaching. While she, too, describes tensions as complex, interrelated and without definitive solutions, she shares an understanding of her practice and perspective as evolving through the examination and articulation of tensions. Her work to identify and describe “how tensions impact practice” (p. 133) is aligned with our perspective on what we see as our dilemmas. We seek to identify dilemmas or tensions in our practice and then move from what Berry, citing Korthagen and Kessels (1999), describes as “situation-specific” (p. 130) knowledge to propositional knowledge. To do so, we begin by identifying dilemmas and exploring the situations that we can consciously identify as containing such dilemmas. By identifying such situations, we can also begin to articulate how our dilemmas operate in the situations. This process allows us to move toward the development of knowledge of self and other that encourages us to operate in ways that are more consistent with our views of self as constructivist teacher.

The acknowledgement that there were differences in our practices in mathematics and mathematics teacher education, resulting from our lack of attention to the prospective teachers’ existing views of teaching and learning, provoked the re-examination of our practice. Comparing our orientation toward our students in mathematics classes with our orientation toward students in methods classes revealed how we manage to give reason to our students even when their mathematical ideas do not match our own. Questions about how we supported our mathematics students and the kind of activity we expected of them highlighted our expectations that they had knowledge that they would develop through interactions with us and with their peers. Their efforts to build understanding began with their thinking and were constructed as a personal project with the goal of creating further understanding. All students were expected to participate in discussions, not from the perspective of another, but from their own perspective. As students shared their perspectives and ideas the classroom community worked to support the emergence of the ideas the student was building, knowing that the interaction would benefit all of us in our own understandings of our personal projects. Members of the class co-constructed knowledge, in that each constructed knowledge through the experience, though the knowledge constructed might be different from that of a peer or a teacher. Ideas would evolve and learners would feel empowered as they created opportunities for the applications of these new ideas.

Reconsidering giving reason in mathematics teacher education
Giving reason in mathematics teacher education requires coming to terms with contradictory views of effective teaching. We struggled to give reason to others when their dispositions and actions were inconsistent with our own, as Dan’s were. Yet, even with Sheila, where we inferred that her dispositions and actions were consistent with our own, we still stumbled. The prospective teacher’s views on teaching and learning had not been used to shape a model of the prospective teacher’s existing constructions. We had to successfully move beyond a corroborator of our image of self, “I’m ok because you reflect me,” to a corroborator of our knowledge (von Glasersfeld, 1995). A corroborator of our image of self is based only on actions of another as reflective
of our own actions. A student like Sheila is taken as a corroborator, without much concern for the development of a model of her as an autonomous constructor (von Glasersfeld, 1995, p. 127). Our experience with Dan illustrates the danger of working with a model that seems to be in conflict with a view of a teacher as in control of his or her own transformation. We were unconcerned with Dan’s perspective and evaluated him as he did his students. Dan existed in our mind as the antithesis of a constructivist teacher and yet we never really invited Dan to examine and explore his ideas with us. The degree to which his thinking was so different from our own, was unexamined. We had not elicited enough information about his teacher self to be able to create a model and thus assess his needs in learning to teach. Because we never elicited information about Sheila’s teacher self, Sheila became an object for us rather than a human being with ideas and concerns relevant to teaching children mathematics. Our models were built of simple images of ourselves reflected in Sheila’s work and a deficit model of ourselves reflected in Dan’s work.

Both cases contain evidence of our difficulty in giving reason to our students with unintended consequences. Our intention was to support the development of the teachers to create a constructivist practice and yet we failed to apply the fundamental notion of humans as autonomous constructors of knowledge to these prospective teachers (von Glasersfeld, 1995). Through the creation of these cases we began to see significant differences in our work with prospective teachers. In working with our students learning mathematics, we would never believe that we could change their thinking by just talking about ideas they had not yet considered. Instead, our goal would be to build awareness of our students’ existing knowledge and differentiate and develop that knowledge through conversations and problems that challenged some existing structures while supporting others. The goals of instruction would always be to build from the existing knowledge and provide opportunities for the development of that knowledge.

Weissglass’s (1990) constructivist listening and our belief in his work, suggested to us a way to build a practice of mathematics teacher education. In particular, in constructivist listening, the listener’s work is in the service of the goals and ideas of the talker. The listener supports the talker, silently, but with empathy, while the talker explores ideas of concern or under examination in his or her own personal project. In this version of listening, the goal of the listener is to support the talker to elaborate on and build a more useful and emotionally satisfying structure.

Our consideration of how we might give reason to our prospective teachers, unearthed a critical element missing in our practices, namely the exploration and understanding of their existing knowledge. In the case of Dan, this effort would include his articulation and examination of his notion of good teaching. With encouragement, Dan might have included justifications for the use of worksheets with the children. To support him and to focus on his ideas, we would provide opportunities for him to elaborate on his description of his teacher self. It would be a goal to further clarify those ideas and help him wonder about the implications of his teacher choices. This would occur by engaging in read-

ings and considering his model of “good teaching” in light of his interpretations of the class readings. Using this method, we would be giving reason to Dan. We would honor his thinking and also gain insights that would allow us to join with him in his project to become a good teacher.

Our work with Sheila too would be affected. What might she see as her personal challenges in becoming a teacher? We would turn to her and ask her this question as a starting point to illustrate our interest in her personal project. Stepping toward her work, not to ignore it or criticize it, but to explore with her, would provide opportunities for us to learn about the journey of becoming a teacher of mathematics.

We gained insights from our analysis of teaching Dan and Sheila, as we sought to minimize or eliminate the living contradiction in our practice. While true to a constructivist view of teaching and learning when teaching mathematics, our constructivist teacher selves were challenged in our work of teaching mathematics pedagogy. We are still true to our constructivist selves, but realize that the most important aspect of constructivist teaching had been violated in our practice, that of giving reason to our students. As a result, we have focused on realigning our teacher selves and our practices in mathematics teacher education. Coming to understand the differences between our teaching of students learning mathematics and our teaching of students learning to be teachers of mathematics, allowed us to build a deeper view of our practice. As we move forward we want to embrace the creation of a learning environment that encourages contradictory narratives and personal projects involving competing understandings that differ from our own. We are encouraged by the possibilities of employing constructivist listening in which the elaboration of richer and more complex personal narratives allows for the further development and co-construction of our teacher selves along with those of our students.

Notes
[1] This article was collaboratively written and both authors contributed equally to its preparation.

References
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I need not […] be a person who cares only for myself in order to behave occasionally as though I care only for myself. Sometimes I behave this way because I have not thought through things carefully enough and because the mode of the times pushes the thoughtless in its own direction. Suppose, for example, that I am a teacher who loves mathematics. I encounter a student who is doing poorly, and I decide to have a talk with him. He tells me that he hates mathematics. Aha, I think. Here is the problem. I must help this poor boy to love mathematics, and then he will do better at it. What am I doing when I proceed in this way? I am not trying to grasp the reality of the other as a possibility for myself. I have not even asked: How would it feel to hate mathematics? Instead, I project my own reality onto the student and say, You will be just fine if only you learn to love mathematics. […] Bringing him to “love mathematics” is seen as a noble aim. And so it is, if it is held out to him as a possibility that he glimpses by observing me and others; but then I shall not be disappointed in him, or in myself, if he remains indifferent to mathematics. It is a possibility that may not be actualized. What matters to me, if I care, is that he find some reason, acceptable in his inner self, for learning the mathematics required of him or that he reject it boldly and honestly. How would it feel to hate mathematics? What reasons could I find for learning it? When I think this way, I refuse to cast about for rewards that might pull him along. He must find his rewards. I do not begin with dazzling performances designed to intrigue him or to change his attitude. I begin, as nearly as I can, with the view from his eyes: Mathematics is bleak, jumbled, scary, boring, boring … What in the world could induce me to engage in it? From that point on, we struggle together with it.