Experiential, Cognitive, and Anthropological Perspectives in Mathematics Education

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There is only one world
but this holds for
each of many worlds.
[Goodman, 1984, p 278]

In the first part of this paper, I summarize the claim that accounts of mathematics learning and teaching involve the coordination of analyses conducted in three non-intersecting domains of interpretation—the experiential, cognitive, and anthropological contexts. For the purposes of the current discussion, the focus in each context is restricted solely to mathematical activity. Readers interested in the relationship between mathematical activity and broader aspects of social and community life are referred to Cobb [in press]. The discussion of the three contexts then serves as a basis for the second part of the paper in which attention turns to the notion of mathematical truth This notion is taken as being practically real. We unquestioningly accept mathematical truths and believe we are making discoveries when we engage in mathematical activity. Although we can step back from our mathematical activity and speculate that mathematics is a mind-dependent human construction, the fact remains that mathematical truth and mind-independent mathematical reality are practically real when we do and talk about mathematics. In this regard, mathematical experience is distinct from philosophical reflection on that experience. In one case, mathematics is discovered and in the other it is invented. The issue is not to force a choice between discovery and invention or to argue we are mistaken when we assume that mathematics is true. Rather, it is to take mathematical experience seriously and explore the constructive activity that accomplishes our experience of mathematical truth and certainty. In short, mathematical truth is a phenomenon to be explained rather than to be denied In the final part of the paper, I try to demonstrate that this issue is of more than philosophical interest by considering two currently popular approaches to the explanation of mathematical learning in light of the analysis

Three contexts

The claim that the experiential, cognitive, and psychological contexts are non-intersecting domains of interpretation [Maturana, 1978] means that constructs used to develop interpretations in the different contexts are mutually exclusive For example, the construct of conceptual operation is relevant only in the cognitive context whereas mathematical culture is an anthropological but not a cognitive construct. The goal is to find ways of coordinating analyses developed in the various contexts.

THE EXPERIENTIAL CONTEXT

The purpose that structures the experiential context is that of attempting to infer what another's experiences might be like As we observe a child doing mathematics or talking with others about mathematics, we strive to understand what his or her mathematical world might be like In doing so, we assume that the child's activity is rational given his or her current understandings and purposes at hand. The trick is to imagine a world in which the child's activity does make sense. In making these inferences, the analyst can only draw on his or her own conceptual resources. Consequently, in attempting to understand the child's mathematics, the researcher frequently elaborates his or her own mathematics

Even within the experiential context, there is a distinction to make—between potentialities and actualities [Steiner, 1987] Sinclair [1988] speaks of potentialities when she says that

> just as for an infant a block is something you can push or put on top of something else and that makes a noise when you throw it, and also something that is not soft, not good-to-eat, not something you can put another object into, so, say, "weight" as an object of thought is no more and no less than the sum of the different operations the subject can perform when dealing with weight. Similarly, "number" as an object-of-thought is what one can do with numbers: Thus there is not one single "concept of number" but an unending series of such concepts. [p. 5]

Sinclair's analysis of weight and number as potentialities stems from her focus on knowledge as something at hand rather than on knowledge as an object of reflection that appears to be separated from human intention and purpose In contrast Hardy is quite explicit about his Platonist assumptions when he separates mathematical knowledge from the knower:

> I believe that mathematical reality lies outside us, that our function is to discover or *observe* it, and that the theorems which we prove, and which we describe grandiloquently as our "creations" are simply our notes of our observations. [1967, pp. 123-4]

For the Learning of Mathematics 9, 2 (June 1989)
FLM Publishing Association, Montreal Quebec, Canada

In a similar vein, Gödel claimed we "have something like a perception . . of the objects of set theory," as witnessed by the fact that its premises "force themselves on us as being true" [1964, p 265]. As a philosophy of mathematics, Platonism has been devastatingly critiqued, particularly by Wittgenstein [1956]. Nonetheless, *as a description of the subjective experience* of reflecting on previously made mathematical constructions, Hardy's and Gödel's accounts ring true. Once we have made a mathematical construction and have used it unproblematically, we are convinced that we have got it right—it is difficult to imagine how it could be any other way Mathematical objects are, for all intents and purposes, *practically* real for the experiencing subject [Goodman, 1986].

An analysis of potentialities, guided by the metaphor of using a tool while acting in physical reality [Polanyi, 1962], attempts to analyze unreflective knowledge in action. Platonism, in contrast, takes objectified physical reality as its guiding metaphor [Bloor, 1976] and deals with how things seem when we reflect on previously made mathematical constructions. In my view, it is necessary to use both metaphors when accounting for students' mathematical experiences. This is particularly so because students operating at the frontiers of their knowledge are in the process of making objectifications The purpose of characterizing what Thom [1973] called the development of the existence of mathematical objects is incompatible with the metaphor of externalized physical reality In effect, one needs a language to talk about what it might be like before one can talk as a Platonist about particular concepts. Attempts to infer what students' mathematical experiences might be like therefore involve inferences about both mathematical knowledge-in-action and objects of knowledge.

THE COGNITIVE CONTEXT

The purpose that structures the cognitive context is to explain how it is that students have the mathematical experiences they are inferred to have. In other words, students' inferred mathematical worlds are the data of cognitive explanation This is in line with Goodman's [1984] exhortation that we ask the hard but inevitable questions about the mental operations required to construct a world like that of modern physics or of everyday life. As Bruner [1986] noted, this characterization of the cognitive context is at odds with mainstream American psychology. Psychologists felt that they

> had to take a stand on how the mind and its mental processes transform the physical world through operations on input. The moment one abandons the idea that "the world" is there once and for all and immutably, and substitutes for it the idea that what we take as the world is itself no more nor less than a stipulation couched in a symbol system, then the shape of the discipline alters *radically* [p 105, italics added]

From the cognitive perspective that Bruner, following von Glasersfeld [1984], characterizes as *radical,* key constructs include scheme, conceptual operation, sensory-motor action, re-presentation, and reflective abstraction [Steffe, 1983]. We note in passing that the inclusion of mathematical objects in models that purport to be cognitive in fact indicates a conflation of the experiential and cognitive contexts. The Platonist experience is something that needs to be explained by asking the hard question of how a student can have the reflective experience of a mind-independent mathematical object From the radical perspective, mathematical objects are the experiential correlates of conceptual operations.

THE ANTHROPOLOGICAL CONTEXT

The purpose that structures the anthropological context is to identify and account for aspects of a culture (or microculture) by analyzing regularities and patterns that arise as, say, a teacher and students interact during mathematics instruction In this context, the teacher and students are viewed as members of a classroom community with its own unique microculture. As Eisenhart [1988] put it, the focus in the anthropological context is "on describing manifestations of the social order in schools and developing frameworks for understanding how students, through exposure to schools, come to learn their place in society" [p 101] From this perspective, "if one asks the question, where is the meaning of social concepts—in the world, in the meaner's head, or in interpersonal negotiation—one is compelled to answer that it is in the last of these three" [Bruner, 1986, p. 122]. This notion of interactional or *emergent meaning* derives from Meade's [1934] analysis of social interaction. On the one hand, we have the participants' interpretations of their own and each other's actions and, on the other hand we have the observer's analysis of their *joint activity*. The observer creates the emergent meanings while attempting to make sense of the joint activity that he or she sees when interpreting the interaction from the outside. This idea is closely related to Krummheuer's [1983] notion of working interim *(Arbeitsinterim)*. A working interim is a period when the participants' interpretations of their own and each others' actions fit together and the interaction proceeds smoothly. The observer, viewing the interaction during the working interim as a joint activity, can talk about the meanings that participants appear to share Krummheuer, like von Glasersfeld [1984], uses the term *fit* rather than *match* to stress that although the participants believe that they understand each other, they might well be ascribing different meanings to their own and each others' actions In other words, there may be differences in the meanings that each participant thinks he or she shares with the others. From the anthropological perspective, meanings are assumed to be shared [Gergen, 1985] and, from the cognitive perspective, they are assumed to be compatible [von Glasersfeld, 1984]

Thus far, we have talked about emergent meanings in general. We can also legitimately talk of emergent mathematical meanings. This and the related notion of institutionalized mathematical knowledge are of value if we wish to address the issue of how "children come to know in a few short years of schooling what it took humanity many years to construct" [Sinclair, 1988, p. 1] in an epistomologically sound way It should be noted that "institutionalized knowledge" does not refer to knowledge associated with what are typically thought of as institutions in society—schools, universities, prisons, the army, or, more generally, large well-bounded organizations with clearly delineated functions. Rather, institutionalized knowledge refers to the physical and intellectual practices that are taken-for-granted by *specific* communities of knowers. The mathematical practices that are beyond justification in one second grade classroom can, for example, differ in significant respects from those in another second grade classroom [Cobb, Yackel, & Wood, in press]. Institutionalized knowledge, as the term is used in this paper, refers to the product of the coordinated activity of members of a community.

The contention that constructs such as emergent mathematical meaning and institutionalized mathematical knowledge are relevant to explanations of mathematics learning and teaching does not imply that they can be taken as solid bedrock upon which to anchor such analyses. It is easy to subordinate individual experience to cultural knowledge by concluding that individuals internalize mind-independent cultural knowledge and that this drives their behavior. Theorists such as Comaroff [1982] and Lave [1988] propose that the relation between the mutual construction of cultural knowledge and individual experience of the lived-in world is dialectical. In this formulation, it can be argued that cultural knowledge (including mathematics) is continually recreated through the coordinated actions of the members of a community This proposed relationship between cognitive and anthropological analyses of mathematical activity is as applicable to the teacher and second graders as an intellectual community and to two or three children working together during small group problem solving as it is to society at large. Each child can be viewed as an active reorganizer of his or her personal mathematical experiences and as a member of a community or group who actively contributes to the group's continual regeneration of the taken-for-granted ways of doing mathematics From the anthropological perspective, these institutionalized mathematical practices constitute the consensual domain mutually constructed by members of the group For example, as I and my colleagues analyzed a corpus of video-recordings of second grade mathematics lessons that we have studied intensively, we (as observers) inferred that the practice of operating with units of ten and of one emerged as a taken-for-granted way of doing things. It became taken-for-granted in that a point was reached after which a child who engaged in this practice was rarely asked to justify his or her mathematical activity It was beyond justification and had emerged as a *mathematical truth* for the classroom community. To be sure, when we adopted the cognitive perspective and interviewed the children individually, it became apparent that this intellectual practice had a variety of qualitatively distinct meanings for them—their meanings were compatible rather than shared. Nonetheless, their participation in a classroom community that negotiated and institutionalized certain mathematical practices but not others profoundly influenced their individual conceptual developments. It is not just that children make their individual constructions and then check to see if they fit with those of others Children also learn mathematics as they attempt to fit their mathematical actions to the actions of others and thus contribute to the construction of consensual domains—as they participate in the process of negotiating and institutionalizing mathematical meanings [Bauersfeld, 1980; Bishop, 1985; Voigt, in press]. From this perspective, the notion of children's uncontaminated natural mathematics is a fiction. The children we observed engaged in consensually constrained mathematical activity In an attempt to coordinate contexts, we can say that the children's and teacher's mathematical activity created the institutionalized mathematical practices that constrained their individual mathematical activities. Conversely, the institutionalized mathematical practices constrained their individual activities that give rise to the institutionalized practices. Acculturation and the institutionalization of mathematical practices would therefore seem to be a necessary aspect of children's mathematics education Analyses that focus solely on individual children's construction of mathematical knowledge tell only half of a good story The issue that needs to be addressed is the form that the process of mathematical acculturation should take and how it can be coordinated with what is known about the cognitive processes by which individuals construct mathematical knowledge It is this issue that will be further explored in the second part of this paper.

Platonism revisited

My purpose in reconsidering Platonism is to suggest that progress can be made in accounting for the Platonistic experience of a mind-independent mathematical reality by coordinating the anthropological context with the experiential and cognitive contexts and using the constructs of institutionalization and negotiation Institutionalization, it will be recalled, refers to the process of mutually constructing the taken-for-granted practices that make communication possible Schutz [1962], speaking from the experiential perspective, put it this way:

> Until counterevidence, I take for granted—and assume my fellow-man does the same—that the differences in perspectives originating in our unique biographical situations are irrelevant for the purpose at hand of either of us and that he and I, that "We" assume that both of us have selected and interpreted the actually or potentially common objects and their features in an identical manner or at least in an "empirically identical" manner, i.e., one sufficient for all practical purposes. [p 12]

In Schutz's view, this *idealization* makes possible the reciprocity of perspectives essential for interpersonal communication. It is in the process of making these idealizations and finding that they work that things are experienced as objective. Intersubjectivity is "inconceivable without naive, reciprocal faith in a shared experiential world . . *Intersubjectivity must in some sense be taken for granted in order to be attained*" [Rommetveit, 1986, pp. 188-189]. Schutz's observation that in communication "it is assumed that the sector of the world taken for granted by me is also taken for granted by you, . . . even more, that it is taken by "Us" " [p. 12] is as applicable to mathematics as to any other topic of conversation. It is when we can objectify the products of our mathematical thinking and proceed unproblematicaly by tacitly assuming that others have made the same objectifications that we have intuitions of a shared, mind-independent mathematical reality and talk of mathematical truth. This is mathematical truth as an existential phenomenon.

> In practice the existence of an external-world order is never doubted. It is assumed to be the cause of our experience, and the common reference of our discourse. I shall lump all this under the name of "materialism." Often we use the word "truth" to mean just this: this is how the world stands. By this we convey and affirm this ultimate schema with which we think. [Bloor, 1976, p. 36]

Bloor's choice of the term "materialism" is appropriate. We have alrady noted that objectified physical reality is the guiding metaphor behind our intuitions of a Platonist mathematical reality.

The tendency to assume that mathematics consists of certain, time-independent truths is, in experiential terms, closely related to our experience of a mind-independent mathematical reality. After all, if we view ourselves as making discoveries about this reality, how can mathematics be other than the way we understand it? The tacit assumption that mathematics comprises ahistorical truths can be so compelling that, as Lakatos [1976] noted,

> mathematics has been the proud fortress of dogmatism. Whenever the mathematical dogmatism of the day got into a "crisis", a new version once again provided genuine rigour and ultimate foundations, thereby restoring the image of authoritative, infallible, irrefutable mathematics [p. 5]

Schutz's [1962] analysis of social reality is again helpful when we consider the issue of mathematical certainty. He contended that knowledge is experienced as being more objective and anonymous to the extent that it is assumed to be shared not only by the partner in a conversation but by everyone who is a member of a particular community. "In complete anonymization the individuals are supposed to be interchangeable" [p. 18]. Wittgenstein [1964] argued that mathematics constitutes anonymous, standardized activity par excellence.

> If you talk about *essence* [i.e., pre-existing mathematical objects], you are merely noting a convention [i.e., institutionalized mathematical practices]. But here one would like to retort: there is no greater difference than that between a proposition about the depth of the essence and one about a mere convention. But what if I reply: to the depth that we see in the essence there corresponds the *deep* need for convention. [p. 75]

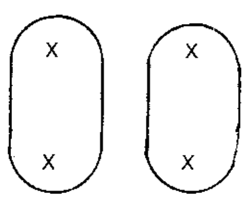
Lakatos' [1976] rational reconstruction of the historical development of Euler's theorem illustrates the deep need for institutionalized mathematical practices. "By each "revolution of rigour" proof-analysis penetrated deeper into the proofs down to the *foundational layer* of "familiar background knowledge". . . where crystal clear intuition, the rigour of the proof reigned supreme and criticism was banned" [p. 56]. As we know, the foundations were never found. This, of course, did not constitute a reason to surrender the notion of mathematical truth. In fact, in the periods of "normal mathematics" between revolutions of rigour, attention routinely shifts from whether theories are true to why they are true. For example,

> *Newton's mechanics and theory of gravitation* was put forward as a daring guess, which was ridiculed and called "occult" by Leibniz and suspected even by Newton himself. But a few decades later—in the absence of refutations—his axioms came to be taken as indubitably true. Suspicions were forgotten, critics branded "eccentric" if not "obscurantist" . . . The debate—from Kant to Poincaré—was no longer about the truth of Newtonian theory but about the nature of its certainty. [Lakatos, 1976, p. 49].

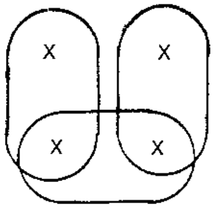
In other words, in the absence of accepted refutations, a community endows a theory that proves useful for its purposes with the aura of certainty. In practice, the issue of mathematical foundations is tangential to the processes by which a theory becomes the way mathematical reaality is until further notice.

To account for these processes and the experience of certainty, Wittgenstein [1964] contended that mathematical activities such as arithmetical calculations are grounded on certain physical and psychological processes taht, with institutionalization, become taken-for-granted. And "the more standardized the prevailing action patterns is, the more anonymous it is, . . . the less the underlying elements become analyzable" [Schutz, 1962, p. 33]. From the anthropological perspective, its certainty emerges in the course of the interactions of the members of the community who participate in the processes of negotiation and institutionalization. From the experiential perspective, it is experienced as mathematical truth.

Wittgenstein said of the following picture that an acculturated member of a community assumes that you only have to look at it to see that 2 + 2 = 4. "The claim is that we



can directly apprehend the mathematical significance of the figure without the need for accepted techniques for analyzing it, and without any agreed conventions for manipulating its parts or synthesizing the information it is meant to convey" [Bloor, 1983, p. 91]. Wittgenstein then said "I only need to look at the figure to see that 2 + 2 + 2 = 4" [1964, p 38]. We might at first reject Wittgenstein's example out of hand and brand him a crank.



> Clearly an aura, a certain feel, surrounds the characteristic patterns whcih exemplify mathematical moves . . . It is the effort and work of institutionalization that infuses a special element and sets apart certain ways of ordering, sorting, and arranging objects A theory which tries to ground mathematics in objects as such (i e., empiricism), and in no way captures or conveys the fact that some patterns are specially singled out (by members of a community) and endowed with a special status, will be oddly deficient [Bloor, 1976, p. 88].

This special aura is such that is is usually difficult to conceive of how mathematics could be any other way, and when a Wittgenstein comes along and illustrates another way we tend not to take him seriously We should perhaps reflect on the development of complex numbers, non-Euclidean geometries, non-Cantorian set theories, and non-standard arithmetics that have themselves been institutionalized by the mathematics community and become true.

The role that Wittgenstein and Bloor, speaking primarily from the anthropological perspective, attribute to social processes and to community in the development of mathematical truths is compatible with recent developments in the philosophy of mathematics In line with Wittgenstein's approach, these developments take the mathematical activity of members of communities seriously. As Tymoczko [1986a] argued, "it is this (mathematical) practice that should provide the philosophy of mathematics with its problems and the data for its solutions" [p xvi]. In Wittgenstein's case, the mathematical activity was that of the elementary school students he taught for five years when he participated in the Austrian school reform movement The philosophical psychology that guided this movement has many points of contact with the work of Jean Piaget and contemporary constructivism. As Bartley [1973] noted, Wittgenstein's "later philosophy suggests that he learned as much, and probably more, from those children than he learned from adults" [p. 85] In fact, Wittgenstein himself asked and answered, "Am I doing child psychology?—I am making a connection between the concept of teaching and the concept of meaning" [1970, p. 74].

In the case of philosophers of mathematics, the activity of interest is that of mathematicians. Even here, teaching is given an increasingly prominent role in that it is a central part of most mathematician's activity [Grabiner, 1986; Tymoczko, 1986b] Further, just as second graders learn mathematics by participating in a classroom community, so mathematicians "are able to do mathematics and to know mathematics only by participating in a mathematical community" [Tymoczko, 1986b, p 45] This is philosophizing within the anthropological context From this perspective, mathematics is a human social activity—a community project [De Millo, Lipton, & Perlis, 1986]. This does not merely mean that mathematicians talk to each other and discuss their ideas. The crucial point is that "a mathematical theory, like any other scientific theory, is a social product It is created and developed by the dialectical interplay of many minds, not just one mind" [Goodman, 1986, p. 87]. It is this social process that determines whether a theorem is both interesting and true. After enough

> transformation, enough generalization, enough use, and enough connections, the mathematical community eventually decides that the central concepts of the original theorem, now perhaps greatly changed, have an ultimate stability. If the various proofs feel right and the results are examined from enough angles, then the truth of the theorem is eventually considered established The theorem is thought to be true in the classical sense—that is, in the sense that it could be demonstrated by formal deductive logic, although for almost all theorems no such deduction ever took place or ever will
> [De Millo et al., 1986, p 273]

It is through this social process of the institutionalization of mathematical knowledge that the theorem gains the special aura of which Bloor spoke From the anthropological perspective, its unquestionable truth *emerges* in the course of social interaction. Emergent meanings are institutionalized and the theorem constitutes a firm foundation for future work until further notice. Much the same can be said with regard to the theorems-in-action constructed in the second grade classroom we observed. The primary difference between a community of mathematicians and the second graders is, of course, their standards of rigour. Like mathematical truths, these standards are themselves social products. For the second graders solving arithmetical tasks, the court of appeal of last resort appeared to be to count physical objects.

Thus far, we have concentrated on the social processes by which mathematics emerges and becomes taken-for-granted as true knowledge. These processes are not specific to mathematics but apply to any community of scholars. However, mathematics appears more certain, more true, than, say, biology or chemistry. To address this issue, we have to differentiate between mathematical and scientific ways of world-making in both the anthropological and cognitive contexts.

As a first step, the assumption that the growth of mathematical knowledge is cumulative can be questioned from the anthropological perspective Kitcher [1986] argued that

> the appearance of harmony and staightforward progress may be an artifact of the histories of

> mathematics that have so far been written Until the history of science came of age, it was easy to believe that the course of true science ever had run smooth Unfortunately, the history of mathematics is underdeveloped, even by comparison with the history of science. [p 223]

In other words, the assumptions that mathematical truth is ahistorical and that the growth of mathematical knowledge is cumulative was so beyond questions to the authors of these histories that they produced portraits of mathematical developments to fit their assumptions They simply took-for-granted that new theorems are added without the need to abandon or reconceptualize old theorems. Both Kitcher [1986] and Lakatos [1976] argue that these assumptions are unwarranted. They give examples to illustrate that developments in mathematics have involved both conceptual and methodological changes and Grabiner [1986] concluded her historical analysis by contending that mathematics is the area of human activity that has the most fundamental revolutions.

The crucial difference between scientific and mathematical developments from the anthropological perspective is that we do not seem to find mathematical analogues of the discarded theories of past science. Whereas a competition between scientific theories ends in the elimination of all but one of the theories, mathematicians did not need to choose between, say, Euclidean and the various non-Euclidean geometries. Nonetheless, a conceptual revolution occurred. Euclidean geometry was originally viewed as a delineation of the structural features of physical space. When alternative geometries of comparable richness and articulation were developed, a new conceptualization of the nature of geometry was negotiated and institutionalized by the mathematics community. The various geometries could coexist because mathematicians institutionalized the interpretation that the geometries delineate the structures of different abstract spaces, none of which was taken to be physical space. In other words, the meaning of geometry that emerged excluded physical space The problem of deciding which particular geometry was most useful for coping with particular physical problems was then left to the scientific community. More generally,

> the old mathematical investigations of light, sound, and space are partitioned into explorations of the *possibilities* of theory construction (the province of the mathematician) and determinations of correct theory (the province of the natural scientist). This division of labor accounts for the fact that mathematics often resolves threats of competition by reinterpretation, thus giving a greater impression of cumulative development than natural science. [Kitcher, 1986, p 225]

Kitcher's claim that mathematics is concerned with the possibilities of theory construction reminds us of Piaget's [1971, 1980] basic premise that mathematics is a conceptual creation constructed by reflective abstraction from sensory-motor and conceptual activity. In experiential terms, mathematical objects are experienced as being practically real

The anthropological distinction between mathematical ways of world-making and scientific ways of world-making does not imply that mathematics as an activity is divorced from the world of practical activity In the second grade classroom we observed, for example, the children frequently solved arithmetical problems by counting available manipulative materials by tens or by ones. The crucial point is that when children explained and justified their solutions, they described or demonstrated how they counted the objects. They did not talk about physical properties of the objects such as their color. This was a mathematical context for the children as members of the classroom community and they focused on their sensory-motor actions on the objects. As part of the taken-for-granted bakground of this mutually constructed context, the children viewed the objects as things to be counted. The meaning of the objects as arithmetical units had emerged in the course of classroom interactions Further, from the cognitive perspective, the distinction between mathematical and scientific ways of world-making does not imply that mathematical activity is separated from sensory-motor activity. In fact, the relationship between the two is continually emphasized in constructivist theories of cognition. Finally, the claim that mathematical ways of world-making are characterized by reflective abstraction does not imply that mathematical activity consists of a distinct set of procedures or techniques divorced from the remainder of a person's activities As the above example of the second graders illustrates, it is the way that interactions with others and with one's physical world are interpreted that makes them mathematical Thus, on the one hand, mathematics is open in that anomalies become apparent when we reflect on conceptual re-presentations with sensory-motor content and discuss mathematical ideas with others On the other hand, it is self-referential in that its anomalies, while often social and quasi-empirical in origin, are conceptual in nature This self-referential aspect of mathematics contributes to its apparent absolute certainty when compared with scientific knowledge.

The above arguments concerning the nature of mathematical truth and certainty can be summarized by the contention that it is as if the effort a community puts into sustaining certain mathematical practices returns to the community's members in the experiential form of objective, mind-independent mathematical structures This view is compatible with Peirce's [1935] claim that the "very origin of the conception of reality (including mathematical reality) shows that this conception involves the notion of a community" [p 186] and with the comedienne Lily Tomlin's suggestion that "reality is a collective hunch." It is this notion of community that is absent in both Platonism and empiricism. Two implications for mathematics education follow. First, if we view Platonism and mathematical truth as experiential aspects of consensually constrained mathematical activity, then, as a constructivist mathematics educator, I want students to experience intuitions of a mind-independent mathematical reality and to experience the discovery of relationships that they believe were there all along This is a crucial aspect of mathematical experience [Davis & Hersh, 1981]. If students do not act as Platonists when they do mathematics they are left with nothing but empty formalisms It is not the Platonist experience of mathematical objects but formalism that is the foe of all who value meaning over rigour

Second, if we are serious about encouraging students to be mathematical meaning-makers, we should view the teacher and students as constituting an intellectual community. The classroom setting should be designed as much as possible to allow students to do their own negotiating and institutionalizing—in short, their own truth-making. This approach contrasts sharply with traditional instruction in which students are presented with codified, academic formalisms that, to the initiated, signify communally-sanctioned truths that have been institutionalized by others

Internalization and institutionalized knowledge

Throughout this paper I have argued that attempts to make sense of the complex of processes that constitute the learning and teaching of mathematics involve the coordination of analyses developed in a variety of different contexts. This idea and the discussion of mathematical truth and certainty allow us to consider two currently fashionable genres of explanations of mathematical learning in instructional settings in more detail. Both genres posit a process of internalization as a primary learning mechanism. The first concerns the use of instructional representations whereas the second focuses on the relationship between social and cognitive processes.

INSTRUCTIONAL REPRESENTATIONS

Certain empiricist variants of information-processing psychology have yielded analyses of the cognitive processes and information structures said to be involved in mathematical understanding Proponents argue that, in contrast to other approaches to cognitive analysis, the computer-simulation approach provides "much more definite and specific hypotheses about the patterns of information that students need to recognize in the (problem) texts and about the cognitive processes involved in that recognition" [Greeno, 1987, p. 69]. This work is thought to be relevant to the challenge of developing instruction that aims "to place learners in situations where the constructions that they naturally and inevitably make as they try to make sense of their worlds are correct as well as sensible ones" [Resnick, 1983, pp. 30-31]. In particular, the cognitive analyses can be used to guide the development of instructional materials that "present explicit representations of the information patterns that students need to recognize in (say) word problems" [Greeno, 1987, p. 69] As Resnick [1983] noted, these instructional representations are supposed to "be "transparent" to the learner (i e represent relationships in an easily apprehended form or decompose procedures into manageable units)" [p. 32].

The basic learning mechanism that makes this instructional approach reasonable is internalization. Mathematical relationships or, in current parlance, patterns of information, are thought to be internalized from concrete materials such as base ten blocks [Resnick & Omanson, 1987], from pictures and diagrams [Tamburino, 1982], and from computer graphics [Shallin & Bee, 1985] At first blush, this approach might seem intuitively reasonable. After all, we can see place value numeration embodied in a set of base ten blocks and relationships between numerical quantities embodied in Tamburino's [1982] diagrams In doing so, we look at the blocks or diagrams as acculturated members of a particular community And as long as we take our acculturated ways of interpreting for granted it is difficult to imagine how anyone could see anything other than the true, correct mathematical objects and relationships that we see Clearly, they are there in the instructional representations in an easily apprehendable form.

This position has been critiqued within the cognitive context elsewhere [Cobb, 1987] From the anthropological perspective, the self-evident nature of the internalization hypothesis becomes problematic as soon as we become aware that our ways of interpreting are the product of our own acculturation. As we have noted, both Wittgenstein [1976] and Bloor [1976] argued that we see certain things and not others embodied in objects and diagrams because we have grown into a culture that has institutionalized these ways of seeing and not others. Fischbein [1987] made essentially the same point when he observed that the productive use of diagrams in mathematics involves the establishment of a number of conventions which are implicit in the meanings of the figures used. "Diagrams belong to the "symbolic mode" (in Bruner's terminology)" [p 158]. We are usually oblivious to these conventional suppositions implicit in our interpretations and assume that our conventional way of seeing instructional representations is the only possible way precisely because we have grown into a mathematical culture. The notion that the mathematics we see in the world exists independently of both our own cognizing activity and institutionalized ways of knowing then appears self-evident And if this is how the world stands, how else could we come to know the truths and certainties of mathematics other than by a process of internalization? We end up with a view of ourselves and of mathematics students as environmentally driven systems and with environmental contingency theories of education [Kohlberg & Mayer, 1972]

The alternative view of mathematical truth and certainty discussed in this paper calls into question the subordination of the individual to institutionalized mathematical knowledge The relationship between the individual's mathematical cognitions and institutionalized ways of knowing is instead seen as dialectical The admonition to develop instructional representations that cause students to make correct constructions is then rejected in favor of a focus on processes such as the negotiation and institutionalization of meaning. This does not rule out the use of manipulatives, diagrams, and graphics in mathematics instruction. In fact, they would appear to play an essential role in helping students construct intuitive re-presentations that make the abstract comprehensible The crucial point is that the meaning of these instructional materials has to be negotiated by the teacher and students. In effect, the teacher has to initiate the students into the interpretive stance he or she takes with regard to the materials. This, of course, is what good teachers do without thinking about it—they simply take the necessity of negotiating interpretations with their students for granted. The so-called instructional representations can then be seen as essential aspects of settings in which to negotiate mathematical meanings. The students acquire mathematical knowledge not by internalizing it from the representations but by reorganizing and elaborating their interpretations in the course of the negotiation process. Materials typically characterized as instructional representations are of value to the extent that they facilitate the negotiation of mathematical meanings and thus individual students' construction of mathematical knowledge In this regard, the notion of mathematics instruction as a delivery system is displaced by a concern for emerging systems of meaning.

INTERPSYCHOLOGICAL AND INTRAPSYCHOLOGICAL PROCESSES

The instructional representation approach remains plausible only if we fail to consider the anthropological perspective and ignore the social settings within which students actually learn mathematics. It is an approach that seeks to reconcile the practically real truths and certainties of mathematics with a focus on solo, isolated learners In contrast, the second genre of explanation stresses the important role that social interaction plays in learning. Nonetheless, there are some interesting parallels between the two approaches.

One of the most frequently quoted passages from Vygotsky's writings is his formulation of what Wertsch [1985] called the "general genetic law of cultural development" [p. 60].

> Any function in the child's cultural development appears twice, or on two planes First it appears on the social plane, and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category. . . Social relations or relations among people genetically underlie all higher (mental) functions and their relationships. [Vygotsky,1978, p 57]

In this general characterization of development,

> internalization is a process involved in the transformation of social phenomena into psychological phenomena. Consequently, Vygotsky saw social reality as playing a primary role in determining the nature of internal intrapsychological functioning [Wertsch, 1985, p 63]

The work of Newman, Griffin, and Cole [1984] provides a clear example of this hypothesized relationship between interpsychological and intrapsychological processes. The researchers provided groups of two and three fourth-graders with four beakers of colorless solutions that had been chosen so that each pair of solutions would have a distinctive reaction. The children were instructed "to find out as much as they could about the chemicals by making all the combinations of two and recording the results" [pp. 179-180] One way to complete the task is to use a procedure that is called intersection in the Piagetian literature.

> This can be understood as treating the single array (e.g four chemicals) as if there are two dimensions that intersect. Each item on one dimension is paired with the items on the other dimension in the manner of a matrix. . . . With this matrix conception, choosing pairs follows planfully from beginning to end. All the children had to do is work through the matrix. [Newman *et al*, 1984, p 178]

This is a description of an intrapsychological process and is formulated within the cognitive context. Only four of 27 children were credited with a complete run-through of the intersection procedure. Instead, "when the intersection procedure appeared, it arose in the talk among the children" [p. 183] "The intersection schema thus regulated the interaction among the children rather than just regulating the individuals' actions" [p 184]. This observation led Newman *et al* to conclude that "the intersection schema is not just or even primarily an internal knowledge structure It is also importantly locatable in the interaction among the children. It is, in Vygotsky's terminology, an "interpsychological cognitive process" [p 185]. Consequently, "a framework that has schemata moving from the interaction to the individual makes the interaction and how it changes over time the central topic of analysis" [p. 193] It is this transition from interpsychological functioning to intrapsychological functioning that was at the center of Vygotsky's research program [Wertsch, 1985]

In this approach, internalization from the social world rather than from concrete objects, diagrams, and graphics is posited as the primary mechanism of intellectual development. This approach appears to be very plausible. We can see a particular process in the social interactions between members of a group Further, this process is often constructed by individual children. It might seem obvious that the children have internalized the process from their social interactions.

Difficulties arise as soon as one notes that the interpsychological and intrapsychological schemata are theoretical constructs developed by the researcher in two different contexts. The interpsychological schema is an interaction pattern constructed by the observer in the anthropological context. Here, the group of interacting children constitutes a community with regard to the analysis. In contrast, the intrapsychological schema is a theoretical construct developed within the cognitive context. The movement that Newman *et al* speak of from social interaction to individual cognition conflates the anthropological and cognitive contexts. Symbolic interactionism [Blumer, 1969; Meade, 1934] constitutes an alternative tradition from which to analyze the relationship between psychological and social process. In the terms of this tradition, people learn in interactive settings by resolving the semiotic challenges that occur as they attempt to fit their activity to that of others and thus mutually construct a consensual domain for joint activity This contrasts with the view that people learn by internalizing constructs that researchers project into their social environment "Simply put, people act towards things (including the actions of others) on the basis of the meanings these things have for them, not on the basis of the meanings that these things have for the outside scholar" [Blumer, 1969, p. 51].

PARALLELS BETWEEN THE TWO GENRES OF EXPLANATION

Both genres of explanation view the learning of mathematics as a process of internalization. In one case, it is internalization from material or figurative entities that have mathematical significance for acculturated members of a community In the other case, it is internalization from material or figurative entities that have mathemattical significance for acculturated members of a community In the other case, it is internalization from interaction patterns that are constructed by and have significance for acculturated members of research communities. In the case of the instructional representation approach, the internalization process results in a copy of what is external to the child inside the child's head. As Bidell, Wamsart, and De Ruiter [1986] noted, this position seems to place undue reliance on the doctrine of immaculate perception For Vygotsky, in contrast, it went, "without saying that internalization transforms the (interpsychological or social) proces itself and changes its structure and functions" [1978, p. 57]. Thus, although the structure of interpsychological and intrapsychological processes are not necessarily isomorphic, there is nonetheless a process of internalization that itself needs to be explained And, as would be expected, this is precisely where work in the Vygotskian tradition has run into difficulties.

Both genres characterize mathematics students as environmentally-driven systems. In one case, the environment is composed of instructional representations. In the other case, it is a social environment composed of theoretical entities constructed by the researcher In both genres, the individual is subordinated to institutionalized mathematical knowledge—to mathematics as cultural knowledge. In one case, the subordination is mediated by interactions with instructional representations. In the other case, it is mediated by interactions with other members of a community. In one case, this subordination of the individual reflects an empiricist position. In the other case, it reflects the doctrines of dialectical materialism. Vygotsky, we should remember, contributed to the institutionalization of socio-historically specific ways of knowing that constrained his own intellectual activity. Thus, he said,

> To paraphrase the well-known position of Marx, we could say that humans' psychological nature represents the aggregate of internalized social relations that have become functions for the individual and forms the individual's structure We do not want to say that this is the meaning of Marx's position, but we see in this position the fullest expression of that towards which the history of cultural development leads us. [1981, p 164]

Vygotsky has clearly made a profound contribution to our understanding of intellectual development, not the least by alerting us to the crucial role of social interaction However, it would be naive to divorce his work from its socio-historical setting and assume that it provides ready-made answers to our socio-historically specific problems.

Complementarities

I have suggested that mathematics learning and teaching can be analyzed in three distinct contexts—the experiential, psychological, and anthropological contexts. This framework of complementary though irreducible contexts was applied to the problem of truth and certainty in mathematics The analysis involved a coordination of all three mathematical contexts. From the anthropological perspective, mathematical theorems can be seen as emergent truths that are institutionalized by the coordinated activity of members of mathematical communities. from the experiential perspective, objectivity, truth, and certainty grow out of the unquestioned belief in a shared external reality that is necessary for and is made possible by interpersonal communication From the cognitive perspective, mathematics as the paradigm case of certainty is related to reflective abstraction from activity as the primary process by which mathematical knowledge is constructed

Most attention in this chapter has been given to the anthropological context because (in blatantly realist language) it is the perspective most neglected by mathematics educators, particularly in the United States. We have severe difficulties if we restrict ourselves to the cognitive and experiential contexts even if our primary focus is on mathematics learning. There appear to be at least four equally unpalatable options. The first is to go with our subjective intuitions and accept Platonism as an explanatory theory despite the fact that it has been demolished by philosophical critiques. The second is to develop Mill's empiricism despite the blows delivered by Frege [1960] and others. This is the approach taken by contempoary information-processing psychologists who talk of developing instructional representations. The third is the neo-Vygotskian position based on dialectical materialism As we have seen, this position posits an inexplicable internalization process as a primary learning mechanism The fourth alternative is constructivism. This is a solipsistic position as long as we restrict ourselves solely to the cognitive context The most inviting way out that I see is to complement cognitive constructivism with an anthropological perspective that considers that cultural knowledge (including language and mathematics) is continually regenerated and modified by the coordinated actions of members of communities. This characterization of mathematical knowledge is, of course, compatible with findings that indicate that self-evident mathematical practices differ from one community to another [Carraher & Carraher, 1987; D'Ambrosio, 1985; Saxe, 1988]. Further, it captures the evolving nature of mathematical knowledge revealed by historical analysis [Bloor, 1976, 1983; Grabiner, 1986; Lakatos, 1976].

This position might at first seem paradoxical; mathematical meaning can be in the world (experiential), in the individual's head (cognitive), and in social interaction (anthropological) This apparent paradox is the result of one attempt to cope with an omnipresent if implicit complementarity in mathematics education theorizing. As Steiner [1987] noted, the idea of

> complementarily is well known . in mathematics education as the cause of many short-lived reform movements and "waves of fashion" that ebb and flow between the extremes of polarized positions . such as skill versus understanding. [p 48]

A complementarity is, then, an expression of the apparent paradox between seemingly opposite positions. Such paradoxes are not, of course, unique to mathematics education but pervade our everyday lives. We have hopes, dreams, and ambitions despite the fact that we know we will die (or, as Woody Allen put it, despite the fact that the universe will contract) Learning appears to involve a paradox. As we make progress and figure out solutions to our problems, we simultaneously construct new assimilatory mechanisms that are our own conceptual prisons Teaching appears to involve a paradox As Lampert [1985] put it, the dilemma of teaching "is an argument between opposing tendencies within oneself in which neither side can come out the winner. From this perspective, my job would involve maintaining the tension between . . pushing students to achieve and providing a comfortable learning environment, between covering the curriculum and attending to individual understanding" [p 183] Lampert goes on to illustrate that in practice it is a matter of repeatedly coping with this tension in concrete situations rather than of resolving the dilemma once and for all

The complementarity that seems endemic to mathematics education theorizing expresses the apparent paradox between mathematics as a personal, subjective construction and as mind-independent, objective truth Accounts of students' mathematical learning typically emphasize one extreme or the other. We seem to have a choice between individual students each constructing their lonely, isolated mathematical realities or students mysteriously apprehending pre-constructed mathematical knowledge in the world As with the complementarity implicit in teaching, we cannot resolve the problem once and for all Rather, we have to learn to cope with it in local situations by reflecting on "the underlying antagonistic relationships and mutual interactions of the two positions" [Steiner, 1987, p. 48] It is for this reason that I have discussed ways to coordinate analyses conducted in different contexts while at the same time arguing that the contexts are non-intersecting domains of interpretation. They are complementary though irreducible.

If this seems less than desirable, we can at least take heart from the observation that "hard scientists" have to cope with complementarities of their own

> The physicist flits back and forth between a world of waves and a world of particles as suits his purpose. We usually think in one world-version at a time . . but we shift from one to the other often. When we undertake to relate different versions, we introduce multiple worlds. When that becomes awkward we drop the worlds for the time being and consider only the versions We are monists, pluralists and nihilists not quite as the wind blows but as befits the context [Goodman, 1984, p. 278]

Acknowledgments

The project discussed in this paper is supported by the National Science Foundation under grant No. MDR-847-0400 All opinions expressed are, of course, solely those of the author

The author is grateful for the helpful comments made by Terry Wood, Erna Yackel. Les Steffe and Jeremy Kilpatrick on a previous draft of this paper

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But the brain is no less complicated than the world There is an immensely complex system of millions of neurons. of chemical transmitters and electrical activity It s not enough to divide the brain into areas with this area more important for *X* and that one for *Y*; we need to know how it works There is not much chance of that in neuropsychology until we have a conception of language and thought that will suggest what kind of structure one should look for Without that there will be as many alternative models of the complexities of the brain as we already have for the complexities of the world.

I ll give you an example of what I mean Some fifteen or twenty years ago a rudimentary filter theory of attention was very popular among psychologists The idea was that unattended inputs were filtered out by special peripheral mechanisms so that only attended inputs reached higher centres. When a person was attending to *visual* stimulation a sort of gate closed against impulses from the ear Given that theory of attention it seemed reasonable to look for specific filter mechanisms in the nervous system You probably know the famous experiment by Hernandez-Peon and his collaborators which seemed to demonstrate this point. They presented a cat with a series of clicks and recorded the amplitude of the click-triggered responses from the cochlear nucleus When they showed the cat a mouse the amplitude of these responses was sharply reduced; it was as if the clicks were being filtered out The experiment has been widely cited but it turns out not to be replicable; cats in other laboratories don t do this The phenomenon was due to some sort of artifact . .

In my view . there was never any chance of finding those peripheral filters Attention is not like that. It would make no sense to close gates on *any* source of information; animals should always pick up all the information they can get. Mice might make noises after all As I have argued in various places . attention is a matter of positive constructive selection, not of negative exclusion But what can a neuropsychologist do except look for the kinds of things that the prevailing psychological theory suggests?

Ulric Neisser