Communications

Being radical

ANDERSON NORTON

In recognizing the human body as the ever-present context for mathematical learning, embodied cognition holds promise for bridging the divide that has developed between situated cognitionists and radical constructivists. Roth's (2010) article, "Incarnation: radicalizing the embodiment of mathematics," does much to fulfill that promise– more than the article admits. This commentary answers questions Roth posed for constructivists, while explaining how radical constructivism and radical embodiment might align in a theory of learning.

Question 1: *"How can the knowing subject know itself?"* It can't. The mind can only know what it constructs, which may include a self-concept but not the constructing agent (the mind) itself. Thus, the sage advice to "know thyself" is truly an invitation to a lifelong quest.

Question 2: *"How can the abstract mind manipulate the body and the senses and test its knowledge in the world?"* It can't. The mind – or "I" – exists in the praxis of living, just as Maturana (1988) has described. It only becomes abstract for the observer (including self) in trying to describe it, so the *abstract mind* manipulates nothing. In fact, anything (bodies included) we try to describe or explain is a model that serves to make sense of experience. Here I define a model as von Glasersfeld and Steffe (1991) have: "a conceptual construct that is treated as though it gave an accurate picture of the real world" (p. 95). It's impossible to know whether these models reflect an ontological reality because we only experience the world through the assimilation of electrical impulses sent to our brains. Even that description is a model, but it is a particularly useful model for explaining learning (McCulloch, 1965).

Piaget, whom von Glasersfeld (1987) credits as the founder of radical constructivism, was a biologist who became interested in explaining how children learn. He took children's reflexes and embodied actions as the primary sources for all subsequent learning. Action in the praxis of living produces the feedback that tests knowledge in the world; but, again, feedback comes in the form of electrical impulses that result from action (*i.e.,* through our senses) and that our brain has to interpret. As Kant might say, the *phenomenal* reality we experience has no direct access to the *noumenal* realm in which we act (Campbell, 2002).

Question 3: *"How, without a plan of what a cube is or looks like, does a constructive mind arrive at a cube from the disparate sensual (visual, tactile) experiences that a learner may have with the object that we know to be a cube?"*

Fortunately, it can, and the explanation a radical constructivist would provide is not so different from the one Roth provided: "knowing an object does not mean copying it – it means acting on it" (Piaget, 1971/1970, p. 15). In other words, we construct objects through our actions. In the case of the cube, these actions would include various rotations, for example. In the case of the square, which (in a sense) *can* be seen all at once, action is still necessary, just as Roth explained and just as Chris (the student in Roth's article) demonstrated when tracing the edges with his fingers.

Perception is no simple matter of seeing, but must be learned through the coordination of action. Piaget and Inhelder (1963) provided a striking example by showing that three-year-old children copy drawings of circles, triangles and squares as the same simple closed figure (roughly, as circles); but they, nonetheless, have little trouble copying drawings of intersecting circles and crossed lines.

To illustrate the role of mental actions in perception, consider the Figure 1. What do you see? … a cube with a corner missing? … a small cube in front of a larger cube? … a cube in a corner? Whichever you saw first, the process of forming the image probably required no conscious effort. Your mind performed its actions outside of your awareness. To experience the kinds of actions your mind used to create the initial image, try changing to one of the other suggested images. These are actions that you no longer need to carry out physically; they are mental actions abstracted from experience, through a process Piaget (1971/1970) described as a process of reflective abstraction.

Roth describes an analogous process in terms of "the flesh," "auto-affection," and the "I can." He explains that students form their knowledge based on coordinations of movements their bodies can perform – actions and coordinations that children eventually learn to carry out imaginatively. Roth claims there is an "*I can* prior to any *I*" because, for him, "I" implies consciousness. However, constructivists recognize that most cognition occurs outside of our awareness so that we can think about the mind as the subconscious "I" in "I can." Radical constructivists might agree that there is a sense in which Roth is correct to say,

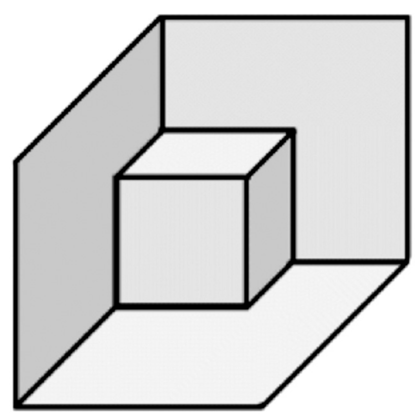

Figure 1. Ambiguous figure

"mind is the flesh itself" (p. 13). If we replace "flesh" with "mind" (and, perhaps "auto-affection" with "auto-regulation", and "I can" with "actions fitting experience") throughout his article, we get a fairly cogent elaboration of radical constructivism that aligns with embodied cognition.

Embodied cognitionists and radical constructivists might continue to disagree on whether mathematical objects (or any knowledge) should be considered abstract, but recognizing commonalities in our approaches to understanding how students learn is fundamental to continued conversation. We might describe both the mind and the flesh as the human central nervous system, so long as we recognize that the nervous system is yet another model that may or may not accurately describe ontological reality. As such, future discussion about disagreements could be mediated by studies in educational neuroscience, which are just beginning to emerge.

Acknowledgement

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Is constructivism a victim of its success in mathematics education?

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What is it with constructivism that prompts so many of us in mathematics education to criticize it in advancing other theories? I've been asking this question for quite some time, and it resurfaced when I began reading Roth's (2010) article in the previous issue of FLM. I reflect on this issue in this short communication. To be clear, I take Roth's article only as a convenient example; many others have inspired in me the same question. My argument is thus not intended as a critique of Roth's paper. I mean only to discuss a more generalized phenomenon that I see as troublesome.

The question arose for me on line 9 of the first page of the paper (p. 8), where Roth referred us to a footnote in which it is explained, "space limitations prohibit a critique of constructivism" (p. 16). For me, this comment seemed to appear out of the blue, and compelled me to wonder why this sort of critique is needed for the ideas developed in the article. I wondered: "Is it announced here that the paper's intentions are to show the limits and pitfalls of constructivism?" Obviously not, since the paper is about proposing a radical explanation of embodiment in learning mathematics. This ambitious project, in my opinion, is sufficient to warrant attention and interest to the paper itself - and, for me, that only amplified the question of why there was any need to mention that constructivism is limited.

Thinking about this, it struck me that, for some time now within mathematics education research, one could easily get the impression that the theory of knowing named 'constructivism' is an accepted standard for the field - almost to the point of being an hegemony. This point is evidenced in the fact that a simple mention that a piece of writing is "inspired by constructivist principles" is sufficient for many journals and conferences. This situation is an unfortunate one for at least two reasons.

A first reason is that any theory needs to be justified to show its potential and power for making fruitful distinctions for the phenomenon at hand and for pushing forward our understanding of ideas as a field of research. Hegemonies tend to do the opposite. Prevailing theories tend to catch thinking in eddies of non-critical usages. Theories need to grow and aid growth. They need to enrich our understandings. If we prevent a theory from being questioned, articulated, justified, or from illustrating its power, then we don't grow as a field of inquiry.

A second reason - and, I believe, a most unfortunate one for our field - is that because of this "taken for granted" place/relevance of constructivism, researchers who work with other *as-pertinent theories* need (or feel the need) to position their thinking against constructivism. With inevitable "space limitations," this felt need leads to caricatured portraits or inventions of weaknesses/limits of constructivism - which seldom bring anything useful, inspiring or generative to the field. The sole function of such truncated criticisms and caricatures seems to be to elevate one perspective by diminishing another. Thus, we then see proponents of other theories (such as embodiment, socio-cultural theories, enactivism, complexity science, *etc.*) finding/creating holes in constructivism in the efforts to legitimate their own theorizations. Constructivism has become the straw man on which to test or compare other ideas.

I react by wondering if we might need to be reminded that constructivism has not always existed and that it is only one theory among many, many theories that address learning issues. Maybe it is a strong one, I agree, but it is still only one among many. And, as any theory, its relevance rests on the extent to which it is useful in helping us toward better understandings of the phenomenon it claims to address. If it does not, why bother with it? This more positive positioning should support our shared work. By contrast, the more negative approach, taken to an absurd extreme, would cripple reporting efforts. In short, if when writing a paper one had to dismiss all theories that do not suffice or enrich interpretations of one's data, writings would only be about discarding: "I do not opt for behaviorism because …." "I do not opt for cognitivism because …." "I do not opt for activity theory because …." "I do not opt for complexity science because …." And so on. I doubt that this would be helpful in pushing the field forward (not to mention the inevitable tedium).

To put it bluntly, within mathematics education research, the principal issue is not whether constructivism is false or wrong, it is whether other learning theories go further, push ideas even more, or simply draw attention to other aspects of learning. Enactivism is one, complexity science is another, and so on. When doing research, one chooses and/or develops a theory that offers fruitful explaining mechanisms and fruitful distinctions to make productive sense of the phenomenon under scrutiny. A theory is viable as long as it attempts to offer new distinctions, new structuring and new mechanisms to explain experience (in this case, mostly, about learning, doing, and teaching mathematics). It is those fruitful distinctions that we need to develop and gain knowledge of as a field of inquiry, in order to better understand, analyze and comprehend the phenomena we study.

But don't get me wrong. I do not presume that all theories are created equal, nor that all are fruitful to the project of understanding mathematics teaching and learning, nor that we should avoid comparisons. These are matters necessitating more than a short communication. The point is that the usefulness and strength of a specific theory must be assessed according to the potential and fruitfulness of the distinctions it enables, not according to whether other theories are bad or wrong or limited.

Coming back to Roth's article, I felt it was sufficient to take on the project of discussing current work on embodiment in order to propose new distinctions and contributions to the theory. That alone should warrant interest in the paper – independent of the merits and shortcomings of constructivism. That Roth's article is relevant is for the readership of FLM to judge, in regard to the new distinctions and ideas offered. That, I believe, should be all that matters.

Reference

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Dinner with "maTHEMATICs": In honOUR of DemYstifyING mathematics

WILLY MWAKAPENDA

Consider the term "mathematics". In terms of the everyday dictionary of mathematics, the forms in which the term "mathematics" is written are often as follows:

a. mathematics
b. Mathematics
c. MATHEMATICS

Clearly, upon reflection on the above, there are "visible" differences in which the term "mathematics" is written. In *a,* the term appears in a constant lower-case lettering, while in *b,* M is the only letter that is capitalized. In *c,* the term appears in constant upper-case lettering. The question that can be considered here is: how do the ways in which the term "mathematics" is written in *a, b* and *c* affect the way in which one reads and understands the term mathematics itself? Does capitalizing one or more letters in the term affect the way it is read and the meanings one can attach to the term itself? One response to this question is that *a, b* and *c* represent the *common* [1] forms in which the term "mathematics" is written. What is being hypothesized here is that *common* ways of writing terms in mathematics are likely to lead to generally common ways of reading and understanding mathematics. However, more importantly, it is suggested that *uncommon* ways in which mathematics and terms in mathematics are written should lead to *uncommon* ways of reading and understanding mathematics. It is the uncommon forms in which mathematics and mathematical terms are written which is the focus of this article. This exploratory article considers some of the following *uncommon* [2] ways of writing the term mathematics, and the extent to which these ways of writing the term shape the way in which learners and educators read and understand mathematics. Table 1 below illustrates this scenario:

Common ways of writing "mathematics"	**Uncommon ways of writing "mathematics"**
mathematics	**mat**hematics
mathematics	**ma**the**ma**tics
Mathematics	**MAT**he**MAT**ics
Mathematics	mat**HE**matics (mat-HE-matics)
Mathematics	mat**HEM**atics (mat-HEM-atics)
MATHEMATICS	ma**THEM**atics (ma-THEM-atics)
MATHEMATICS	ma**THEMATIC**s (mat-THEMATIC-s)
Maths	mathema**TICS** (mathema-TICS)
…	…

Table 1. Common and uncommon ways of writing the term "mathematics"

As can be seen in Table 1, the first column shows the term "mathematics" appearing in forms that are quite familiar (to learners and educators particularly) because these are the forms in which they often appear in written textbooks concerning school mathematics. Figure 1 below illustrates the common forms in which the term "mathematics" is written in school textbooks.

As noted earlier, the form in which the term mathematics is often written in school is "common", "conventional" and lends itself to being read in traditionally common and unproblematic ways. However, as can be seen in column 2 of Table 1, it is important to acknowledge that there is a need for teaching to attend to mathematics in unconventional

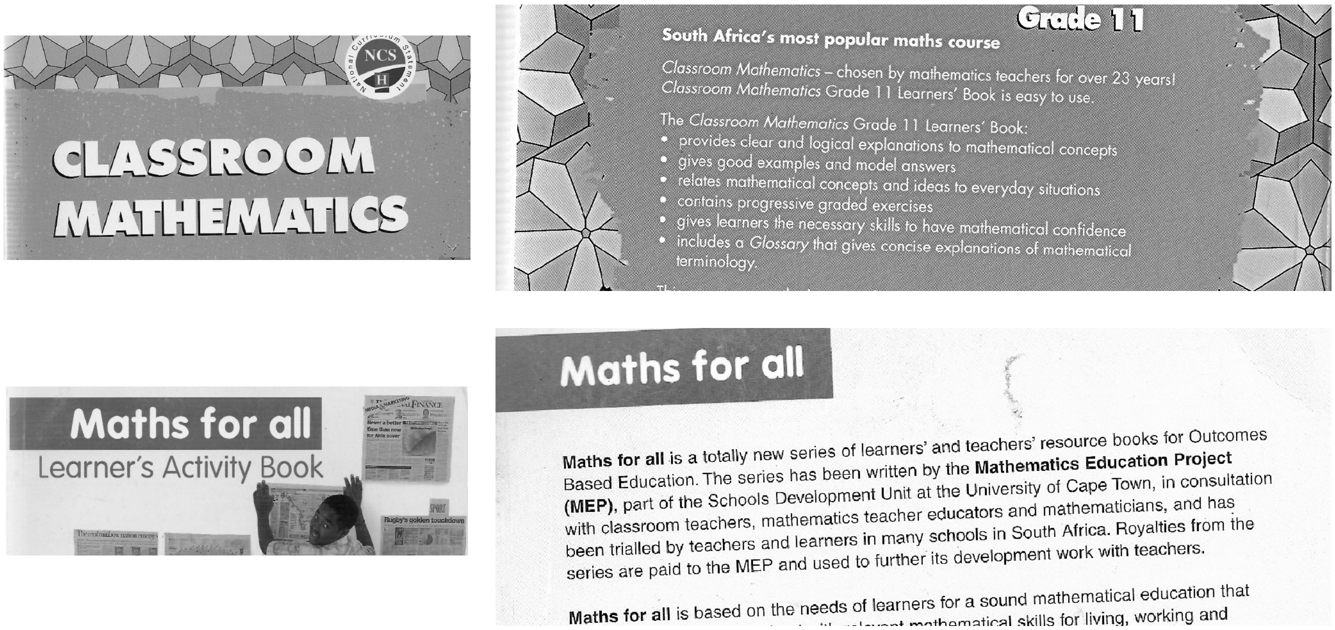

Figure 1. Forms in which the term "mathematics" is written in school textbooks

approaches if we are to circumnavigate the desperate need for change in learner understandings of school mathematics. The approach that is being explored here concerns a need to examine how we write in mathematics: that if we reconsider how mathematics is written, we should provide opportunities for learners to shift the ways they see and read mathematics. For example, consider the following way in which the term "mathematics" has been re-written: mathematics → mat**HE**matics (mat-**HE**-matics). Does writing "mathematics" in the form "matHEmatics" make any difference to the term "mathematics"? If so, what is the difference? And how does that difference shape the way we make sense of the term "mathematics"? What is the significance of writing mathematics in the "newer" version: "mat**HE**matics"? What is the significance of the term "**HE**" in the term "mat**HE**matics"? What comes to mind when our attention is drawn to the term "**HE**"?

What is being proposed in this article is an approach to writing that provides opportunities for learners and educators to see mathematics in newer, fresher and less common ways. It is being suggested here that the term "**HE**" in "mat**HE**matics" should be able to draw the reader to an important association or **connection** of mathematics with people: HE and THEM, as can be see in column 2 of Table 1. What makes it possible to see mathematics in this connected way stems from the way the term has been written.

There are numerous ways in which mathematics and terms in mathematics (*e.g.,* rectangle) could be written, and it is being anticipated that the ways in which different individuals write or re-write a specific term depends on what they see in the term which may not be "obvious" in its conventional form. Hence one of the aims of a possible research agenda that emerges from this exploration is (i) the identification and documentation of various ways in which different individuals (mathematics learners and educators, mathematicians, *etc.*) re-write a specific term in mathematics, and (ii) an exploration of the different meanings, associations / connections the new forms of the terms hold for different groups of individuals. It is suggested here that re-writing mathematics and mathematical terms in this way provides the possibility of making more connections visible – *i.e.,* connections within mathematics, between mathematics and other disciplines, and between mathematics and the everyday world. I argue that in so doing, we begin to demystify the nature of mathematics itself. In the following section of the article, I present a poetic demonstration (Staats, 2008) of this demystifying dimension of the approach that I have suggested in this article. As will be seen, I invoke the tools of language and linguistic play [3] to reveal some of the connections that can be explored by considering the term "mathematics".

Dinner with "mathematics": a poetic demonstration

Where is the dinner taking place?
Look 11 [4]:
Hotels with 11,
Hotels at 11,
Hotels on 11,
Hotels on the 11; Hotels on the 11th,
Hotels on the 11th of the 11th floor.

Who is at the gate?
Ma**them**atics → them
mathe**ma**tics, math**ema**tics → mama
"them" is in the company of mama at the gate to the dinner.

Any parking space? Where does the limo park?
Mathematics → m **at hem** atics → at hem *i.e.* paCk at the hem

(edge) of the pARKing lot, using the **h.m.e.** (home made equation) as a guide.
Mathematics → mathem **at ics** → **at ICS, IKS** *i.e.* at the cultural museum (where the people say: "we have eaten", when they have eaten indeed and eaten in need.

What is on the menu?
Mathematics → **M**ath**E**m**AT**ics → meat.
Mathematics → matHeMAtics → HMA → HAM [highly advintage-d (advantaged) mathematics].
Mathematics → mAtheMAtIcS → A M A I S AMASI (the menu includes "water" and "milk" as essential extras)
AMASI → MASI → SIMA, SIgMA. The meat is served with sima (sigma).

Who is in the kitchen? Who could have been in the kitchen?
Mathematics → math**EMA**tics → EMA
Mathematics → **MAT**hem**A**t**I**cs → MATAI
Mathematics → **maRth**em**a**tics: It has to be a name other than Martha.

Who else is present at the dinner?
Mathematics → Mama Thetics [the First Lady (lay-day!) of mathematics)

And will the dinner be good? What is the assurance?
Mathematics → mathe matics → mathe matic s + k → mathe ma**ticKs** → ticKs. [there will be many ticks (indicating yes yes) in the customer-service complaint envelope. NB: No live "ticks" on the dinner menu! They have already been bracketed from the skin of the dead cow-meat that is being served. So no adead (added) ticks.

Where are the guests sitting?
Mathematics → MAThematics MAT. Being a weekend in the sub-album [sub-urban!] village, they have the luxury (lucksury!] of sitting on a mat. But being in a village, there is always the chance of an uninvited guest (actually everyone is welcome any time anywhere anyhow, in the village). That is why mathematics has a spare MAT (**MAT**he**MAT**ics). Beware: Meaning of "matmat" in XiVenda? Ndebele? Other languages? [be culturally sensitive).

MAThe**MAT**ics → **MAT** by **MAT** → MAT → MAT = MAT^2. The MAT has a square dimension.

What will happen to the one you fail to invite to dinner with mathematics?
Mathematics → ~~M~~athematics → athematics → a**s**thematic. He/she will have a contract with asthma. Mathematics is supposed to be "mathematics for all" (MA3MA4 – mathematics frees and flows!)
Dinner with mathematics begins when the menu is presented.
Menu → MEnU → Me n U → Me and yoU → "me" and "you" are inseparable from mathematics. Everyone is invited.

What is the dress code? And why dress up anyway?
Mathematics → matHEmATics → H.E.A.T. It is cooling hot on the 11th floor. That floor is closer to the sun than the sum!
Mathematics → matHemATics → H.A.T[5], not H.A.RD! (Mathematics is not supposed to be hard but hat (had-it! You are allowed to come full at the dinner village. You do not walk to dinner with an empty stomach. You are supposed (surely) to have eaten something before you came. If not, how do you even press the lift (power) button to the 11th floor?).

Who is paying?
mat**HE**matics → HE. He is the usual suspect, followed by H I S (mat**H**emat**I**c**S**).
But remember, this is in the 11th floor village. So "He" (the individual) comes after "them" (the people). So the people (ma**THEM**[6]atics - them) must have paid for that dinner. Having dinner with mathematics is a rare opportunity, so they might as well donate every penny in the box so that mama can dine with MATH. It is a sacrifiSH.

In which vessels is the dinner served?
Mathematics → mathe**m**[7]atics. "m" is the centre (median) of "mathematics" (the word). So "m" is h**OLD**ing the "mathe" and "atics". Dinner with mathematics will require that you are served with "m" at the centre. So the size of the vessel to be used will depend on how non-e**m**pty one's stomach already is. (At a Chinese dinner, "forKs (fourKs) and sTICS would not be allowed).
In case of tear emergency (resulting from overdose of pepperly sauce), soft towels are also provided. The sauce has a tearing (teaRING) effect on the guest's eyelid.

Who gets the top tip?
Be reminded that the dinning room is not exactly the same as the examination (theatre) room. Eating is more than just putting food in the mouth so that it can be taken away the next mile!
Mathematics → maThEmaTicS → T E T S → T E S T
Mathematics → mAThEmaTicS → A T E T S → T E S T A → T A S T E. Dinner with mathematics does not involve any tests. [8] It is about tasting, taSTING (enjoying the sting in the peppered (un-paper-ed) steak). The waiter is tipped top not for testing dinner guests, but for her ability to know the STATE of the TASTE in the "ati" of the "c" (cooked). The currency ("c") of the tip in the 11th floor dinner room is w-haff: "whatever is affordable".
[If learners were to give marks to invigilators for invigilating, how many marks would examiners get for examining? Hint: Think mathematicaLLY. Let L = learner, I = invigilator, and E = examiner. You guessed right. The value of L+I+E is "infinity" (where infinity → in f unity → in for unity].

END OF MENu. STARterS TwENty over [stars ten (turn) over for salads. Salads → sal ads → sal adds → sal ADDs → DDAlas → Dalas: The next dinner appointment with mathematics will be at a hotel in Dalas! What an adDventure!]

Notes

[1] *common:* this term refers to the traditional ways in which terms are written, according to everyday experiences in school and various print settings. In their common forms, terms are likely to be read in common ways and will evoke meanings and understandings that are frequently encountered in everyday experiences. The meanings of terms written in common ways

are often taken-for-granted and consequently unproblematised.

[2] *uncommon:* This is used here to refer to unusual forms in which the term is written, forms that are not often encountered in conventional school or everyday settings.

[3] The constructive forms of "play" being used here involve: capitalizing letters in a word; breaking a word into two or more parts; inserting a letter or letters into a word; deleting a letter in a word; inserting and deleting letters; reversing the order of letters in a word; replacing a letter with a number, *etc.* This play is used as a "methodology" for creating and recreating words from other words. This approach is not often reported on in the field of education, and mathematics education in particular. It has similar elements to the work of Brooke and Eckler (1976) in the field of recreational linguists. This form of play opens our minds to the issue of how words become words (their etymological dimensions) and also the structure and forms of words (linguistic morphology – morphological dimensions).

[4] Dining and counting go together. There are 11 letters in the term "mathematics". 11 is a prime number, suggesting that mathematics is "prime" property, and that the venue for the dinner with mathematics has to be a prime hotel.

[5] Mathematics is a H.A.T Mathematics is an umbrella subject. This underlines the integrated and intra-disciplinary nature of mathematics.

[6] While no one can leave **for** another, everyone can learn from one another. The benefits of one's learning can flow to everyone involved.

[7] An "after-dinner" game often played at Hotel 11 is called "mapping" (ppaming → palming!). In the palming game, the alphabet (a, b, c, …, z) is mapped to natural numbers (1, 2, 3, …, 26), respectively. So in the term "mathematics", m → 13, a → 1, …, s → 19. This implies that the term "mathematics" could be represented numerically as: 13 1 20 8 5 13 1 20 9 3 19 *i.e.* **m**athe**M**atics = **13**12085131209319. Notice that the significance of having two m's – *i.e.,* two 13's, is that the number 1312085 is bigger than the number 1209319. So we need one 13 to "hold" mathematics at the centre and the beginning of the number in order to achieve a balanced mathematical menu (milieu!). So we have a balance at the beginning of mathematics and in the middle. What about at the end of mathematics? Do we see an m at the end of mathematics? [Hint: mathematics → mathematicS. So the question turns out to be: Does S contain m? Can it be established, math-linguistically, that s contains or can be made to contain m? Consider the case where s = sigma = sigMa]

[8] Dinner with mathematics involves no tests. But the converse: "mathematics with dinner" is a dinnerlet (*i.e.,* an offspring of dinner, as in pig and piglEt). So mathematics with dinner involves e-testing (electronic testing!) The waiter at the dinner is entitled do as many e-tastes (of the food being served) as they please. Remember, there is no charge for electronic tasting. No orders are necessary. It is a paperless (pepperless) activity, and so no tearing and tears can be observed!

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Plank or log?

You're in the unfortunate circumstance of being on a ship that has just been boarded by pirates. Their practice is to force all but one of their captives to walk the plank. The lucky survivor is selected by the following procedure: all the people on board are made to form a circle, facing inward. Standing inside the circle, the executioner points at the ship's captain and says, 'For now you live'. Moving clockwise, the pirate points at the person to the captain's immediate left, barking 'To the gangplank!' That person is taken out of the circle immediately. Continuing in the same direction, to the next person in the circle the pirate says, 'For now you live'. And the fourth he sends to the gangplank. He follows the same pattern around and around the circle until only one person – the lucky survivor – is left.

Assuming you want to be that person, where should you position yourself at the start, relative to the captain?

(source unknown; selected by Elaine Simmt)