Ethnomathematics: a Dialogue

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Instead of each of us writing a paper for this special issue, we decided to have a dialogue. We met for a number of hours in January 1993 in San Antonio and engaged in a conversation about ethnomathematics. Without planning the subjects to be treated in advance, we let our ideas flow. It was a most rewarding experience. We taped the conversation and worked from the recording, trying to keep the transcript as close as possible to the original informal style of our talk together.

Ubi: I learned of your work in a curious way. Ten years ago, visiting the beautiful Library of Arts at Indiana University I saw, among the current periodicals, a journal *Visible Language*, with a paper authored by Marcia Ascher and Robert Ascher on the quipus as a form of language. I had been concerned with ethnomathematics for many years and the quipus as they were in my mind had a much narrower meaning. My vision of ethnomathematics surely went into new dimensions as a consequence of that beautiful paper. And, of course, later on I reviewed your joint book, *Code of the quipu,* which I consider a masterpiece. It was of much help in those years when I was forming my concept of ethnomathematics. Although our views on ethnomathematics are not entirely the same, I believe these views are convergent and I am sure this will emerge in these conversations

My early ideas of ethnomathematics came when I joined UNESCO in a project for a graduate center in Mali, the "Centre Pédagogique Supérieur", in 1970. There I was fortunate to meet frontier research in such areas as architecture, religion, history, language, environmental studies, and of course in the natural sciences and mathematics. These were, in a sense, my years of initiation to ethnomathematics.

Just to get started on our conversation, tell me your initiation to ethnomathematics I would like to know some background to your work with the quipus.

Marcia: The work with the quipus emerged; that is, first there was the question: Was there something mathematical in them? What could one see in artifacts? We (Bob and I; he is an anthropologist) started the quipu work just to do something together. A lot of the work was figuring out how to go about such a study. Since I had a very traditional mathematics background, I wasn't at all sure that ideas expressed on knotted strings could be "read" or could have significance.

In a sense, with artifacts, one can only deal with structure, that is formal aspects, such as internal logic, consistencies and relationships, rather than with meaning. As the work went on, I began to see that many things could be "read" from these artifacts. An important aspect of the work was that I didn't start with knowledge of the Incas—I tried to work only with the artifacts. But then, from Bob's knowledge of the culture, we were constantly amazed that the structural characteristics I was coming up with had resonance in other parts of the culture. The change that you describe definitely took place. When we began to write the book (*Code of the quipu* [1]), in order to write it, we would sit down and have very long discussions on what we both saw—Bob in Inca culture and I in the quipus—and how these linked together. The fact that I did *not* start with a knowledge of the culture was very important to our approach.

Ubi: You were looking for the structure that was there, in the quipus. All the artifacts you have examined, out of the analysis of these artifacts, you came to the perception of the cultural issues behind the manipulation of these artifacts. Trying to understand the artifacts led you to penetrate the culture of the Incas, something very difficult. How do we understand what were the modes of thought of these people five hundred years ago? These modes of thought were implicit in the artifacts. How do we see these artifacts with our perception (cognitive tools) today? Methodologically what you did was marvellous. You did not try to fit the artifacts to your perception of their culture, but you tried to understand their culture through your understanding of the artifacts.

Then after your work with the quipus, how did you work with ethnomathematics? Was it the result of your stay in the Getty Center?

Marcia: No, basically all my thinking started with the quipu work. The concluding chapter of *Code of quipu* broadens to these other ideas and discusses what is there called "implicit mathematics " I had come to believe that the issues raised were not confined to the Incas; that is, there was a lot that wasn't known, that could be known, about mathematical ideas in other cultures. And so I began looking for other substantial cases. I wrote a few articles and then the stay in California gave me the opportunity to start building on them towards a book, and also the time to devote to looking for more cases.

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Ubi: Through the work with the quipus and your further work in other cultures you were able to generate a new conception of mathematics.

Marcia: Yes, I think that is very important for us to discuss. One of the hardest problems I encountered, particularly because I began by working with a person who was not a mathematician, was that he felt I should or could state for him a definition of mathematics. But there is no clear definition of what mathematics is. The problem then becomes how to say that something is mathematical or a good illustrative case. It meant resorting to a belief that, from my training as a mathematician, I would know it when I saw it even if I couldn't say what it was. So I worried a lot about a working definition of mathematics and it kept changing a bit. It wasn't until I was satisfied with my statement that I could sit down to write the *Ethnomathematics* [2] book.

I believe that what is used as a working definition of mathematics is very important for the various aspects joined together under the umbrella of ethnomathematics. The word "mathematics" for many people is linked very closely to what they learned in school. Depending on their level of schooling, the word means quite different things to different people. Mathematicians, as such, rarely define mathematics. When they do, it is either quite broad and includes everything or it is utterly narrow. But surely, to most of them, it started with the Greeks. As a result, the word "mathematics" starts arguments of ownership—that is, who *owns* mathematics? Is it mathematics if people weren't doing proofs?, and so forth. So, to avoid those tangential arguments, I concern myself instead with *mathematical ideas*. That focuses on, and talks to, what people have in common. Those ideas have to do with number, logic, and spatial configuration and, very important, the combination or organization of those into systems and structures.

I am concerned that some of the other working definitions of mathematics that are used by the group of people who talk about ethnomathematics are much too specific to the elementary school level and, as a result, minimize the mathematical ideas that are really out there in the non-school settings.

Ubi: They start to work with something and out of this the discipline is identified. Others doing things that are not exactly what they have identified, although with the same objectives, the same purposes, and even achieving the same results, they don't recognise as mathematics

Marcia: And they don't seek it. If you say, for example, that mathematical ideas have to do with counting and then you go out and look for culturally embedded mathematical ideas, what you will find is counting. That is too small; it is much too small. Certainly mathematical ideas of number, logic, and spatial configuration organized into systems, include counting. But counting doesn't include all those wonderful things that are in mathematics and that are mathematical ideas. This is a worry to me because so many people who are inspired by the concept of ethnomathematics are involved in the elementary school mathematics level and they are, perhaps, looking for too restricted a set of ideas.

Ubi: They look at mathematics in a narrow way. We may express it differently, but there is much coincidence in our views. When I look at the etymology of "mathematics" I recognize, in *mathema* or *mathemata*, what is usually recognized as explaining, understanding, a broader sense of coping with the many aspects and challenges that reality presents. We know very little of Ancient Greek but the texts where something that looks like what we now call mathematics appear support my interpretation of the *mathema*. The intellectual venture of explaining it is the result of a challenge and generates ways of facing these challenges. When you use mathematical ideas you are broadening the merely technical way of solving a problem. It is more than the technique: here I prefer to use the Greek word *techné* that stands for both the technique and the art. This is why I see your use of the term "mathematical ideas" as very close to my interpretation of *mathema*. Clearly all this is attached to particular cultures, which suggested to me the use of the prefix *ethno*. In this way I coined the word *ethno mathema tics*! Although the origin of the term is different from yours, the idea is the same and the link is your concept of mathematical ideas.

Most people, among them mathematicians, are subordinated to the same way of interpreting and doing mathematics—to the same techné, in my language—and they fail to recognize that others are also doing mathematics.

Marcia: Yes. Let's take symmetry as an example. Symmetry is around people in the natural world. We observe symmetry, create symmetry, impose symmetry. It has aesthetic components; it may have practical components. But the idea of symmetry, the concern for symmetry, the recognition of symmetry and, of course, in mathematical theory building, the observation and seeking of symmetry or even, sometimes, the feeling that some theory may be a bit wrong it if doesn't exhibit certain symmetries—that doesn't have to do with technique. If you pin it down just to counting, measuring, and solving problems, you lose the broad concept of symmetry.

I also use the example of symmetry in order to bring in aesthetics. I believe that many mathematical ideas are totally unnecessary By unnecessary I mean that we do not need beautifully patterned mosaics on the floor in order to walk. It doesn't make walking any easier. But people put in a great deal of effort to create these beautiful floor mosaics. One, therefore, has to enlarge the idea of problem solving All human beings, all cultures, order the space in which they live. They *need* order on their space This is not problem solving in the strict sense.

I don't think that there is disagreement on this; I think, instead, it points to a need for broadening the way things are expressed.

Ubi: The idea of problem solving is too narrow If you focus on mathematics as the solution of problems you will never justify the decoration of a floor and the important branch of mathematics called symmetry Thus mathematical ideas are much more than solving problems. This is why I give so much importance, in developing my ideas of ethnomathematics, to history. I see the challenges presented in the current histories of mathematics as much broader than mere problem solving—in particular, say, the Greeks dealing with the classical problems: trisection of the angle, quadrature of the circle, and duplication of the cube. For example, there is the huge importance given by them to trisecting an angle with ruler and compass. If you really want to solve the problem of the trisection of an angle, you can easily do it, as many people did But the moment you limit yourself to the use of ruler and compass you are at another level of reflection, trying to go beyond, to transcend, the immediate needs. This is the same for several cultures that have ritual geometries For example, in Amazonian tribes color plays an important role This is much more than a matter of their immediate needs. This is why aesthetics plays an important part in raising and building up what you call mathematical ideas and I refer to as ethnomathematics

Let me elaborate on the importance given by the Greeks to the famous problems and to their solution strictly with ruler and compass: the only explanation for this is that they were interested in something else. The same with the so-called philosophical impasses of infinity and the irrational It is a big mistake to look at these out of context. We cannot separate Greek philosophy from Greek mathematics—or Greek medicine or Greek drama. Indeed, in no culture, can it be supported that a remote observer two thousand years later makes these separations.

Marcia: And, one of the—you can call it prejudices if you will—that we are brought up with as we become mathematicians is that it is really mathematics when it is for itself. On the one hand it is prejudice but, on the other hand, it is, indeed, what separates the mathematician in Greece from the carpenter. The carpenter definitely is dealing with a mathematical idea; the mathematician who set those strictures on the problem was dealing with an idea. They are *both* important, but they are *different*. And, they are *linked*.

Ubi: Yes, that's important

Marcia: Our discussion, I think, underscores the kind of broadening needed. Referring simply to problem solving can miss all these different ramifications we are elaborating.

Ubi: The problems that were philosophical impasses to the Greeks are indicators that Greek mathematics was very close to the idea of a game. These intellectual games were related to divine activities, activities which were typical of the gods. I am very much influenced by the scenario given by Herman Hesse in his "Magister Ludi" The most important intellectual activity in Castalia was the glass-beads game, which has much of the same characteristics of the mathematical games the Greeks were playing. This is related to physical games, the Olympic games, which again followed strict rules, as was appropriate for the perfection attributed to the gods. And the same with the sense of beauty which led to these physical exercises. By playing these games they were approaching the gods, both the intellectual games—mathematics, in the sense of the *mathema*—and the physical games. The big objectives of athletics were not to win the medals as a reward for fitness but to get closer to the gods. The athlete, as an athlete, and the philosopher are following the same practice, though with different drives. It's the same thing that doing mathematics was not aimed at solving daily, everyday, problems—this was done with another kind of mathematics—but to be closer to the gods.

Marcia: To me, the idea of intellectual games is a very important part of mathematics in any culture. For example, I've written about logical puzzles—the challenge of a logical puzzle and the fact that these can occur in different cultures with different stories around them. One can say that the story puzzles are being used to educate, or as amusement, but they are, in any case, logical play. As another example: among the Malekula there are paths one must trace to get to the Land of the Dead This, too, is intellectual play Not play in that it isn't serious. These are how one—to use the word you used—transcends—which I think is very apt. Not that mathematics is better than other things but it does include a *transcending* of the absolutely corporal. In different cultures that is evidenced in different ways. Going back to the mosaic on the floor—that extraordinary amount of work to make the mosaic is an intellectual

Ubi: Yes, we can use the word play in this broader sense of transcending, which is pure existence. It leads to the concepts of religion, which are clearly of the same nature as those of the arts, sciences, and everything else. In every culture you find some form of god. Every culture tries to reach above the mere earthly needs of survival. We have found in every culture these kinds of intellectual exercises, these plays and these games, as practices that allow people to approach the considerations that go beyond pure survival. This I see as the need to transcend one's existence. This is always associated with the search for explanations, for understanding and meeting the challenges, which I call the *mathema*. Mathematical ideas, in your sense, are clearly associated with this. Regrettably, the history of mathematics, and history in general, has put so much emphasis on the need of man to survive, as if survival and transcendence were separate states of human behavior. The originality of man among the other species is the association of drives towards survival and towards transcendence; man's behavior reveals both components. For example, man eats ritually, man plays ritually. Other species have eating and playing habits, but none associated with transcendence, none ritualized. Historical shortsightedness has led to philosophical dichotomies such as "to know/to do", "manual work/intellectual work", "mind/body", which are so prevalent in Western civilization. I call Western civilization that which developed, with so much cultural interchange, in the Mediterranean region. This led to a sort of ethics and social values reflected in modes of property and production—that is, in the division of labour—and the consequent economic systems.

But this kind of discussion would lead us into another strand of ethnomathematics—very ample—which I would rather leave to another discussion. The important point is that the *mathema* was responsible for some body of knowledge within a certain context, and this has not been clearly recognized in current historiography which is so impregnated with the dichotomic style. Ethnomathematics relates to life in all its aspects. Indeed, it is a description of the evolution of mankind through its diverse ramifications—that is, the civilizations, cultures, communities, families, and individuals. This calls for deeper recognition than is found in most anthropologies. It is quite interesting that the publishers of my book *Ethnomathemática* [3] had trouble knowing whether to place it as a book about mathematics or history or anthropology. I felt tempted to say it was a book about values. The ultimate goal, I think, is to propose attitudes deprived of arrogance and of bigotry.

Marcia: The important point that ethnomathematics adds, however, is that, nonetheless, expressions of mathematical ideas *do* have content and cultural context. In the past, this has been a large source of misunderstanding. An excellent example is the re-study by Sylvia Scribner [4] of Kpelle understanding of logical implications and syllogisms. An earlier study concluded lack of understanding because of unsatisfactory responses. In one instance, the Kpelle respondent would not answer when given the premises "All Kpelle men are rice farmers," "Mr. Smith is not a rice farmer," and then asked whether Mr. Smith was a Kpelle. One part of the respondent's explanation was that if you know a person and were asked about him, you could answer, but that was not the case if you didn't know him. The logic of the response was fine, but, clearly, answering the original question conflicted with Kpelle values. That is, to the Kpelle, the form ***and the content*** of the syllogism were both of importance.

Ubi: This goes back to the beginning of our conversation. People from a different culture, particularly those Westernized by such an important mode of thought as mathematics, do not recognize in Kpelle reasoning any form of logic, mainly because syllogisms are intrinsically both form and content for them. Their inferences are loaded with ethics—or, we might even say, with passion—while in the Western mode of thought passion and even ethics and rationality are mutually exclusive.

Marcia: The concern for context is also evident in the many cultures whose languages have numeral classifiers—where different endings or different number words are used depending on the category of the things that are being counted. With classifiers, one somehow still relates the number to the context so that the context is not forgotten or overlooked. Most mathematicians believe that what gives mathematics its power is the manipulation of symbols standing for anything and having no context. In a problem, x is x and nothing more. However, I believe mathematics could be even more powerful by retaining some recognition of what the symbols stand for and gearing the approaches used to that. If, for example, you are dealing with x's referring to numbers of human beings, you should only be seeking integer solutions and selecting solution methods accordingly.

Ubi: ... something that has a past, and seems more natural. The mind of a child seems more receptive to the idea of number as attached to something else. This is true too in many cultures. This has been characterized as a lack of capability for abstract thinking, which is an obvious historical perversion. Simply said, we use numbers according to what we want to design or classify with the number. The fact that we deal with numbers by insisting on their abstract meaning may reveal a distortion in our civilization, the distortion of decontextualization, essentially, which is a form of reductionism. This criticism leads to what some have called a holistic view.

Marcia: I also think that, especially in the education of children, values are being taught through our emphasis on contextless numbers. Very often school examples are phrased in terms of money. In the examples, numerical equivalents are stated for labor, for objects, for health care, for food, etc. Then the students are told to focus on and manipulate only these numerical equivalents. As such, it is more than just teaching the mathematics for itself; it is teaching the students to view the quantities and their manipulations as contextless and divorced from meaning. I think this is a very important issue for people in education to consider.

Ubi: The absolute suppression of context, and the quantification of values for comparison, in order to value something more than another, is what I consider a sign of the philosophical damage done to modern thought; it leads to a world deprived of human values, even of human feelings. Everything valuable has to be quantified.

Marcia: One of my favorite examples is from linear programming in which you are minimizing or maximizing some function under linear constraints. One early application was the creation of a diet for pigs. The constraining equations were their daily nutrient requirements and then, using current prices, the most economical diet was found. But the pigs wouldn't eat it! There are a number of similar examples where the mathematics was absolutely correct but the solutions couldn't be used because the audiences and their tastes and values were ignored during the mathematical formulation.

Ubi: This has also much to do with the problems we face with the environment. The way we look at our behavior, in general, trying to quantify it, without attaching any value—in the ethical sense—to the quantifiers, is probably the main reason why we have been so unwise in dealing with the environment.

Marcia: I also think that this is one of the causes of the dislike of mathematics among young people. There is a feeling, often articulated by those alienated by mathematics, that it is emotionless and lacks feeling. Even students who like mathematics seem to associate it with a certain inhumanity.

Ubi: We do mathematics systematically out of context. We put ourselves inside the discipline, look at a few parameters, and apply the solving techniques according to those parameters. But we know that the real situation is so complex, has such a multiplicity of parameters, that in simplifying it we inevitably limit the overall view.

Marcia: I agree that this is a very important issue. Particularly in school settings, people try to make the ideas relevant to the students' world by creating word problems or story problems. I generally find that students can more easily grasp the more theoretical topics like number theory. It is at the interface between mathematics and unreal "real" problems that the greatest confusion and oversimplification occurs. It is exceptionally difficult to take abstract mathematical statements and apply them in a real world context. It would, perhaps, help if more time was devoted to teaching about the creation of mathematical models. Then students could learn to be more critical of them and could learn to distinguish between the validity of the mathematics and the sufficiency of the model. When mathematical models were based primarily on physical theories, the omitted variables represented smaller effects and so what resulted was a first approximation. As the applications are moved into social, economic, or global settings, the problems become more difficult. There are far more variables, far less clarity on the hierarchy of their effects, and more value judgements are involved in deciding which effects to consider or ignore.

Ubi: Putting all this in the context of ethnomathematics: this is the reason I call ethnomathematics a program in the history and philosophy of mathematics. It's a program with a holistic approach, much broader than current historiography and epistemology which have clearly selected only a few variables for analysis. This program has implications for pedagogy. I think we do not disagree on this.

Marcia: I very much agree. This is why I believe there are two distinct aspects of ethnomathematics which are definitely related but sometimes have to be more clearly separated. One aspect is seeking understanding of the relationship between mathematical ideas and culture. For this understanding, more investigations, study, and research, and more thinking about these historical/philosophical issues are needed. Once there is this deeper understanding, *then* can come the educational aspect which addresses the question of how to incorporate it; how should we or how do we modify education? The two aspects share in seeking a broader view of mathematics in which mathematical ideas are not restricted to any one group, profession, culture, or historical time. But they differ in that modification of education depends largely on the goals of the educator and the setting of the education. This is not a question of research but of clarifying one's goals specifically enough to develop methods or practices that move towards those goals

Ubi: The essential point on this is that there has been a big misunderstanding of the way mathematics has developed since the 16th century. People fail to realize that mathematics then was suitable for very limited purposes, the systems appropriate to those purposes were very rough, very limited. The mathematicians of those times were dealing with things, with systems, that could be understood with a very rough degree of approximation. Their perception of the world was very narrow. It was our big mistake to build up a curriculum based on this kind of mathematics. This mathematics was important in the 16th and 17th centuries when it was essential for that world, with many limitations in practices and in knowledge. To focus, to reduce down, and to center the educational system on this narrow perception of the world was a mistake.

Marcia: But it is also a narrow perception of mathematics because the scope of mathematics has broadened and mathematics changes. In many ways, much of the educational system is left with 17th/18th century mathematics. For example, there have been many changes incorporating probabilistic thinking. I am shocked when I talk with college students who have never heard of quantum theory. They still have a Newtonian view of the world

Ubi: There has been an evolution in mathematical thinking, an evolution that has incorporated all that resulted from the technology that was developed from those same theories—mainly the capability of making observations. We have to take into consideration the fact that much of Newtonian science, and the mathematics developed to deal with it, was the result of a sophisticated scheme of observation, experimentation, and mathematization with rudimentary tools. Of course these tools improved over the following centuries, but the school system insisted on almost ignoring that fact. We are now able to see things that were impossible to perceive before, and we can create experimental conditions that were unthinkable before. And the same goes for the treatment of the information obtained. There is much obsolescence in schools. There is a gulf between the modern world and the school world, particularly in mathematics. And probably this is the basic factor causing the disaster in current educational systems.

Marcia: And there are the other aspects we were talking about earlier that have been greatly downplayed because of the extreme rationalistic approach. For example, in the non-Western math course that I teach, one of the topics is magic squares. I talk about all of the time and devotion that was put into constructing the squares by people in diverse cultures and that, when they did this, it was to order the universe or with some other religious motivation and feeling. Students are very surprised that such feelings are involved because most things in school are presented as rational—and the emphasis is that rational is better. In class, we often get into a discussion as to whether they believe in magic. They immediately say "No!" because in college you are not allowed to admit this; you are supposed to be scientific. But then there emerge examples of lucky pencils for exams, special objects or words associated with certain sports events, and so on. All of this is a big problem *for* mathematics because so many people point to it as the ultimate in rationality. people in other fields, for example, show surprise when mathematicians put less credence than they do in judgments based on numerical evaluative schemes.

Ubi: Yes, we know the limitations.

Marcia: Instead they think we are the advocates.

Ubi: I think the richness of ethnomathematics, and the reason for the acceptance of it, is that we are looking at mathematics in this much broader way.

Marcia: And also because we are adding the idea of cultural variety. There is no question that no culture has remained pure or unchanged. The students are more diverse and many students are part this and part that—a wonderful, beautiful mixture. The more we ignore the variety of cultures, the less the students will understand what we are talking about. And, here I use the word "culture" in a very broad sense

Ubi: We should address another area in which I think we have agreement. Ethnomathematics in schools is not an attempt to replace mathematics. Our observations apply to other forms of knowledge too. This misunderstanding is a danger we face.

Marcia: I, too, am quite concerned when I realize that some people think ethnomathematics is trying to replace mathematics. I have heard comments saying, for example, that students coming from traditional cultures could excel in modern mathematics but that ethnomathematics was trying to bring them down to a lower level. This is part of why, in my work, I stress broadening the history—the global and ongoing history. On several occasions I have used the example of graph theory. When most students learn about graph theory, they are told about the Konigsberg bridge problem. That is an excellent opportunity to tell them instead, or in addition, about the Malekula who posed essentially the same problem in trying to get to the Land of the Dead. In part, it is like Claudia Zaslavsky's goal of bringing the world into the classroom, but, more particularly, it is to tie into humanity. This is a question that *human beings* have been interested in—not just some mathematician or some people in Prussia in 1730. In my view, it makes the question more wondrous because it is the same question. Or, as in my earlier example about magic squares; it is that people do these things for intellectual reasons, emotional reasons, aesthetic reasons, religious reasons. It broadens the realm of the possible for students. As a byproduct, it also provides a different way to introduce students to mathematical ideas even if they were put off by the algebraic/symbol manipulation aspects of school mathematics. The point you raise is true and very serious. How do you think this danger can or should be dealt with?

Ubi: We shouldn't try to define ethnomathematics—or mathematics. Your use of the concept of mathematical ideas and my use of *mathema*, which have such an affinity, recall man's efforts to transcend his existence through explaining and understanding what is going on, looking for new ways of coping with what reality is imposing on him, but at the same time going a step beyond the mere solution of the problem. This can restore to the mathematics in every cultural system—that is, to ethnomathematics—its breadth: it goes beyond the solution of the problem, it restores the higher dimension of an intellectual exercise.

Marcia: But at the same time as we value the aspect of mathematics that transcends daily problem solving, we should recognize and value the daily aspects. There are many groups of people who engage in mathematical thinking but who express it in their specific ways. I include them when I refer to the need for more studies in ethnomathematics. As you frequently have mentioned, there are, for example, people in particular fields of endeavor who share mathematical ideas and who transmit them to each other. That is part of their apprenticeship as in architecture, boat building, carpentry, or sailing. There has to be more understanding of their ideas, as they are culturally embedded, so that their mathematical aspects can be recognized. Take weaving for example. That surely involves geometric visualization. It not only requires the creation and conception of a pattern, but also requires knowing what moves to execute or colors to use to cause the pattern to emerge. In effect, the weaver is digitalizing the pattern. The weaver expresses the visualization through actions and materials.

Ubi: It is not a lower level of thinking, it is a different way of thinking. If an Eskimo and an Amazonian Indian face a similar problem they will follow different approaches to it. Not only are their gods different, their everyday lives are different—and these two major differences are related. The way they live their everyday lives is related to the way they try to transcend them. We concern ourselves with arts, morals, religions, sciences, production, and so on, often without paying attention to the fact that all this comes together. Breaking down these different forms of thought into separate disciplines may be the cause of most of our troubles in dealing with civilization as a whole. It naturally leads to mistakenly recognizing the prevalence of one form of thought over another, and even to the valuation of one form over the other. We see manual work less valued than intellectual work, leading to an unjust social ethics; we see material assets more valued than cultural assets; we see a dichotomy between material and spiritual concerns: at the one extreme we see people regarding material values as absolute while at the other we see people treating mind as disembodied, as if the mind and the body were separate entities.

Marcia: Within the mind/body distinction, I think of it more specifically as drawing too rigid a line between creations of the hands and creations of the mind. They are both creations and they are related through visualization and planning. As with the quipu—they embody a logical-numerical system but creating them and "reading" them involves the tactile.

Ubi: This leads to some reflections about cognition, which has been studied in such a narrow way. The process of creation, typical of human beings, is the result of a conjugation of the sensorial with the intuitive and with the emotional and with the rational dimensions of our behavior. We do not create, hence we do not build knowledge, and we cannot act, without the conjugation of all these. But we tend to place each manifestation of culture in a single dimension. Religion is purely emotional, science—and particularly mathematics—is purely rational. This is not so, and has not been so in history. Yet historiography and epistemology both disregard this pluridimensional aspect of building up knowledge. Look into Greek geometry, for example. A triangle comes from three sticks and a string joining them. Greek geometry is always regarded as having ignored the sticks and the strings. And, surely, these sticks and strings are made of some materials, have colors in them. These are suppressed in the abstract version of geometry. Amazonian geometry is full of colors. The geometric properties are there in both the Greek posture and the Amazonian posture, but they are regarded as different expressions of human mind: one is science, the other is at best art, and is more commonly called folklore. And if you move to the Hippocratic writings, this is early scientific medicine, while the use of rituals and roots by the Amazonians is called sorcery. I see ethnomathematics dealing with all these considerations. This is why I like to classify it as a program in history and philosophy. History and philosophy of mathematics? No, not only of mathematics, but of everything. And since mathematical dimensions are just about everywhere, in the cosmos, in nature, in our actions, the name for such a program is not inappropriate.

Marcia: I would like to raise one final point about an educational aspect of ethnomathematics. It is generally assumed that grade-levels in school go with particular ages. However, there are a growing number of people seeking education or reeducation who are not in those age grades. You find, for example, more students in college who are 35 or 45 or 65. Whether on the college or elementary level, in the U.S. or in other countries, I believe that the teaching of the adult learner is of growing importance and needs special attention. At all levels of schooling, the teaching of subject matter is intertwined with the teaching of culture. But these people are already fully enculturated. As a result, ethnomathematics may have even more important ramifications for their education. If part of the pedagogical purpose of ethnomathematics is a revitalization of mathematics education, that aspect would be particularly important for this older group. And, since they have had many life experiences, some of which may have involved utilizing mathematical ideas, they could make significant contributions to our understanding. Have you thought about this?

Ubi: I have been thinking a lot about this. My view of the future of education points to a different concept of the levels of schooling. Labor will be something very different from what it is now, offering learning opportunities which are nowadays not even considered. I see every moment of life as a learning experience, and this is different from individual to individual. So I see school as a kind of meeting place where people with different experiences come together to socialize their experiences. Thus they begin another experience, which is to put their capabilities together to function at a common task. Ethnomathematics is a most suitable pedagogy for this kind of school, an institution which addresses not individual action but cooperative action. Because ethnomathematics is not passive it is loaded with critical components. But most importantly, the gains and advancements will be collective and not individual. While keeping capabilities very individualized (each individual is different from the other) we have to generate, through this socializing school, respect for the other with all his/her differences, solidarity with the other in her/his pursuit of satisfying the needs of survival, and transcendence of their material and spiritual needs, learning how to act in cooperation with others, putting together the physical and intellectual resources to reach common goals. These three components: *respect, solidarity,* and *cooperation,* constitute an ethics for a global civilization and serve as the basis for my model of the school of the future.

Of course this is intrinsic to my view of ethnomathematics. I entirely agree with you that the revitalization of mathematics through ethnomathematics will be the result because ethnomathematical pedagogy is an active one. One is in each moment putting all one's efforts, intellectual and manual, to the performance of a common goal. The place of adults in this scheme is essential. But we have to treat them, these adults, as individuals with lifetimes of experiences.

We have covered many points in this conversation. Now, even more than before, I think we are very convergent even though from different roots. And I am very happy to see this.

Marcia: Yes, I agree. And we are particularly convergent in our effort to see ethnomathematics as a revitalization of mathematics in school. We do not want to see mathematics disappear as a school subject

References

[1] Ascher, Marcia and Robert Ascher. *Code of the quipu: a study in media, mathematics, and culture,* University of Michigan Press, Ann Arbor, 1981

[2] Ascher, Marcia, *Ethnomathematics: a multicultural view of mathematical ideas,* Brooks/Cole, Belmont, California, 1991

[3] D'Ambrosio, Ubiratan, *Etnomatemática: Arte ou técnica de explicar e conhecer.* Editora Atica, São Paulo, 1990

[4] Scribner, Sylvia, "Modes of thinking and ways of speaking: culture and logic reconsidered," in *Thinking: readings in cognitive science,* ed. by P.N. Johnson-Laird and P.C. Wason, Cambridge University Press, New York, 1977, pp. 483-500