MATHEMATICAL ANALOGY AND METAPHORICAL INSIGHT

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My interest in this topic grows out of a long-standing, hands-on engagement with the making of metaphors. As a poet, and as an editor and reader of poetry, I have often been struck by the power of good metaphors to change my stance in the world, to alter in a profound and, it seems, permanent way how I look at things. Whence this power? And further, how is it that we are able to distinguish such 'good', world-altering, metaphors from metaphors that are merely *outré* or arcane – surprising linguistic constructions that lack, or seem to lack, genuine ontological depth?

It is difficult to provide examples without quoting whole poems or paragraphs. A metaphor is like a depth charge: if you know nothing about the material in which it is embedded, it can be difficult to evaluate its force. Some very powerful metaphors can speak to us *sans* context; but often metaphors in the shallow and mid-ranges won't yield up their full meaning on their own. As a consequence, almost any candidate for a shallow metaphor that I might offer without context can appear as a challenge to invent a context in which it would appear effective.

With this caveat, let me offer without further commentary the following: "the eyes are the windows of the soul" (a good or strong metaphor); "the table fizzed like a platypus" (a weak or shallow metaphor); "the river/Is a strong brown god" (good); "the luggage resembled/godly" (weak); "the road was a ribbon of highway, perfect for Pekinese" (weak); "If I have exhausted the justifications I have reached bedrock, and my spade is turned" (good). My interest, as I say, is in what appears to be an intuitive capacity many of us have for being *struck* by certain metaphors, and for being left cold by others. What is involved in the comprehension of good metaphors? *How* do metaphors mean? (It will be clear from the preceding that I am using the term 'metaphor' broadly to cover any linguistic expression of focussed analogical thinking. Thus what we would strictly regard as a simile is also 'metaphorical' in the sense I am concerned with.)

In reflecting on these issues, three things struck me more or less simultaneously. The first was that metaphors involve what Wittgenstein called "seeing-as", a seeing of one thing in terms of another. [2] The second was that, as a working poet, I find that understanding a metaphor *feels like* understanding certain kinds of mathematical demonstrations: I am aware of features of various figures or expressions, or various images or ideas, being pulled into revealing alignment with one another by the demonstration or the metaphor. The final observation – or, in this case, idea – was that seeing-as involves a kind of re-cognition, and, as such, is what we mean when we say we *understand* something. This last claim is essentially empirical in nature. If we ask "When do people *say* that they 'understand' or 'get' or 'see' or 'grasp' something?", it turns out that the experience of 'getting it' seems to involve a reconfiguration of an initially problematic array or scenario – a redirection of emphasis that somehow affects the overall shape of the problem. And the emergence of this new way of looking at things is often accompanied by a feeling of astonishment, or of things falling into place, of their coming home.

In sum, I began my investigations with the intuition that both metaphors and certain kinds of mathematical demonstrations are species of analogical *reasoning:* both say, in effect, "Look at things like this, if you want to understand them". But how close is the connexion between mathematical analogy and metaphor? (And here I should perhaps emphasize that my use of the phrase 'mathematical analogy' is intended to be at least as broad as my use of 'metaphor' – it embraces everything from certain visual proofs of the Pythagorean theorem to Euler's result about the sum of the reciprocals of the squares.) Mathematics clearly involves reasoning – but poetry? Aren't literary metaphors simply inventions – airy nothings, loose types of things, fond and idle names? Can understanding in mathematics really be compared with understanding in literature? These are the questions I wish to explore in what follows. First, I will offer more detailed testimony from mathematicians and poets to support the claim that there is an important correspondence between metaphors and analogies in mathematics. Then I will look briefly at two points of apparent non-correspondence. I will conclude by suggesting that the answer to our initial questions about the power and recognizability of good metaphors lies with the phenomenon of what we might call *metaphorical insight:* to grasp a good metaphor is, like understanding a fruitful mathematical analogy, to experience the significance of a newly seen alignment for what the figure, concept, or thing actually is. Or, to put this another way, a good metaphor changes the way we see the world because it is not a mere linguistic fiction, but is in some sense – a sense analogous to that which attaches to mathematical demonstration – true.

Evidence for a correspondence

Because of the third idea mentioned above – that seeing-as is at the root of our experience of *understanding* – I was led to the work of Max Wertheimer, one of the leading figures in the development of gestalt psychology. [3] There I found elegant and thoroughly researched descriptions of the phenomenon of 'getting it,' couched in terms of re-arranging 'internal' structural relations [4] – in effect, re-seeing an initial configuration in a different way. For example:

> In this square with a parallelogram strip across it (Figure 1) the lines a and b are given. Find the sum of the contents of the two areas. One can proceed thus: The area of the square is a^2, in addition that of the strip is …? But suppose that one hits upon the idea:
>
> [Sc1=] (square + strip)= (2 triangles, base a, altitude b) [=Sc2]
>
> [Sc2=] ……………… $=\left(2\,\frac{ab}{2}\right) = ab$ [= P]
>
> The solution has thus been attained, so to speak, at a single stroke. [5]

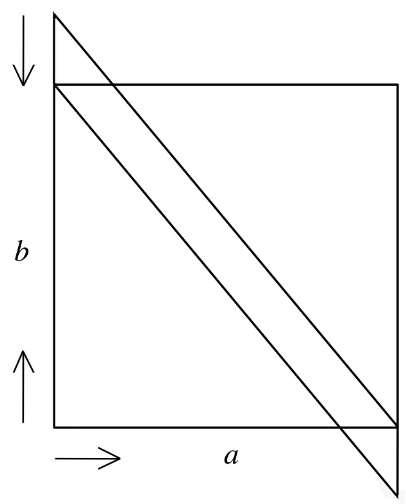

Figure 1.

Indeed, most of Wertheimer's examples were drawn from elementary geometry or arithmetic, a few from music – and none from poetry. But his summary characterization precisely captured central features of the experience of grasping a metaphor:

> In general we see that in [trying to discern whether S is [S´]], the object (S) … is given *as* [something defined by a certain set of characteristics] – but there is no direct route from S to [S´] … It frequently occurs that [seeing] the required relationship to [S´] is only *possible* when [S] has been *re*-formed, *re*-grasped, *re*-centred in a specific way. And it is not less frequently the case that to effect this process *a deeper penetration into the nature and structure of S is required.* [6]

Subsequently, in Poincaré, I found descriptions of the process of mathematical creation that appeared to echo the process of actually *making* metaphors.

> To create consists precisely in not making useless combinations …, [in choosing to study facts] which reveal to us unsuspected kinship between other facts, long known, but wrongly believed to be strangers to one another.
>
> Among chosen combinations the most fertile will often be those formed of elements drawn from domains which are far apart. Not that I mean as sufficing for invention the bringing together of objects as disparate as possible; most combinations so formed would be entirely sterile. But certain among them, very rare, are the most fruitful of all. [7]

This could easily be Robert Hass, talking about metaphor: "Metaphor, in general, lays one linguistic pattern against another. It can do so with a suddenness and force that rearrange categories of thought". [8]

But it was ultimately Kepler's remarks on analogy that seemed to me most suggestive of all:

> The geometrical voices of analogy must help us. For I love analogies most of all, my most reliable masters who know in particular all secrets of nature. We have to look at them especially in geometry, when, though by means of very absurd designations, they unify infinitely many cases in the middle between two extremes, and place the total essence of a thing splendidly before the eyes. [9]

The phrase "by means of very absurd designations" seems designed to invoke the image of metaphor, which, of course, in its strict sense proceeds *via* an 'absurd' designation. Kepler's claims that mathematical analogies "unify infinitely many cases" and "place the total essence of a thing splendidly before the eyes" both underscore the connexion with gestalt thinking [10] and are echoed in the remark of poet Anne Michaels that "metaphor unifies separate components into a complex whole, creating something greater than a sum of parts". [11] (Kepler does not say that the "total essence" that is placed "splendidly [*luculenter*] before the eyes" is greater than a sum of parts, but the rhetorical construction here suggests that he experiences it as something other than a computed arithmetical average or simple mean.) Also I believe Kepler's sense of what analogy *does* is surprisingly close to poet Jane Hirshfield's sense of what metaphor does: both are, in Hirshfield's words, "central devices for ordering the plenitude of being". [12] Finally, Kepler's suggestion that these mathematical analogies "know all secrets of nature" could be a paraphrase of poet Charles Simic's claim that, surprising though it sounds, metaphor is "the supreme way of searching for truth". [13] And in this connexion, we should note that Kepler is not alone in thinking analogy is of vital importance to discovery in mathematics. Eberhard Knobloch points out that both Bernoulli and Leibniz made similar claims. [14] More recently, George Pólya has argued that analogical thought is fundamental to both mathematical insight and pedagogy. [15]

A final point of correspondence between metaphors and mathematical analogies concerns an awareness on the part of practitioners that they can lead us astray, but a refusal to cede pride of place either to analytic description or logicist investigation. Both Kepler and Leibniz explicitly acknowledged the potential of analogies to mislead, yet they remained advocates of analogical reasoning. Pólya, though one of its most vigorous champions, notes that it is "hazardous, controversial, and provisional". [16] (Their attitude is nicely captured in Butler's observation that "though analogy is often misleading, it is the least misleading thing we have". [17]) The poet Charles Wright is speaking of a similar difficulty in literary composition when he discusses the discipline of learning to distinguish between true and false images [18]; Simic also alludes to it in discussions of the epistemology of poetic composition. [19]

Apparent points of non-correspondence

No one is seriously going to maintain that mathematical analogies and metaphors are essentially the *same thing.* There are several obvious points of non-correspondence, of which I will discuss two. My aim here is to concede differences, while arguing that the most significant among them need not damage our impression of fundamental similarity. Indeed, I wish to suggest that these apparent differences point to deeper connexions we have not considered, and that they will help us begin to formulate answers to our original questions about metaphor.

The first dissimilarity arises as a direct consequence of consideration of the last point of correspondence mentioned above. Precisely because analogical reasoning can be misleading, in mathematics one often does construct "analytic" or "logical" proofs to back up one's analogical or visual intuitions. One's reasoning remains suspect until one can produce the four-lane axiomatic deduction that leads to the same destination as the leap of gestalt imagination. There seems no comparable procedure or demand in the case of metaphor. Books of poetry do not usually come organized like Euclid's *Elements* or with appendices that parse and defend every metaphor in the body of the text. Indeed, while we can imagine providing such explanations or elaborations, they would seem to be antirequisite if the metaphor is to remain literarily pleasing or effective. In this, metaphors closely resemble jokes, as Ted Cohen has pointed out. [20]

The second point of non-correspondence focuses on the relations between metaphors and mathematical analogies and the world. Mathematical analogies are in some robust, though perhaps intuitive, sense *true,* and are perceived to be so, even by members of the general public. This is an honour rarely, if ever, accorded to metaphors except by poets themselves. Pólya, for example, suggests that poets "feel some similarity [when they compare a young woman to a flower], … but they do not contemplate analogy. In fact, they scarcely intend to leave the emotional level or reduce that comparison to something measurable or conceptually definable". [21]

I agree that most poets would resist attempts to quantify or schematize their metaphors, but I think Pólya is just wrong to suppose that poets don't "contemplate" analogy. The points of correspondence, and the cited testimony, point to a genuine concern with truth on the part of many poets. The key question, I think, is how we can best make sense of this concern – and how our intuitions about truth in mathematics can assist us in making sense of it. To this end, let me first suggest ways of accounting for the points of apparent non-correspondence in which the disanalogy with metaphor does not appear so severe. Then I will return briefly to the issue of the nature of necessary truth which is, I believe, at their root.

With respect to Apparent Disanalogy No. 1 – the absence of linear or 'analytic' proof in metaphoric contexts – it is important to reflect a moment on how analogies function in mathematical contexts. Guldin, in his discussion of Kepler, comments, "I consider [his] analogies to be useful for the invention of things more than for their demonstration". [22] That is: the analogy expresses the insight; the proof, by contrast, establishes the incontrovertibility of the insight. It is a sentiment one finds echoed in implicit and explicit forms throughout the literature on analogy and proof. Interestingly, it appears to repeat the first point of non-correspondence and then collapse it into the second: no proof, no truth. But in so doing it points once again to the deep *similarity* between mathematical analogies and metaphors. True, there is nothing corresponding to linear or algebraic proof in metaphoric, *i.e.,* literary, contexts – but this absence (and its corresponding presence in mathematical contexts) is a feature of the *context,* not of metaphor itself. And what Guldin's (and Pólya's, and, arguably, Kepler's own) view underlines is that the same holds for mathematical analogies: they are vehicles of insight, not proofs themselves.

This leads directly to a question about the nature of proof: how is it that a proof '*establishes* the incontrovertibility' of an insight? Isn't its 'incontrovertibility' precisely what makes us call something an *insight* in the first place, rather than a guess or a hypothesis? (Even when we're wrong? – And note that we can be wrong about proofs as well as analogies.) What, exactly, is a proof anyway?

As with most fundamental notions in most disciplines, the answer is unclear. I am inclined to follow Hardy:

> I have myself always thought of a mathematician as in the first instance an *observer,* a man who gazes at a distant range of mountains and notes down his observations … when he sees a peak he believes that it is there simply because he sees it. If he wishes someone else to see it, he *points to it,* either directly or through the chain of summits which led him to recognize it himself. When his pupil also sees it, the research, the argument, the *proof* is finished. The analogy is a rough one, but I am sure that it is not altogether misleading. If we were to push it to its extreme we should be led to a rather paradoxical conclusion; that there is, strictly, no such thing as mathematical proof; that we can, in the last analysis, do nothing but *point;* that proofs are what Littlewood and I call *gas,* rhetorical flourishes designed to affect psychology; pictures on the board in the lecture, devices to stimulate the imagination of pupils. [23]

But if Hardy is right – if a proof is a "rhetorical flourish" designed to get other people to see what you see – then there is a clear parallel with poetry: as proof is to mathematical analogy, so the poem is to metaphorical insight. The poem itself is a 'rhetorical flourish' that positions its reader or auditor in such a way that she or he sees what the poet saw.

An example may be helpful. Consider the claim "The human heart is a red pepper". Yeah, OK, we say: it's about the right size, is in roughly the right colour range (assuming the heart is alive or relatively fresh, and has been exposed to the air). But at first blush, this looks like one of those mid-range or even overtly shallow observations – it's not that interesting. Here, however, is its full context, a poem by Sue Sinclair [24]:

Red Pepper

Forming in globular
convolutions, as though growth
were a disease, a patient
evolution toward even greater
deformity. It emerges
from under the leaves thick
and warped as melted plastic,
its whole body apologetic:
the sun is hot.

Put your hand on it. The size
of your heart. Which may look
like this, abashed perhaps,
growing in ways you never
predicted.

It is almost painful
to touch, but you can't help
yourself. It's so familiar.
The dents. The twisted symmetry.
You can see how hard it has tried.

The analogy becomes more and more resonant as we proceed through the second and third stanzas, the final line 'explaining' the deformities observed in the first stanza and clinching the bewilderment, the almost-pain, and the odd familiarity of the second and third. (That the claim, as articulated above, is not fully explicit in the poem, is a feature of what I'd like to call the gestalt-rhetoric of much lyric poetry. An exploration of this phenomenon would take us too far afield; but it is worth noting that we also see such implicitness from time to time in mathematical proofs, as for example in Bhāskara's visual demonstration of the Pythagorean theorem. [25])

"But still", one may wish to protest, "isn't it the case that Kepler's analogies, or Pólya's analogical presentation of the Pythagorean theorem, express TRUTHS, in a way that metaphors never do? Aren't metaphors creatures of the imagination rather than delineations of reality?" Here, I think a couple of assumptions, one about literature and one about mathematics, dovetail to produce questionable prejudice. I'll address the literary side of the matter first.

Note that the phrasing of Apparent Disanalogy No. 2 includes the phrase "are perceived to be [robustly true] *even* by members of the general public". This is one clue that we are dealing with a phenomenon conditioned by, perhaps even expressive of, culturally determined levels of literacy and numeracy. My suspicion is that if, as members of either the general or expert public, we were all equally and highly literate and numerate, we would be less inclined to imagine all mathematical demonstrations were *transparently* true and more inclined to be struck by the profundity of certain metaphorical claims. The poem as 'proof' – that is, rhetorical flourish that positions its reader or auditor to see what the poet has seen – works by a subtle interplay of rhythm, assonance, denotation, and connotation, much of which our schooling does not prepare us to pick up. Just as we are rarely informed in grade school of the existence of contested proofs, or that mathematicians themselves debate the *nature* of mathematical truth.

I do not wish to deny, however, that most of us at some time or another have been struck by what I called earlier the incontrovertibility of some mathematical demonstrations. Consider the visual proof that Plato offers in *Meno* that a square double in size is built on the diagonal of a given square (Figure 2). I have taught this proof countless times, mostly to students who are less interested in lifting it off the page of Plato's prose than they are in the socio-political drama of Socrates' interrogation of a slave. But the proof, once drawn on the board, is easily grasped.

It is also elegant and very powerful. And time after time, there is someone in the class who experiences it with a physical shock – an audible gasp or an involuntary "Oh!" as the light dawns. What underlies this experience, I think, is not only 'getting' that the square double in size is built on the diagonal, but that this *has* to be the case. What impresses is not simply the claim's truth, but its *necessity*.

This – *necessary* truth – would seem to be something a metaphor cannot possess. Metaphors, we think, are creatures of linguistic play, not deductive logic; surely there is nothing *conceptually* necessary about a claim like the heart is a red pepper. But let us think hard about this for a moment. Good poems – including poems like Sinclair's "Red Pepper," which elaborates a single metaphorical insight – are

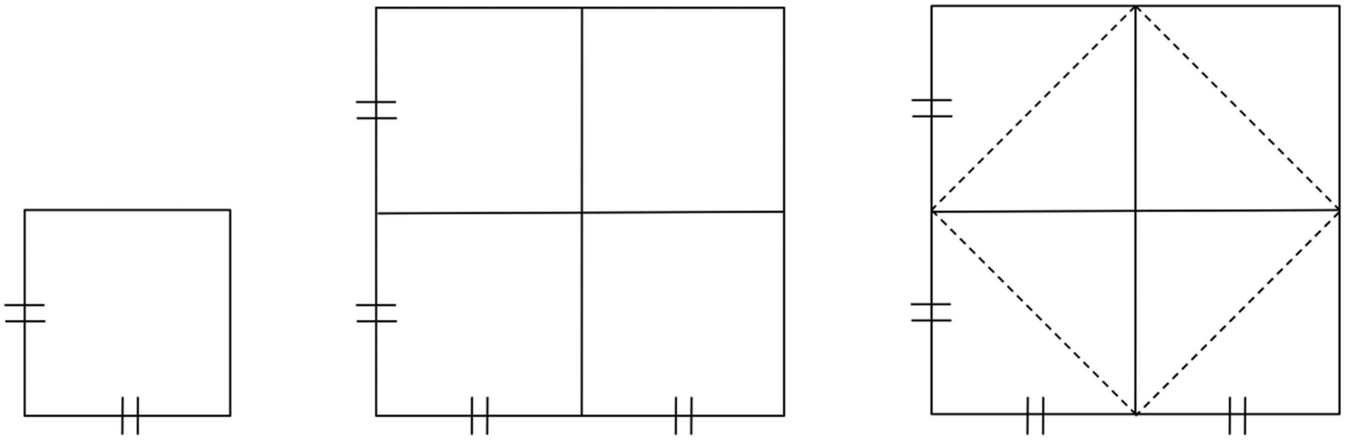

Figure 2. The square double in size is built on the diagonal of the original

notoriously difficult to teach: ask any poet or sensitive English professor. You can build a tolerable lecture around a mediocre poem, which often requires lots of external information to make it comprehensible; but one often has the sense with a good poem that everything that *can* be said *has* been said, and perfectly, in the poem itself. Either you get it or you don't. In this, it seems to me, good poems resemble the simple visual proofs we try to teach students in ancient-philosophy classes. Yes, there are some who grasp the *Meno* proof with a gasp; but there are others who don't see it the first, or even the second, time. If they don't get it, there's little I can do but say the same thing – walk through the demonstration, read the poem – again. And *when* they get the poem, grasp its central metaphorical insight, there is often an expression of astonishment just as there is with the theorem: a sudden stillness in the room, occasionally tears. These are not in all cases the *same* acknowledgements of necessity as we find in mathematics, but that is nonetheless what I believe they are. Their differences stem from the nature of the necessities compassed by the two domains: mathematics, I believe, shows us necessary truths unconstrained by time's gravity; poetry, on the other hand, articulates the necessary truths of mortality.

What the correspondence suggests

It is time now to return to our initial questions: Whence the vision-altering power of some metaphors? And how is it that we are able to distinguish between such metaphors and arresting, but mere, linguistic confections? Metaphors, I suggest, can be insightful in just the way that mathematical analogies can: they reveal to us "unsuspected kinships" between "facts long known, but wrongly believed to be strangers to one another." And, as in mathematics, "the most fertile will often be those formed of elements drawn from domains which are far apart." This does *not* mean, as Poincaré notes, that we simply bring together the most disparate objects we can think of – such a tactic can produce surprise, but it is surprise without depth, "sterile," is Poincaré's word. It means that, as in the case of mathematical analogies, metaphorical power is a product of *discernment,* to borrow again from Poincaré.

But discernment of what? I propose that the correspondence between metaphors and mathematical analogies suggests that we distinguish between profound and shallow metaphors along the lines of a mathematical distinction between important or fruitful mathematical conceptions and unimportant ones. [26] Here, it is helpful to note that even a realist like Kepler, who believes that good mathematical analogies reveal truths about the actual universe, argues that the *value* of analogy lies in the "most spacious" field of invention that it opens. [27] In other words, discernment in mathematics, and, as part of this, the development of 'true' analogies, consists in perceiving connexions that point the way to yet other connexions. The power, the value, of an analogy lies not in a definitive mapping of some territory, but, paradoxically enough, in its freeing of the imagination for further discovery. To put this yet another way: a fruitful or important analogy is one that establishes a deep field of resonance.

It might be objected that this proposal clouds the issue more than it clarifies it. The notion of a fruitful or important conception is so contested in mathematics that it cannot usefully form the basis of a parallel account of good and weak metaphors. For underlying questions about which conceptions will prove fruitful or important is a debate about the *nature* of mathematical creativity: is it a form of discovery, or a type of invention? In metaphysical terms, the question is whether mathematical entities and truths have an existence independent of the human minds that eventually discover them, or whether they are – as metaphors are often supposed to be – simply constructs of human discourse and imagination. Neither argument nor experience has yet been able to settle the issue. As Demidov notes, the experience of the working mathematician supports both claims. [28]

It is not my intention to settle the debate here. My aim is to suggest that, rather than rendering the comparison between powerful metaphors and fruitful mathematical analogies problematic, it may point yet again to a fundamental similarity, obscured yet again by literary prejudice we are disinclined to examine. For what the existence of the debate between realism and constructivism in mathematics should suggest to us, given the correspondences we have noted, is that a similar debate might be joined with respect to metaphorical insight. It may be that the world does exist independently of human activity and discourse, and that writers whose metaphors are consistently strong are not just good at manipulating language, they are good at perceiving the way that world actually *is.* Yet, as noted earlier, we tend to think it is obvious that metaphor has no purchase on what I have previously called a 'robust' conception of truth – that (to paraphrase Nietzsche) metaphor's self-conscious use marks a *liberation* of the human understanding from the stultifying effects of naïve (or even sophisticated) realism. If, however, there is reason to construe the power of metaphors along the lines of the importance or fruitfulness of mathematical conceptions, there is reason to think the realism-constructivism debate might be a live one for metaphorical insight. In that case, we must accept that we have been given grounds for a radical reconstrual of the rôle of metaphor in our lives: we must take seriously the possibility that metaphors are not invented but are perceived, and that the true ones among them limn the structure of a resonant, mind-independent universe.

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Notes

[1] This article was originally published in 2006 in *The Mathematical Intelligencer* (**28**, 5, pp. 4–9), © Springer Science+Business Media, Inc. It is republished with kind permission from Springer Science+Business Media.
[2] Wittgenstein, 1972, Pt. II, § *xi.*
[3] Ellis, 1938; Wertheimer, 1959.
[4] Wertheimer, 'Dynamics and logic of productive thinking', 1959, pp. 235–236.
[5] Wertheimer, 'The syllogism and productive thinking', in Ellis, 1938, p. 279.
[6] *Ibid.,* p. 280.
[7] Poincaré, 1929, p. 386.
[8] Hass, 'Images', 1984, p. 287.
[9] Kepler, 1604/1939, p. 92.

[10] *Cf.* Wertheimer's description of the "fundamental 'formula'" of Gestalt theory in 'Gestalt theory' in Ellis (1938).
[11] Michaels, 1995, p. 179.
[12] Hirshfield, 1997, p. 111.
[13] Simic, 'Notes on poetry and philosophy', 1990, p. 67.
[14] Knobloch, 2000.
[15] Pólya, 1954, p. *v.*
[16] *Ibid.,* p. *v*.
[17] Butler, 'On the making of music, pictures and books', 1919, p. 94.
[18] Wright, 'The narrative of the image: a correspondence with Charles Simic', 1995, p. 59.
[19] Simic, 1995, p. 72. See also Simic, 1990, p. 64, p. 78.
[20] Cohen, 1978.
[21] Pólya, 1954, Note 14, p. 13.
[22] Guldin, 1641, p. 327. Passage translated by Gordon Shrimpton.
[23] Hardy, 1929, p. 18.
[24] Reprinted by permission of the author.
[25] 1817/1973, §§146–147. [Editor's note: Colebrooke's editors follow a different orthographical convention in their Englishing of Sanskrit.]
[26] Demidov, 2000; Maddy, 2000.
[27] Kepler, 1615/1960, p. 37.
[28] Demidov, 2000, pp. 379, 382.

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