Graphing Calculators in a "Hybrid" Algebra II Classroom

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> I think basically to be good in math, good in science, all that stuff, you have to be willing to pay a certain price. It's not gonna come easy, but then I think (mathematics) teaches you a lot of life values, because a lot of things don't come easy, and I think you have to be willing to pay a certain price. I think we've gotten to the point, up until recently, where the idea of problem solving by many, many people was to do those twenty-five problems, then go home and do problems 1 to 49 the odds. Come on! You're asking a kid to go home and in a half hour do twenty problems or so, which means I expect you to do these things two minutes at a time. Well that's a mechanical thing. How much thought has there been when you're doing something like that? I mean, I'm still not good at getting away from that, but I'm getting better at it.
>
> Lee Brook (high school algebra teacher)

What can cause a teacher to make real changes in his or her teaching? At times, national recommendations are handed down that deliver a philosophy that can drive real pedagogic changes. Teachers can also take a wait-and-see attitude when such documents occur, and follow the recommendations when they become a bit more popular. But following a set of recommendations does not necessarily imply that the true spirit of the reform is captured [Reys, 1992]. If textbooks and other curricular materials that support such reform are not easily attainable, then the teacher's role in operationalizing these ideas in a classroom can become exhaustingly difficult.

Teachers can also become attracted to instructional aids and technologies that have the potential to affect real change in the classroom. The graphing calculator (CG) is an instructional and learning technology that has assisted in current mathematics education reform. The GC is becoming extremely popular in secondary and postsecondary classrooms throughout the world. For example, a glance at the program of the 1995 annual NCTM conference in Boston reveals that 41 presentation titles specifically referred to graphing calculators. The CG can affect the nature of the instructional environment and the avenues for content delivery, but, perhaps more importantly, it can also affect the nature of the mathematics that is being discussed.

However, it is clear that instructional reform initiatives are running well ahead of the data, and instructional changes are often based more on theoretical than on empirical support [Hiebert and Wearne, 1993]. Although the research base in the area of algebraic pedagogy has been growing considerably, a major focus has been on issues of student understanding, and research specific to graphing calculators is still quite sparse.

Setting

Just what do we gain by using a GC in an algebra classroom? And what do we lose? The following study illuminates some of the effects of introducing a novel pedagogical aid into a somewhat traditionally taught classroom. Information on the instructional process in two high school Honors Algebra II classrooms at a private, mid-Atlantic high school is provided. Most of the students in the courses were sophomores at the time of the study and their performance on standardized tests and their past academic performance suggest that they should be classified as above-average-to-gifted students. Some of the students in the algebra course had experience with the graphing calculator in Algebra I, but those who did not were required to purchase one. Prior use of the graphing calculator in the Algebra I course was minimal compared to its use in Algebra II.

Method

This study focuses on the instructional practices of Lee Brook (pseudonyms are used throughout), an experienced classroom teacher who has received several teaching awards. Year-long data were collected in one section of the course which contained 18 students. Periodic classroom observations were conducted throughout the school year which focused on classroom discourse, uses of the GC, the nature of instructional tasks, and instructor-questioning patterns. Three units involving linear, polynomial, and exponential functions were analyzed more closely in regard to these features. The instructor followed the traditional sequencing of topics in most Algebra II courses: a progression from linear, quadratic, polynomial, exponential, to trigonometric functions, with statistics and probability mixed in.

An initial-response-evaluation (IRE) discourse analysis [Cazden, 1986, 1988a; Mehan, 1979] was conducted on videotaped lessons of the three target units to provide detailed information on the kinds of interactions that took place in the classroom. An example of an IRE segment would be:

Teacher: So what do you think we should do now? *(Initiation)*
Student: Use the TRACE to find the zeroes. *(Response)*
Teacher: Use the TRACE, huh. OK, we'll do that. *(Evaluation)*

The types of questions asked during lessons were also examined. These included 1) *factual questions*, which were perceived to require brief responses that merely continued an existing line of discourse, 2) *procedural questions*, which were perceived to ask for aid in following one or more steps of the problem solving process, and 3) *analytic questions*, which were perceived to involve conceptual issues or to allow room for new lines of discourse to emerge. Only questions asked during the content portions of the lesson were coded, and a repeated or rephrased question was not counted. A trained independent investigator coded three lessons to assist in measures of reliability. Acceptable agreement was achieved on the identification of questions (83%), question types (93%), and IRE discourse segments (87%). After disagreements were discussed, I coded the data that was used in the analysis of the instruction.

Use of the graphing calculator

Although probability, matrices, and other mathematical topics were discussed in the course, this study focussed on functional algebraic topics. From this perspective, three main uses of the GC were noticed:

1) The use of the GC as a means of providing alternative problem solving strategies;
2) The use of the GC as an alternate means of transmitting information in support of symbolic explanations; and
3) The use of the GC to introduce functional properties and concepts, often in an investigatory fashion.

To this end, Lee Brook made extensive use of the ZOOM (which allows changes in the parameters of the viewing screen) and TRACE (which provides numerical values of coordinates of a graph) features of the GC to focus on one particular aspect of the graph, and then analyze it numerically. This approach allowed him to evaluate functions (including finding zeroes) and solve for extrema. On a more global level, the graphic images made the behavior of the function visually apparent, and the TRACE feature incorporated a numerical perspective to this visual approach. Specific functional properties and general characteristics of the graphs could then be discussed. Most of the graphical investigations performed during the lessons made extensive use of these two features of the GC in either the local or the global manner just described.

	U1#1	U1#2	UI#3	Total
Instruction With the GC	53.0	57.3	47.9	56.2
Instruction Without the GC	55.8	51.7	56.0	54.5
Total	55.0	55.0	54.8	55.2

Table 1
Percentage of instructor transmission time per lesson

The following data illustrate how Lee Brook's instructional patterns using the GC changed drastically as the course progressed. The percentage of time that the GC was used during an average class session was calculated for each of the three target units. Early in the year the GC was used sparingly, just 29% of the time during the linear unit. Observation data show that these uses normally involved providing alternative solution strategies. Usage of the GC increased to 56.7% during the polynomial unit. Moreover, despite the fact that the composition of the discourse maintained an average of approximately 50% for instructor transmission (Table 1), student initiations were far more frequent and of longer duration in this unit. Student initiations ranged from requests for clarification, questions about a problem solving process, factual questions, and conjectures. The majority of the initiations were of the first two varieties.

By the time of the exponential unit, Lee Brook was introducing topics through the use of the graphic images supplied by the GC. Symbolic manipulations were introduced after a discussion of functional properties using graphic representations. Although the amount of time that Lee Brook spent using the GC decreased sharply during the exponential unit (26.5%), its pedagogical power was still being utilized because topics were now being introduced using graphic images supplied by the GC. In other words, it was only at the end of the year that Lee Brook decided to allow the students to use the GC to investigate functional topics before providing them with information in a more traditional fashion.

But what role can the graphing calculator have in shaping the kinds of teacher-student and student-student interactions that take place during the course of a lesson? Since the students possessed their own GCs and could make use of them in their own manner, there existed great potential for an impact on the nature of the discourse in this classroom. We will now take a closer look at the instructional patterns and classroom interactions in each of the target units.

What really happened

Linear Unit. The first month of the course involved finite mathematics and general pattern finding. The linear unit began in mid-October and was the first occasion that Lee Brook significantly used the graphic features of the GC (statistical and programmable features were used earlier). He slowly incorporated the GC into the lessons, and usually only in a procedural fashion. This benefitted those students unaccustomed to using the GC, as several students made comments during the first part of the year regarding their inability to "find the right buttons." These problems soon resolved themselves through in-class explanations by Lee Brook on an individual basis and through student practice.

The graphing calculator can introduce numerical uncertainty and inexactness into the problem solving situation. Lee Brook allowed the students to discuss, among themselves, differences in their answers to a problem which required ZOOMing in on a portion of a graph or using

TRACE to make a numerical interpretation. One student commented "the window is not precise enough" upon seeing that the cursor was not laying precisely on the desired point on the graph. Lee Brook followed this with his own comments on the different limits for x and y brought about by different ZOOMing procedures, eventually commenting "you can get as much accuracy as you want, and that's good enough", a theme he repeatedly echoed throughout the year. This particular discussion seemed to have the effect of increasing student participation in the lesson, and the students began to suggest appropriate ways to analyze the given graph. The acceptance of inexactness by Lee Brook appeared to be important in the increased student participation.

Polynomial unit The unit began with a symbolic discussion of the division algorithm for polynomials, with Lee Brook using long division in arithmetic as a procedural comparison. A discussion of the Rational Root Theorem then followed. When the students were somewhat familiar with symbolic methods of attacking polynomial equations, Lee Brook introduced the GC into the unit. As we will discuss later, he noticed that his students were becoming quite comfortable with the GC at this time.

Symbolic methods were introduced first to discourage the sole use of graphic methods by his students.

Lee Brook: I'll do the (symbolic) algebraic techniques first because otherwise they'll go right away to the graphing calculator and not pay any attention to the algebraic

Field notes indicate that this was a correct assumption, and student suggestions for problem solving strategies were consistently in terms of graphic representations by the end of the unit.

	U1#1	U1#2	UI#3	Total
Instruction With the GC	.35	.27	.70	.40
Instruction Without the GC	.20	.32	.16	.21
Total	.24	.29	.30	.28

Table 2
Average number of analytic questions asked per minute

After having discussed the above topics for three days, the students were given in groups an open-ended writing task characterizing third- and fourth-degree polynomials. The students made significant use of the GC during this task, and were allowed to discuss their results during the next class period. As previously discussed, Lee Brook increased the use of the GC during the polynomial unit, but Table 1 also shows increases in the amount of transmission time when the GC was being used. In addition, the number of analytic questions asked during the periods of GC use also fell (see Table 2). Despite this, as Table 4 indicates,

	U1#1	U1#2	UI#3	Total
Instruction With the GC	1.43	1.22	.80	1.19
Instruction Without the GC	1.41	1.12	1.19	1.25
Total	1.41	1.18	1.08	1.22

Table 3
Average number of factual questions asked per minute

	U1#1	U1#2	UI#3	Total
Instruction With the GC	.43	1.13	.81	.88
Instruction Without the GC	.41	.20	.15	.27
Total	.41	.73	.34	.49

Table 4
Average number of student initiations per minute

the number of student initiations during the polynomial unit rose very sharply when the GC was incorporated into the lesson. These findings were, in part, a result of student-led discussions which followed a group activity on the characterization of quadratic and cubic polynomials (see below).

It is not uncommon for teachers to feel the need to restate student comments [Cazden, 1988b], and Lee Brook was often reluctant to let a student's explanation stand on its own. In fact, many of the evaluation discourse contributions made by the teacher were simply restatements of student comments. Although the students took a significant role in the direction of the discourse, Lee Brook continued to feel the need to reshape it in his own manner. The following excerpt is from the discussion on the characterization of polynomials:

LB: Now, the calculator is going to become an important part of what you're doing. Um, let's let Group 1 read their comments first, just on the third degree polynomial, so CH, since you have it in front of you, do you want to read it?

CH: (reads the group's work)

LB: OK, uh, a third degree equation crosses the x-axis or the coordinate plane either 1, 3 or 0 times. OK, let's start with that, is that accurate, do we all agree, MD?

MD: It can't cross 0 times because it's gotta go from infinity to infinity.

LB: So does everybody else agree to that, does your group, I mean that's a very good point, if in fact it goes from negative infinity to positive infinity, or positive infinity to negative infinity. And we talked about this idea of continuous, so it would have to make sense that someplace it would have to cross the x-axis

These restating tendencies were especially prevalent when the student's explanation revolved around the graphic images of the GC. Lee Brook wished to ensure that everyone followed along, feeling uncertain that a student explanation could stand on its own. He restated the student's comments while recreating the appropriate graphic images using the overhead projector.

LB: OK, what's your next statement that goes with that? Uh, you have 1 or 3 or 0. OK, we ruled out the 0, so what you're saying now is a third degree polynomial can cross the x-axis 1 time or 3 times, OK, LJ?

LJ: That is, um, if you're considering that when it barely touches the x-axis, then that's considered only twice

LJ is stating that this tangency case could be considered as one in which a third degree polynomial has two roots, a statement contrary to what Lee Brook had just stated

LB: OK, that's a good point, I'm glad you thought of that because that's because that's exactly what I was going to bring up. Suppose we took a polynomial function, or let's say I'm going to make up a third degree polynomial. How can I make up a third degree polynomial? I want to make one up where I know my roots ahead of time. DB?

DB: Well, . .

LB: I want to know my roots

DB: If you're going to make it up make sure each of your roots is, um, an x with degree of one

LB: OK, but, for example, give me a third degree polynomial with all my roots integers, give me an example of one, help me make it up.

DB: You'd just have x plus or minus an integer times the quantity x plus or minus an integer multiplied by .

LB: OK, . . isn't this going to be a third degree polynomial $(x - a)(x - b)(x - c)$? (writes this on the board) If a, b, c are integers then I have three solutions that are all integers, right? And I could just multiply it all out, and I'd get x cubed, whatever. Uh, I'll put it into the calculator in that form, OK, so (gets calculator out) let's go in and graph a polynomial function, let's do $(x - 1)(x - 2)(x + 1)$.

Lee Brook put this into the GC and graphed it.

LB: OK, if we graph that polynomial, here it is, and it's got three solutions. Now LJ made a comment, he considered it . . How did you say that, LJ, that was pretty good the way you said that?

LJ: I said if the graph touches the x-axis then it has two roots

LB: OK, so, could you give me, like . What would I change up here to have one that would do that?

LJ: Make the first two have the same zero

LB: OK, so make them the same, right? Change the first one and make it negative 2. So we have $(x - 2)(x - 2)(x + 1)$ and now we look at that (Lee Brook graphs this function on the GC), and what's happening here on the calculator when we look at the graph is that this point right here at 2 is just touching, it's not actually going through the x-axis, right? We call that a double root.

The discussion continued in this manner for the remainder of the period, with Lee Brook allowing a group of students to discuss the results of their explorations with the GC, allowing other students to offer additional comments or corrections, and then recapping the discussion in his own fashion. We should also note the manner in which Lee Brook used the GC to extend the comments of the students. The graphing technology allowed him to quickly change the parameters of the function under discussion, enabling him to easily illustrate the comments involving double roots brought up by LJ.

This activity, along with the overall increased use of the GC in the classroom, may have had the effect of stimulating student exploration with the GC as well as increasing the amount of student initiation in the classroom discourse. Field notes indicate that the students seemed to be exploring problems more on their own during classtime, but still to a limited degree. For example, only a few students seemed to investigate a function different from the one Lee Brook would display; but most students performed the graphical analysis in their own manner, perhaps different from the analysis used by the teacher. This is in contrast to the beginning of the year, when most of the students simply mimicked the keystrokes he used. Interviews with case study participants confirm these results. After the polynomial unit, the following dialogue with Erica, a sophomore, occurred:

DS: OK, while he does something up on the board, do you pretty much follow along (with your GC) or do you investigate on your own?

E: Both. Sometimes when I'm not too sure I just do what he does, but other times I do some other experimenting

DS: OK, can you give me an example of when you did that?

E: Well, like the ranges, sometimes he just has these outrageous ranges and it's hard to see, so I change them to try to make it easier to see the graph

The increased willingness of the students to explore a given graph at their desk helped promote their increased access to the classroom discourse. On several occasions when Lee Brook began to explore a problem symbolically, students would remark, "Can't you just graph it?" This would be followed by other suggestions, such as "ZOOM in on the axis", while he worked the problem. As noted in the classroom excerpt, connections between the graphic and symbolic representations were also made by the students. Hence, the students' individual explorations with the GC provided avenues for entering the current mathematical discussion

This also suggests that Lee Brook's suspicions regarding student tendencies to focus on graphic solution strategies were correct. When planning strategies for the polynomial unit, he was aware of his students' increased tendencies to use the GC, so he decided to introduce polynomial functions from a symbolic perspective:

These kids have the luxury now, pretty much, the graphing calculator is going to be available to them to use . . I'll do the algebraic techniques first because otherwise they'll go right away to the graphing calculator and not pay any attention to the algebraic, and I want them to see those techniques . . One of the problems is if they just have graphing techniques and they put the numbers in and what's in the window isn't representative of what the function might be doing, they might be totally clueless. Yeah, they're liable to take a fifth-degree equation and think it's a straight line, or something like that . But when I put the graphing calculator to use I'm going to talk about: Let's draw the picture and think what can happen; What should a function like this look like? What are its possibilities? Do you in fact get a picture that fits those possibilities? And now can I just go in and find the solutions to the problem? So we'll put all of those in, and then it's basically whatever tools are the best for you at this time are the ones you're going to use

It seems that Lee Brook is recognizing a familiar dilemma, but in a novel context. Many researchers have previously worried about children routinizing symbolic manipulations without reflecting on their meaning [Brown and VanLehn, 1982; Erlwanger, 1975; Hiebert, 1992]; Lee Brook is doing precisely this, but in a technologically-enhanced graphic context.

Exponential unit Although the amount of time that the GC was in use during the exponential unit decreased sharply compared with the polynomial unit, its pedagogical importance did not Recall that during the linear unit Lee Brook introduced linear topics using symbolic representations, and that during the polynomial unit certain topics were introduced symbolically, while others were allowed to be discovered by the students using graphic representations (including the Fundamental Theorem of Algebra) During the exponential unit, Lee Brook decided to introduce exponential growth from a numerical perspective and then illustrate this behavior graphically Again this was accomplished through the use of a group problem, where the students were given the choice of receiving either a) $1 million, or b) one penny on the first of the month, two cents on the second, four cents on the third, etc. Part of the student response to this problem was developing an algorithm to add these amounts, and several of the groups used the function $f(x) = 2^x$ in this process After a classwide discussion of the solution processes, Lee Brook used the GC to graph the function $f(x) = 2^x$. Using phrases such as "See how it shoots up over here", he was able to relate the graphical images to the students' numerical problem solving experiences. One student then noted that, as x gets negative, "The graph is getting closer and closer to $y = 0$." This prompted Lee Brook to use ZOOM and TRACE in an effort to explore these behaviors further It was only after these discussions that he used the words "exponential function" and began to focus on the symbolic forms of this function class However, with the introduction of each function equation, Lee Brook immediately displayed a graphic representation on the GC, with the students following along at their desks. Discussion of the general behaviors of these graphs ensued, as well as discussion of interesting local phenomena, such as the nature of the y-intercepts. This emphasis on functional properties was common to the class discussions involving graphical images supplied by the GC. Later, specific points (e g , $x = -1, 0, 1, 2$ for the function $y = 2^x$) were used to further discuss the general growth properties of these functions. Lee Brook again related these points to the previous penny-doubling activity.

As the unit progressed, Lee Brook paid increased attention to symbolic manipulation, and the amount of GC use decreased sharply. As each manipulation was introduced, an application of it followed, such as logarithmic manipulation in an analysis of the Richter Scale. However, these applications generally focussed on local, point-wise functional properties and a discussion of individual x and y relationships. For example, the logarithmic property $\log(x) - \log(y) = \log(x/y)$ was used to find the Richter Scale value for the 1906 San Francisco earthquake This is in contrast to the analysis of global functional properties earlier in the unit, which were performed using graphic representation This attention to procedures in different representations is in line with the instructional strategy of the polynomial unit. However, the global approach to exponential functions, particularly a discussion of growth behavior, was performed prior to any symbolic manipulations within the unit.

The number of analytic questions asked by Lee Brook during the exponential unit was much higher than in the previous units (Table 2), as was the amount of student initiation during periods of GC use. As can be seen from Table 4, the students were more eager to take an active part in the classroom discourse when the GC was being used, and an analysis of the IRE patterns suggests that this was partly a result of the increase in the number of analytic questions posed by Lee Brook during these periods The number of analytic questions asked during the polynomial unit was relatively stable whether the GC was and or was not used, but the number of student initiations was higher when the GC was involved in the instruction This suggests that the GC also played a role in evoking student initiations in the mathematical discussion. In fact, over the course of the year, the students were three times as likely to initiate discussion during a lesson when the GC was incorporated into the instructional sequence, and Lee Brook was twice as likely to ask an analytic question during these times Hence, the instructional features of the GC seemed to provide an environment that allowed the teacher and his students to "feed off each other" and raise the level of the classroom discourse.

Discussion

Very few reports of the instructional process and classroom interactions in alternative mathematics instructional situations have been published This is especially true in the case of instruction incorporating graphing calculators, since very few reports of any kind have been made. The algebra curriculum that was the focus of this study was somewhat traditional, and Lee Brook seemed to modify his teaching strategies and the curriculum to accommodate the teaching strategies and topics that arose from the use of the GC What resulted was a blend of traditional and alternative approaches to the teaching of basic elementary functions.

The evolution of a "hybrid" classroom seems a natural occurrence, given the circumstances described above. This teacher was faced with the task of fitting a technology that specializes in connecting symbolic, numerical and graphical representations into a curriculum that was partly driven by a textbook that made limited use of graphical and numerical situations More importantly, he had to modify over 25 years of teaching experience to accommodate to the presence of the GC. Because he was excited about the potential of the GC, the modification of his attitude was not a problem. However, the presence of the GC caused changes in the nature of the mathematics being discussed, and changing the nature of a curriculum is a large chore for any teacher. The lack of support from the textbook made this even more difficult

The data indicate that the use of the GC was associated with higher levels of discourse in the classroom, including higher-level questioning by the instructor and more active learning behaviors by the students. The IRE discourse analysis revealed that both the number of analytic questions and student initiations increased during periods of GC use There may be several explanations for this connection. The higher-level questions used by the instructor were due in part to his perception of the topics as more complex or novel Therefore, some of the increase in the level of discourse was simply due to the increased level of the topics being discussed. In this case, the GC was only coincidentally involved

A second explanation involves the capabilities of the GC as an aid to the instructor in modifying or creating problems which were relevant to the lives of the students. The use of these problems during periods of GC use may have also played a role in the increased student initiation in the discourse In this case, the GC was facilitative because the instructor depended on the GC when creating and solving these problems.

A third explanation is that the higher levels of discourse may have resulted from the explorations taking place within a multi-representational environment The GC allowed the instructor to investigate problem situations from graphical and numerical perspectives while relating these to the symbolic form. The capabilities of the GC also made the nature of these investigations broader and more flexible than when more traditional means are used For example, when exploring the effects of increasing gas mileage on the cost effectiveness of cars, the multi-representational capabilities of the GC allowed additional aspects of the problem to be quickly analyzed in a "representationally-connected" fashion. The GC also allowed for several function parameters to be used, increasing the breadth of the investigation The ease with which the GC allowed for changes in the functional parameters helped to make these investigations efficient and smooth If this explanation is even partly appropriate, it would implicate the capabilities of the GC directly, since the instructor depended on them to carry out these activities

It seems that, even in conjunction with a somewhat traditional curriculum, the use of a graphing technology can help to stimulate student involvement in the classroom discourse, particularly in regard to the inclusion of students in the problem solving aspects of the instruction. However, evidence for such claims is just beginning to accumulate, and additional studies which document these effects are still greatly needed Reform efforts that involve changes in the nature of the content as well as in the vehicles of instruction need both strong empirical support and supporting instructional materials to facilitate the teacher in operationalizing their spirit in the classroom If not, hybrid classrooms can result

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References

Brown, J.S., & VanLehn, K. [1982]. Towards a generative theory of "bugs' In T.P Carpenter, J M Moser, and T A. Romberg (Eds.) *Addition and subtraction: A cognitive perspective* (pp 117-135) Hillsdale, NJ: Erlbaum

Cazden, C B. [1986] Classroom discourse. In M.C. Wittrock (Ed.), *Handbook of research of teaching* (pp 432-463). New York: Macmillan

Cazden, C B. [1988a] The structure of lessons. In C B Cazden *Classroom discourse: The language of teaching and learning* (pp 25-52). Portsmouth, NH: Heinemann

Cazden, C B [1988b] The teacher-talk register. In C B Cazden, *Classroom discourse: The language of teaching and learning* (pp 159-182). Portsmouth, NH: Heinemann

Erlwanger, S.H. [1975] Case studies of children's conceptions of mathematics. Part 1. *Journal of Children's Mathematical Behavior* 1(3), 157-283

Hiebert, J [1992]. Reflection and communication: Cognitive considerations in school mathematics reform. *International Journal of Educational Research*, 17(5), 439-456

Hiebert, J., & Wearne, D [1993]. Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic classrooms *American Educational Research Journal*, 30(2), 393-425

Mehan, H. [1979]. *Learning lessons: Social organization in the classroom*. Cambridge: Harvard University Press

Reys, R.E [1992] The "Standards"? We did them last fall *Arithmetic Teacher*. 39(1), 3