Genre Analysis as a Way of Understanding Pedagogy in Mathematics Education

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In this article, I argue that genre analysis is a form of discourse analysis which can provide useful and sometimes surprising new perspectives on understanding teaching and learning. An analysis of the written and spoken ‘texts’ of schooling, drawing on linguistic and literary analytic methods, can highlight relationships between educational genres and other culturally recognizable forms. The study of such related genres may allow researchers to uncover hidden cultural meanings, assumptions and intentions inherent in the generic forms of schooling.

I will use two examples to illustrate the application of genre analysis in mathematics education. The first draws on a study of mathematical word problems (see Gerofsky, 1999), analyzing their genre features and suggesting a relationship between word problems, parables and riddles. The second focuses on the language of initial calculus lectures at a university, uncovering generic similarities with the language of the conjurer, the salesperson or nurse. Both studies question the messages, intentional and unintentional, carried by generic forms employed in mathematics education.

What is meant by ‘genre’?

Many writers (for example, Buscombe, 1970/1986; Sobchack, 1975/1986; Palmer, 1991) credit Aristotle with outlining the first notions of genre in western culture. Contemporary ideas about genre in the second half of this century, however, can be traced to the work of Russian literary analyst and linguist Mikhail Bakhtin (1986), in particular his notion of ‘speech gemes’. Bakhtin stresses the extreme heterogeneity of speech genres, oral and written, and cites as examples of speech genres everything from:

- short rejoinders of everyday dialogue [and] everyday narration [to] business documents, [...] the diverse forms of scientific statements and all literary genres (from the proverb to the multivolume novel). (pp. 60-61)

His basic unit of analysis is the utterance, a unit of language in use and in context, and he stresses the dialogic nature of all oral and written language – even monologue is framed as a form of dialogue – and sees the quality of addressivity, that of addressing a known or imagined other, as key to the understanding of utterances. Bakhtin sees the listener or audience not as a passive partner in dialogue, but as a force constantly shaping the utterance of the speaking or writing subject through the listener/reader’s forthcoming or anticipated response. In Bakhtin’s theory, speech and writing can never be removed from a context of addressivity, of dialogue.

For him, there is no possibility of utterances that exist outside of genre:

all our utterances have definite and relatively stable typical forms of construction of the whole (p. 78)

Like Molière’s Monsieur Jourdain, who had no idea he was speaking in prose, Bakhtin claims:

[we] speak in diverse genres without suspecting that they exist. (p. 78)

What does genre study offer to mathematics educators? Ideas from genre studies in other fields

Bakhtin’s concept of genre has been adopted and developed in literary, linguistic, film and folklore studies. Theorists in other cultural fields have raised issues which are also useful for a discussion of educational genres. For instance, the film theorist Andrew Tudor (1973/1986), in a foundational essay on film genre, puts forth the problem of the ‘empiricist dilemma’ in identifying genres in this case, the ‘western’ genre in popular films:

To take a genre such as a western, analyze it, and list its principal characteristics is to beg the question that we must first isolate the body of films that are westerns. But such films can only be isolated according to their principal characteristics. We are caught in a circle that first requires that the films be isolated, for which purposes a criterion is necessary, but the criterion is, in turn, meant to emerge from the empirically established common characteristics of the films (p. 5)

Tudor’s suggestion for a way out of this dilemma is to avoid establishing a priori criteria, and instead to:

- lean on a common cultural consensus as to what constitutes a western and then go on to analyze it in detail (p. 5)

This solution presupposes a high degree of shared culture and the likelihood of shared patterns of recognition and meanings that are part of that culture. He suggests that:

from a very early age most of us have built up a picture of a western. We feel that we know a western when we see one. (p. 5)

Similarly, mathematical word problems are familiar enough...
in our culture to be immediately recognizable in popular culture contexts as varied as a TV episode of The Simpsons, a newspaper cartoon, an advertisement or a joke. To call a film a western, or a math question a word problem, is more than to define it as sharing certain principal characteristics with other westerns or word problems; it also suggests that the item would be more or less universally recognized in our culture as belonging to that particular genre category.

Rhetorician Karen Jamieson (1975), in an article on rhetorical genre, shows that generic forms carry with them a history of culturally and linguistically coded intentions. The use of a generic form may bring with it intentions that are not exactly the same as those of the current writer or speaker. Jamieson gives the example of a papal encyclical which borrows from the rhetorical forms of a Roman imperial edict. She argues that contemporary readers interpret a papal encyclical and its Latin prose style as pompous, turgid and overbearing in part because the encyclical is written like a message from a Roman emperor.

Word problems, as a very old generic form, also carry with them intentions which act as a rhetorical constraint to the intentions of contemporary mathematics educators. Riots exist between the conscious, stated intentions of educators and the force embedded in the word problem genre itself. Contemporary writers and teachers may consciously intend to refer to their students' lived lives, to 'real-life' situations when they offer word problems to their students. The genre itself may constrain or subvert these conscious intentions through intentions carried by the generic form. Most particularly, this includes the intention not to refer in a straightforward, literal way to the 'things' talked about in the word problem, but in a coded and ambiguous way to both those real-life things and, more strongly, to the 'things' or objects of a world of mathematical concepts different from the world of everyday experience.

For this reason, educators' stated intentions with regard to the use of word problems may be read as a justification or an alibi, in terms acceptable within our culture, for a form which does not necessarily fit that culture. The generic form carries with it intentions that are throwbacks to cultural norms of mathematical education from earlier times and other places. Word problems are interesting as artifacts in an archaeology of mathematics education, sedimented as they are with meanings from other worlds.

The rhetorical scholar Carolyn Miller (1984) writes:

To consider as potential genres such homely discourse as the letter of recommendation, the user manual, the progress report [and I would add here 'the mathematical word problem' and other educational genres] is not to trivialize the study of genres; it is to take seriously the rhetoric in which we are immersed and the situations in which we find ourselves [ ] what we learn when we learn a genre is not just a pattern of forms or even a method of achieving our own ends. We learn, more importantly, what ends we may have (pp 155, 165)

This observation speaks most cogently to the acculturating power of forms within mathematics education and education generally - that generic forms, which are familiar and recognizable but often existing below the threshold of our conscious recognition, serve to define us in our relationship to our worlds. Through genre forms, we learn what may be asked and what is beyond question in our culture, what we may aspire to and what is outlandish or forbidden. Questioning and exerting pressure on genre is a way of questioning particular unspoken boundaries of culture, and this is especially important when working in a culture like that of
school mathematics, with a strong tradition of conservatism and exclusion.

Case study 1: mathematical word problems as a genre

St. Peter talking to a man at the gates of heaven: “OK, now listen up. Nobody gets in here without answering the following question: A train leaves Philadelphia at 1:00 p.m. It’s traveling at 65 miles per hour. Another train leaves Denver at 4:00 p.m. Say, you need some paper?” (Gary Larson, Far Side cartoon)

If a joke leaves New York at 11:30, what time does it reach Vancouver? (Advert in the Vancouver Skytrain for the local television rebroadcast of the David Letterman show)

A man leaves Albuquerque heading east at 60 miles an hour. At the same time, another man leaves Nashville heading west at 65 miles an hour. Which man is closest to Nashville when they meet? (Joke told by an eight-year-old to her mother and father, overheard in a restaurant in Thunder Bay)

Mathematical word problems are instantly recognizable to most people in our culture who have been to school, so much so that they can be used without comment or explanation in jokes, cartoons and advertising. There are many aspects of their form which are familiar, including sentence structure, the sequence of story elements, data, and questions, and the kind of stories and imagery typically used. Aspects of the uses of word problems in mathematics classes are also familiar to most people. These problems are to be translated into mathematics (arithmetic, algebra, geometry) and the mathematical question is to be solved using taught methods to find the right answer. Enough data is given in the statement of the problem itself, and the problem solver may not demand more data, although often ‘red herrings’ may be thrown in to the problem to trick inexpert solvers. Adults’ emotional reactions on being presented with a word problem are usually intense: intensely negative (in most cases) or intensely positive. Gary Larson plays on our shared recognition of word problem phobia in the cartoon text quoted above.

In my study of word problems in mathematics education as a genre (Gerofsky, 1999), I used analytic theory from linguistics, literary criticism and mathematics education and considered the question: “What are word problems?”, trying to answer it by ‘taking a walk’ around the word problem genre as an object to see it from many points of view, including the linguistic, the historical and the pedagogical. A consideration of genre leads to questions of addressivity and intention. In this case, ‘intention’ includes educators’ conscious, stated intentions when writing and assigning word problems, but also intentions carried by the word problem genre form itself and students’ uptake of their teachers’ intentions, of which educators may not be aware. I suggest that an exploration of genre in mathematics teaching and learning can be a source for innovation and renewal in mathematics education practices.

Characteristic features of the word problem genre

In a previous article (Gerofsky, 1996), I looked closely at the constellation of features that characterize word problems as a class of genre. Moving among points of view grounded in linguistics, literary criticism and mathematics education, I found that the following features typify mathematical word problems:

- A three-component, sequenced rhetorical structure (a ‘disposable’ story element, followed by data and then a question).
- Indeterminate ‘locutionary force’ – that is, the referents for the nouns used in word problems are ambiguous ‘Deixis’ (pointing with words) in word problems is problematic because the nouns used do not refer in any but the most tangential way to their usual ‘real-world’ referents. Words in mathematical word problems point to some other world than that of our conscious, lived, real-world experience.
- Strongly imperative ‘illocutionary force’ (i.e. coded intention of writer): “Solve this problem to get the right answer, using only the information given and the mathematical methods I have just taught you!”
- ‘No truth value’ As with fiction, it is impossible and unproductive to assign the statements in word problems ‘true/false’ status (Frege’s notion of truth value, adopted into linguistics), because their existential presuppositions are false. However, word problems are very poor quality fiction at best, and are deliberately constructed so that their stories are considered interchangeable with a great many mathematically equivalent stories.
- An anomalous use of verb tense Word problems combine verb tenses in ways that would be jarring in ordinary speech. This use of verb tense is apparently ‘tenseless and non-deictic’ – in other words, the strange mixing of verb tenses is another indication that the words in mathematical word problems point to some other world than that of ‘real life’, so that we are not bothered by their lack of consistency when referring to time.

Related genres

Analysis of the linguistic and rhetorical features of a particular genre may be suggestive of other genres within the culture. This consideration of ‘intertextuality’ of a genre which, in its form and addressivity, recalls other familiar genres may bring to consciousness the hidden ground and intentions embedded within the genre. If one genre is recognizably like another, then echoes and resonances among them can inform a reassessment or reconfiguring of genre-related cultural practices. Rather than being forced into a binary choice of either approving or disapproving of a particular genre (as with contemporary calls either to do away with or focus primarily on the use of word problems in mathematics classes), it becomes possible to look at the...
genre with a sidelong gaze, and consider questions like: “What if we treated word problems as we do parables? How would we use them for teaching then?”

(a) Mathematical word problems and parables

‘Teaching stories’ are familiar in Western religious cultures, in such forms as Biblical parables and Talmudic stories, and in Eastern religious cultures in forms like Sufi teaching tales and parables from Buddhist sutras. In this century, parables have been adapted to non-religious contexts: for example, writers like Franz Kafka, Søren Kierkegaard and Flannery O’Connor have taken up the parable for secular, philosophical purposes.

Parabolic teaching stories share with word problems a particular, and peculiar, non-referential use of language. Or perhaps the term ‘non-referential’ is too absolute: rather, their language refers only in the most tangential way to real-life referents, but primarily points to some ‘other world’ – in the case of parables and other teaching stories, to the world of religious, spiritual or philosophical entities, and in the case of word problems, to the world of mathematical entities. In both cases, seemingly referential language is used to express otherwise inexpressible feelings and ideas through concrete images, which could just as well be substituted by innumerable alternate concrete images, so long as they served to point to the same abstract entities in the primary domain of reference. Parables share with word problems an indeterminate use of verb tense and interchangeability of characters, situations and scenes for structurally equivalent referents within their proper domain of reference. As with word problems, the assignment of truth or falsity to teaching tales is irrelevant, since the tales are not referring to an everyday world where truth values can be established.

Kafka, in his book Parables and Paradoxes, talks about the non-deictic nature of parable in the form of a parable:

Many complain that the words of the wise are always mere parables and of no use in daily life, which is the only life we have. When the sage says: “Go over,” he does not mean that we should cross to some actual place, which we could do anyhow if the labour were worth it; he means some fabulous yonder, something unknown to us, something too that he cannot designate more precisely, and therefore cannot help us here in the very least. All these parables really set out to say merely that the incomprehensible is incomprehensible, and we know that already. But the cares we have to struggle with every day: that is a different matter.

(1961, p 11)

Clearly, the parables do not refer in a non-ambiguous, straightforward way to the usual referents from our lived experiences. When a parable refers to fishermen, the intention is not to talk about the particularities of fishing. In fact, too great an emphasis on the details of real-life fishermen’s work would be a distraction from the point of the parable, which may be to talk about lost souls, spiritual guidance, and so on.

In a similar way, mathematical word problems may purport to talk about real-life situations, but in fact too great an emphasis on the details of real-life fishermen’s work would be a distraction from the point of the parable, which may be to talk about lost souls, spiritual guidance, and so on.

an insistence on the contingencies of the experiential situation distracts from the point or intention of the word problem. One such example is cited by Keitel (1989) She recounts observing a lesson in which a teacher wanted to teach ratio and proportion in a practical way.

She offered the following question: “Somebody is going to have his room painted. From the painter’s samples he chooses an orange colour which is composed of two tins of red paint and one-and-a half tins of yellow paint per square metre. The walls of his room measure 48 square metres altogether. How many tins of red and yellow are needed to paint the room the same orange as on the sample?” The problem seemed quite clear and pupils started to calculate using proportional relationships. But there was one boy who said: ‘My father is a painter and so I know that, if we just do it by calculating, the colour of the room will not look like the sample. We cannot calculate as we did, it is a wrong method!’ In my imagination I foresaw a fascinating discussion starting about the use of simplified mathematical models in social practice and their limited value in more complex problems (here the intensifying effect of the reflection of light), but the teacher answered: ‘Sorry, my dear, we are doing ratio and proportion’ (p 7)

When a word problem is offered that appears to deal with painting, for example, it is important not to introduce much real-life contextual knowledge about painting. That is not what the word problem is about; it is not what it is ‘pointing to’ with words. The mathematical word problem is about mathematics in the same way that the religious parable is about religion, and the same lesson about proportion could be taught through a story problem about mixing cockie dough or cement, about the ratio of girls to boys in a school, or any number of stories that could equally well serve to illustrate proportional thinking.

Word problems are also similar to parables in their non-deictic mixing of verb tenses. Where the use of verb tense in English would be governed by certain norms of grammar and logic in ordinary speech and writing, these norms are suspended for both word problems and parables.

Word problems differ most markedly from teaching tales in their illocutionary force – that is, in the intentions of those writing or offering them to learners. Mathematical word problems are typically used in pedagogy to practise established, recently-taught solution methods. In this style of pedagogy, there is value placed on quick, correct, economical solution of the purportedly problematic, without reference to extraneous knowledge of the world outside the stated problem, without further questioning of the contingencies of the story, and without undue puzzlement, irritation or contemplation. When a word problem has been solved in mathematics class, the problem is usually discarded.

Unlike word problems in mathematics education, parables, koans and other teaching tales demand no solution, and often ask no direct question. They are not meant to be solved, and generally present an insoluble, paradoxical dilemma. Teaching with parables and other teaching tales traditionally involves a discussion of the contingencies of
the story, of the problems of human life that relate to the story, of the sources of its paradox. Teaching tales are certainly not meant to be disposable exercises. They are made to be held onto, to irritate, to resonate. A good parable will inspire contemplation, and will be recalled in times of difficulty as a way of trying to make sense of a seeming impasse.

(b) Word problems, riddles and puzzles

Even when word problem stories appear to refer to aspects of the 'real world', their links to the world of lived experience are ambiguous at best. So why are these rather fanciful stories included at all?

The importance of unlikely and attention-catchign stories seems clear when mathematical problems are intended for recreation and entertainment rather than for 'serious' educational purposes. Mathematician David Singmaster, who has done extensive research into the history of recreational mathematics, claims:

Many problems in recreational mathematics are embellished with a story which is often highly improbable and this is partly what makes the problem memorable and recreational (personal communication).

Similarly, Hoyrup (1994), the Danish historian of mathematics, quotes Hermelink (1978, p. 44) as describing recreational mathematics as "problems and riddles which use the language of everyday but do not much care for the circumstances of reality." Hoyrup goes on to write:

'Lack of care' is an understatement [...]. A funny, striking, or even absurd deviation from the circumstances of reality is an essential feature of any recreational problem. It is this deviation from the habitual that causes amazement, and which thus imparts upon the problem its recreational value (pp. 27, 29).

Using "the language of everyday" but "not much caring for the circumstances of reality" is also a very apt characterization of the non-referential nature of word problems, parables and riddles as genres. A lack of "care for the circumstances of reality" has been a feature of word problems as early as the Old Babylonian period. Hoyrup posits a continuum of non-referential story problems ranging from the most delightful mathematical recreations to the dullest of school exercises.

One function of recreational mathematics is that of teaching [...]. This end of the spectrum of recreational mathematics passes imperceptibly into general school mathematics, which in the Bronze Age as now would often be unrealistic in the precision and magnitude of numbers without being funny in any way. Whether funny or not, such problems would be determined from the methods to be trained. [...]. Over the whole range from school mathematics to mathematical riddles, the methods or techniques are thus the basic determinants of development, and problems are constructed that permit one to bring the methods at hand into play (pp 27-29).

Riddles, folktales, recreational mathematics and pedagogic mathematical word problems appear to have developed and spread in similar patterns. In many cases, the boundaries between riddles, folktales and word problems are determined more by the context of their use than by their form or content. For example, the familiar children's riddle in English, "As I was going to St. Ives", has been traced back to a problem in the Rhind papyrus from ancient Egypt and is related to a problem from Sun Tzu in ancient China. The well-known 'crossing the river' problem (as well as the 'hundred birds' problem and a number of other widespread problem types) have been reported as a source of village riddling contests in the Atlas Mountains of Morocco and in locations in eastern and southern Africa. In an interview I conducted recently, an Iranian-born mathematician told me about being 'riddled' as a boy with a story problem about weighing flour which he later saw as a specific instance of a more general theorem in number theory.

Hoyrup relates the very wide distribution and longevity of a number famous mathematical word problems to an oral tradition of recreational problem riddles transmitted by merchants along the Silk Route:

Like other riddles, recreational mathematics belongs to the domain of oral literature. Recreational problems can thus be compared to folktales. The distribution of the 'Silk Route group' of problems is also similar to the distribution of the 'Eurasian folklore', which extends from Ireland to India [...]. Recreational problems belong to a specific subculture - the subculture of those people who are able to grasp them. The most mobile members of this group were, of course, the merchants, who moved relatively freely or had contacts even where communication was otherwise scarce (mathematical problems appear to have diffused into China well before Buddhism) (pp. 34-35).

Is there a distinction between riddles, recreational puzzles and school word problems? Since the same problems can be found contextualized in all three settings, the difference seems to lie mainly in the intentions surrounding the problem. Riddles are contextualized in a setting of pleasurable social interaction. They can be part of a process of building social solidarity and, simultaneously, a source of competition, as in the village riddling contests. Although riddles have been collected in written form, their primary use is in oral culture, and good riddlers can draw from a large, memorized repertoire upon which a certain degree of improvisation is possible.

In contrast, word problems in school mathematics are traditionally assigned as a sort of bitter medicine that will make you better. In North American mathematics textbooks, they usually come at the end of a series of 'easier' numerically or algebraically stated problems related to a mathematical concept introduced in the preceding chapter. The word problems represent a final test of students' competence in recognizing problem types related to that chapter and translating those problems into tractable diagrams and equations which can be solved using taught algorithmic methods. School word problems are not social events, nor part of an oral culture. They are ideally meant to be solved silently, individually, using pencil and paper. Students are certainly not encouraged to memorize a repertoire of word problems.
for later enjoyment. On the contrary, once solved, they are generally discarded by teacher and students.

Riddles often retain strong links to folktales and parables and other teaching tales in their invocation of paradox and ambiguity, through their use of puns, hyperbole, nonsense, etc. Like parables and word problems, they point to two worlds at once - the world of their literal referents and another world invoked by word play or unexpected associations and structures. In riddles and parables, this ambiguity is embraced as essential to the enjoyment and philosophical import of the genre.

Contemporary writers of mathematical word problems, on the other hand, work hard to make their problems realistic, relevant and unambiguous. In this pursuit of singleness of meaning and relevancy, they are stymied by both the genre's history and its form, which carry with them the intention to create paradox and at best a shifting relationship to everyday reality

Case study 2: initial calculus lectures as a genre
I recently studied first-year university calculus lectures as a genre, comparing a number of taped lectures by four mathematics professors at Simon Fraser University (pseudonyms Brown, White, Green and Black) Using clusters of linguistic features, I found parallels between the 'initial calculus lecture' genre and several other culturally-significant speech genres.

Some prominent features of the 'initial calculus lecture' genre included unusual uses of the first person plural pronoun ('we', 'us', 'our') - see also Rowland, 1999), extensive use of rhetorical questions and tag questions, the attribution of questions or opinions to the audience and lecturers 'answering' these unasked questions or objections, and the structuring of the lecture as an inexorable chain of logic that could lead to no conclusions but the ones given. I was struck by the similarity of some of these features to the language of persuasion, particularly a 'hard sell' sales pitch, and of other features to infant-directed language or 'baby talk'.

In a 'hard sell', the salesperson's job is to forge an inexorable chain of logic that leads to one conclusion: that the prospect would be crazy not to buy what the seller has to offer. Typical tools used in the hard sell include demonstrations (think of the vacuum cleaner salesman who pours dirt on his prospect's best carpet), making claims for the marvelous qualities of the product for sale, voicing possible objections or questions the prospect might have and then answering them with prepared responses, identifying oneself with the prospect's disbelief ('I didn't believe it either at first!'), and above all, creating such an unstoppable stream of talk that the prospect has no chance to consider any objections or questions other than those already suggested by the salesperson before the seller has moved in to 'close the sale'.

I suggest that many of these features are typical of the math lecture genre, particularly in calculus lectures addressing students in their first year of post-secondary education.

These features are also typical of other genres that involve 'selling' or convincing - religious proselytizing, political speechmaking, 'hard sell' advertising in all media. What is being 'sold' is not always a tangible commodity; it may be a belief, an opinion, an idea. I think it is in this light that I see the similarity between a math lecture and these other forms of persuasion.

Like other 'persuasion artists', the math lecturer is often making a 'cold call' - trying to convince an often unwilling and unprepared audience of the truth and efficacy of certain mathematical beliefs. The lecturer is the representative within the lecture hall of the whole field of mathematics - 'mathematics personified'. The lecturer's problem, as seen through this lens, is to use the lecture hour in the most efficacious way to engage students' interest in the wares that are 'for sale' (in this case, mathematical ideas), to convince them of the truth of the arguments the lecturer is to present, and to persuade students to accept those arguments so as to be able to function as if they were self-evident (and use the mathematics involved in problem-solving, exercises and exams). If generic parallels may be used as evidence, then this is a 'hard sell'.

Goffman (1981) defines lectures in general as follows:

A lecture is an institutionalized extended holding of the floor [...]. Constitute statements presumably take their warrant from their role in attesting to the truth, truth appearing as something to be cultivated and developed from a distance, coolly, as an end in itself (p. 165)

The lecturer's monopoly on the right to speak (or to decide who will speak), the presumption that truth exists and that the lecturer can represent and deliver it to an audience are features of all lectures; it is my claim that they correspond in some ways to the salesperson's extended holding of the floor during the 'pitch'. The lecture genre, whatever its subject content, is already a mode of persuasive talk that tries to 'sell' its audience on both the truth of the ideas presented and, implicitly, the authority and status of both the lecturer and the sponsoring institution as purveyors of truth and knowledge.

Within the 'persuasive' framework of the lecture, math lecturers use other discourse techniques that further the impression of lecturer as pitchman. All the lecturers in my study used the rhetorical question to some extent, and most used the old sales trick of raising 'fake' questions and objections on behalf of the listener and then answering them with prepared responses, as if the audience's real questions had then been addressed.

You might think it's just square root of one plus one over x plus one over x squared, but it isn't, it's the negative of that. Let's see why it's the negative of that rather than just the square root (White)

Somebody's going to say: "Why not have infinity on both the input and the output sides?" Well, why not? (White)

So you've got a big numerator over a big denominator and at first glance you might say: "I haven't the faintest idea what's happening" But you should be careful because this x fourth is more ambitious than the x cubed. I think if you think about it for a moment you can sense that the x fourth is sooner or later going to completely outstrip the
x cubed and it's going to be more or less like one over x (White)

And now you've got two things and they're getting large and they're working at cross purposes and now it's not clear what's going to happen and maybe somebody's going to say: "It's getting large positive, this is getting large negative, you're going to get zero". No, you're not To see what you get this time, again we pull our famous stunt of dividing top and bottom by the same thing, so here goes (White)

Now you want to know: "Where is this function maximum?" And the first thing you might think to do, well, I know that the turning points or the points when that slope is zero is when something happens on the function. OK? (Green)

And then you might want to say: "OK then I'll take the second derivative and use that to tell whether or not I have a maximum or a minimum". Now that works fine a lot of the time. But a lot of the time it doesn't help you So we're not going to do the second derivative. We're just going to do the first derivative, then we're going to think about what's happening to this function. (Green)

Schmidt and Kess (1986), in their study of linguistic persuasion techniques in television advertising and televangelism, cite experimental studies on the effect of rhetorical questions on persuasion. It seems that the effect varies with the listener's degree of involvement:

In the case of rhetorical questions [... ] high-involvement subjects actually found the use of this type of question distracting, making them less sensitive to the quality of argumentation than they were when the same arguments were presented as assertions. Low-involvement subjects, however, showed greater sensitivity to the quality of argumentation when the message contained rhetorical questions than when it did not. This effect was explained by the fact that rhetorical questions essentially ask the hearer to think about the topic, thereby increasing the level of message-based thought for low-involvement subjects, a processing strategy which they would not normally have engaged in to the same degree. (p. 31)

These findings fit well with the argument that math lecturers use 'hard-sell' persuasion techniques: the whole idea of a 'hard sell' is that the prospect is hard to sell to, an unwilling listener, a "low-involvement subject". Rhetorical questions are effective in persuading such a listener, although counter-effective in persuading a listener who has greater interest and commitment to the topic.

The particular style of rhetorical questioning noted in the examples above, which we could call 'fake dialogue', has been noted in other types of lectures and other discourse styles as well. Leith and Mcyrston (1989), discussing a university English lecturer, note that:

- in effect he holds a dialogue with imagined voices... articulating the opposed position, and he assumes that some of his students at least some of the time may identify with those voices (p. 16)

In a later discussion of a political speech, they find a similar example of 'fake dialogue':

The speaker holds a dialogue not only with the audience, not only with opposite but absent voices, but also with the previous speaker at the conference. (p. 23)

Goffman (1981), in his discussion of radio talk, finds radio announcers engaging in 'fake dialogue':

So announcers must not only watch the birdie; they must talk to it. Under these circumstances, it is understandable that they will often slip into a simulation of talking with it. Thus, after a suitable pause, an announcer can verbally respond to what he can assume is the response his prior statement evoked, his prior statement itself having been selected as one to which a particular response was only to be expected. Or, by switching voices, he himself can reply to his own statement and then respond to the reply, thereby shifting from monologue to the enactment of dialogue. In both cases the timing characteristics of dialogue are simulated (p. 241)

A further example of 'fake dialogue' has been noted by Ervin-Tripp and Strage (1985) in interactions between parents and pre-verbal children in which parents:

interpret burps, or single- or two-word constructions as if children have elaborate intentions, confirming, expanding and elaborating them. (p. 73)

All these examples of 'fake dialogue' have in common the fact that they are addressed by a speaker to an audience that cannot respond, either because they are not present (in the case of the radio announcer or the political speaker), because they are incapable of responding (in the case of infant-directed talk), or because they are socially constrained from responding (as are the audiences for the university lecturer or the speaker at the political conference). The pedagogical question then becomes: "Should students learning mathematics be treated as if they cannot or must not respond?"

The lecturers made extensive use of tag questions ("Right?", "OK?", etc) to elicit tacit audience consent. These were used to obtain both agreement with a statement made by the lecturer ("It's positive ... OK?") and permission to move on to the next section of the lecture ("Very stringent requirement. Alright? So now I'd like to do this case.").

Now another thing you want to make sure to understand is that, hey look, two ordered pairs are the same if and only if their first co-ordinates have to be the same and their second co-ordinates have to be the same. Right? Very stringent requirement. Alright? So now I'd like to do this case. (Brown)

When theta is equal to zero I get this point. When theta is equal to pi over two you get this point. The in-between points, just take my word, OK? (Brown)
The farmer has to, with whatever fencing he has to, has - perhaps we should say she - wants to build a rectangular pen. OK? (Green)

When } is negative the square root of } is in fact minus } because it’s the positive square root, OK? And } is negative. Think about it. (White)

This use of tag questions is typical of the persuasive talk of both the salesperson and the conjurer (of which more later). Tag questions have several effects in terms of persuasion. When an audience is addressed with such tag questions, and if there is no loud and vocal audience protest, the assumption is that there is tacit agreement with the point the speaker is making or that permission has been granted for the speaker to move on to another topic or example. Presumably, the speaker is ‘reading’ the faces of the crowd and seeing approval there, in the subtle form of slight nods of the head, hints of smiles, approving eye contact. In fact, the members of the audience are seated so that they cannot see one another’s faces and are taking it on trust that the speaker has received approval from the majority of the audience. Short of a minor rebellion in the ranks, the audience is carried along on the lecturer’s unstoppable stream of talk. The implication is that any rational person, having signalled acceptance of one link in the lecturer’s chain of logic, will be led on inexorably to the next link, and the next, and finally reach the one inevitable conclusion.

There are other phrases that seem to be taken straight out of a sales pitch that appear in the examples above and elsewhere in the tapes. Brown’s ‘just take my word’ is nearly the same as the salesman’s ‘trust me’, White’s ‘think about it’ (with no pause allowed for people to really think about it) has much the same ring.

It may be appropriate to make a further comparison here with the ironically persuasive discourse of the conjurer, ironic because we are persuaded at the same time as we know that we are being tricked. (In some ways, this is not very different from being persuaded by a very skilful salesperson whose ‘tricks of the trade’ are very evident - we are persuaded in spite of our better judgment.) Brown’s and White’s lectures bear particular similarities to conjurers’ discourse. At several points Brown slowly and laboriously builds up audience approval of very elementary and obvious statements (like the conjurer’s ‘test’ of his materials with a volunteer from the audience: “Is this a solid table? Pass your hand underneath it. No holes in this scarf?” etc.), then, like the conjurer, he performs a series of rapid moves and, presto, something unexpected has appeared.

Now another thing you want to make sure to understand is that, hey look, two ordered pairs are the same if and only if their first co-ordinates have to be the same and their second co-ordinates have to be the same. Right? Very stringent requirement. [and a very obvious point] Alright? So now I’d like to do this case – I’d like to represent this parabola very simply like this [followed by rapid and unfamiliar algebraic formulation] (Brown)

Both White and Brown hint at the mechanics of illusion - disappearances, levitation, things mysteriously expanding or shrinking - which are metaphors common to the magician and the mathematician. Both make marvellous claims or set challenges to the audience in the course of their lectures, then carry out demonstrations to prove their claims are true.

So what I’m saying when I say “the limit of one over } squared is plus infinity” is, really, I’m saying this: “You give me a high horizontal line, and I’ll show you how to get the graph to stay above that line”. (White)

You give me any number no matter how big a number you give me, I can show you how to make one over } squared even bigger than that, and stay bigger than that. (White)

Now in this case, you see, the difficulty we faced here disappears! (Brown)

Now I claim this is a function! You see [demonstration follows] (Brown)

To see what you get this time, again we pull out famous stunt of dividing top and bottom by the same thing, so here goes. (White)

Who are ‘we’ anyway?

Pimm (1987) has discussed the use of the first person plural pronouns (‘we’, ‘us’, ‘our’) in the mathematics classroom. I would like to apply some elements of his analysis to the non-standard uses of ‘we’ which appear frequently in calculus lectures. I also draw some conclusions about the use of ‘we’ in persuasive discourse.

Pimm (p. 67) outlines four familiar contexts in which the non-inclusive we (a we that does not really include both the speaker and the listener) is used. These are baby talk (“We’re getting you dressed, aren’t we”), hospital and doctor’s surgeries (“Now we are going to take your temperature”), formal discourse (“In our attempt to analyse addition ... we are thus led to the idea ...”) and school use (“Susan, we never bite our friends”).

Ervin-Tripp and Strage (1985) note that features of the baby talk register:

are also found in speech to pets, dolls, hospital patients, lovers, and the elderly [and hypothesize that] they may indicate affection or protectiveness of the weak. (p. 72)

Some conclusions Pimm draws are that this atypical use of ‘we’ may indicate a hearer who is unable to respond (a use that is functionally related to the ‘fake dialogue’ discussed earlier), or may signal a relationship of power and dependence and carry an implicit message of condescension, indicate social convention being conveyed, and/or point at the existence of an ‘in-group’ of mathematicians who carry authority within the field of mathematics. He also mentions the use of we in textbooks as attempting “to enroll the (at least tacit) acquiescence of the reader in what is being expressed” (p. 68), a use parallel to that of tag questions discussed above.
He questions the effect of this ‘we’ on the personal or the self within mathematics, and cites several examples where there is confusion about the referent for ‘we’ I have found a number of similar examples in the math lectures in this study.

Now what we want to talk about today is parametric equations (Brown).

But before we do anything we have to figure out ways to describe the curve in a more satisfactory manner. (Brown)

But we will try to show you people a better way of describing a curve (Brown).

So now, here is the idea that will save us. (Brown)

And we say this limit is plus infinity. Which is just a way of speaking. All it means is this: You give me any number no matter how big a number you give me, I can show you how to make one over x squared bigger than that… (White)

There is a good deal of confusion in these examples about exactly who is included in ‘we’ Consider the ‘we’ who want(s) to talk about parametric equations: is it just the lecturer, the lecturer and the students (unlikely!), the lecturer and the rest of the math department, or mathematicians in general?

In Brown’s next two quotations, there is a contradiction in the referent: The first statement (“But before we do anything…”) seems to include the listeners, perhaps in a condescending way as a non-responding audience, in the group of those who “have to figure out ways to describe the curve in a more satisfactory manner.” But in the subsequent quotation (“But we will try to show you people a better way of describing a curve”), the listeners have been separated out as ‘you people’ and are excluded from the ‘we’ Who, then, is the referent?

As in the first example, it seems possible that it could be the lecturer, the lecturer and the math department, or the depersonalized authority of mathematics in general. Similarly, White’s “And we say the limit is plus infinity” must exclude the listeners; otherwise there would be no need to explain the statement further. This ‘we’ seems to refer more clearly to mathematicians in general (an in-group which excludes first-year math students). Brown’s “So now, here is the idea that will save us” is a bit more difficult to interpret. Does it mean “Here is the idea that will save you, the students, when you try to work on this problem”? Or does that ‘us’ refer to “me, you and mathematicians in general”?

(There is also the problem of interpretation of the word ‘save’ here. In its ordinary meanings, it carries a hint of evangelism (being saved by an idea or a belief – ‘eternally saved’?) There is also the connotation from mediaeval science of ‘saving the appearances’ – finding analytic processes that allow a fit between theory and observed phenomena. Or it may just mean that a conventional practice may save students from having to work out a more complicated solution to a class of problems.)

Many of the examples of non-standard use of we fall into two patterns: let’s, and we + do. The use of let’s has an air of invitation about it, as well as the implication of condescending baby talk register mentioned earlier. The use of we + do seems to fall within Pimm’s category of “a common fear in mathematics of involving, and hence exposing, the self” (p. 70). In these examples, we + do usually seems to mean I + do, and the confusion this creates is evidenced by White’s midstream changes of pronoun (“we still need to do something…” and what I decided to do…”). Green’s “It’s easy to see…” is worth remarking. Elsewhere in her lecture she uses other encouraging phrases (“It’s easy, right?”, “Now we’re almost done”, “You don’t even have to think any more – well, a little bit”), which again recall the somewhat condescending, power-dependence relationship typical of baby talk, doctor-patient talk, and so on: Tag questions, rhetorical questions, non-standard use of we, and “making encouraging noises” co-occur in many of these examples, as they do in the baby talk register:

Let’s just take a look. When t is equal to zero, what is my x? x is equal to minus one, y equal to minus one, that’s this point here. So let’s say t equal to one, let’s see what it looks like. (Brown)

Let’s draw a picture over here. (Green)

In order to see this – initially, we have to do it by brutal force. (Brown)

Now what happens on the other side of 75, when t is between 75 and 150? It’s easy to see that the slope is now negative. So if we look at our picture, our tangents go in the other direction. (Green)

OK, now we’re going to get into lots of trouble I think if we start doing this, because all these things – we’re not going to be able to control the variables very well. (Green)

Now let’s see if we can see what’s going on and if we can’t, we’re going to have to do more to it. (White)

And we still need to do something with it. And what I decided to do with it was to divide numerator and denominator by x, but carefully. (White)

There is a link between the non-standard use of ‘we’ and the language of persuasion in the mathematics lecture. Charles Ferguson, the linguist who first defined the parameters of baby talk, writes:

The BT [baby talk] register often serves the purpose of coaxing or persuading […] This ‘wheedle’ use, as Read calls it, can even be extended to objects or to unknown addressees. Read offers, among other examples, Alexander Woolcott’s amusing habit of using BT in addressing his dice at backgammon […] Another example of extension is its use in certain forms of advertising. (1977, p. 231)

Again, an analysis of discourse features of the mathematics
lecture genre leads to an interpretation of the genre as persuasive talk

One final point I would like to consider in this discussion is the role of metaphor and neologisms (new coinages) in the genre of the mathematics lecture. The lecturers in this study all used metaphor extensively, along with metaphorically-based neologisms. The metaphors I found fell into two categories: personification and metaphors of physical movement over time (usually to describe a static graphic image or a numerical sequence)

As soon as \( x \) gets a decent ways away from zero it's safe to divide by it And if it's approaching minus infinity it won't be long before it leaves zero way behind (White)

The limit as \( x \) approaches zero from zero's left hand side (White)

Now a vertical asymptote is simply a vertical line like in this case the \( y \)-axis that the graph snuggles up to because it's having some kind of an infinite limit And it's alright if it just snuggles up to it on one side, it's still an asymptote And it doesn't have to snuggle up to both the positive and negative extensions of the line, just the one end of it (White)

The feeling about that, now you've got a war between a term that's blowing up and a term that's going very very negative and our feeling is I think that this one wins And that feeling is right because you could write it this way. (White)

Inside here, the one over \( x \) and the one over \( x \) squared also die out, but the one survives so you get minus one (White)

So the thing is not blowing up, it's blowing down! (White) [a neologism based on the metaphor of 'blowing up' meaning approaching infinity - 'blowing down' means approaching negative infinity]

A couple of squares went flying! [...] By that time everything was OK. (Black)

Metaphor is such an accepted feature of mathematical talk and math teaching that it would be hard to imagine a metaphor-less mathematics. I think it is clear that metaphor and neologisms play a role in describing, by association, new relationships and phenomena (and indeed it has been suggested that all natural languages are made up mostly of extended metaphors). Bergmann (1982) describes some metaphors as 'fecund' or having 'organizing power' (p. 491), and this power can be very useful in mathematics (or sometimes counter-productive when the metaphor is stretched beyond the limits of its organizing power). Having taken all this into consideration, I would like to suggest one other use of metaphor in the context of the mathematics lecture - its use as a tool of persuasion.

Sandell (1977), summarizing research into the persuasive effects of linguistic style, writes:

The use of metaphors and similar figures has been subjected to some research Bowers and Osborn (1966) varied the final parts of two different speeches in two versions: one literal and one metaphorically intense [...] On both speeches, the effective difference between the versions was significant, the metaphorical ending increasing attitude change in the advocated direction. Perception of competence, trustworthiness, and ingenuity was affected in a very complex way by interactions between alleged speaker, topic, and type of metaphor [...] In a recent study, Reinsch (1971) investigated the effects of the metaphor and another related figure, the simile, in persuasive discourse. Supporting Bowers and Osborn, he found metaphors aiding persuasion to a significant degree, compared to a literal version. As he predicted, the effects of similes was [sic] between those of the metaphor and the literal versions (p. 77)

And in their study of television advertising and televangelism as persuasive language, Schmidt and Kess (1986) identify neologisms as a key feature of persuasive discourse. Lakoff (1982), in a discussion of persuasive discourse, noted that an essential identifying feature of persuasive communication is its quest for novelty of expression. As she puts it:

persuasive discourse wears out; ordinary conversation does not (p. 31)

Evidence for the role of novelty in persuasive communication was taken from examples of television advertising which were found to exhibit the following types:

1) lexical novelty or neologism (e.g., devilicious)
2) morphological or syntactic novelty (e.g., the soup that eats like a meal) [...] Lakoff accounts for this extensive use of linguistic novelty as follows. First, anything neologistic, because it violates the Maxim of Manner [one of H. P. Grice's Principles of Conversation], draws attention to itself, and by capturing the hearer's attention increases the impact of the message. Second, through this violation of the Co-operative Principle, neologism forces the hearer to interpret, and therefore to participate in the discourse. According to Lakoff, this active role played by the hearer, in turn, enhances learning and retention, and consequently also persuasion (Schmidt and Kess, 1986, pp. 30-31)

Again there is a strong connection possible between the language of the mathematics lecture and the language of persuasion.

Postscript: how has genre analysis affected my own teaching practices?

Beyond specific classroom activities in the high school where I teach, there is a change in the tone of teaching that has come as part of my awareness of pedagogical genre. For example, I do not see myself as a salesperson for mathematics, nor do I think of the act of teaching as a form of hard-sell persuasion. In my classes, I consciously work against the hard-sell lecture genre (although I cannot say
whether some elements of this genre may still unconsciously be present!).

I expect that my students will often conceptualize things differently from me, and I am interested in learning about their concepts. I expect to be surprised and to continue to broaden my learning about both mathematics and education by listening to students’ ideas and concerns. I expect that I will make mistakes, not know things, and behave foolishly from time to time. In the role of teacher, I see myself as inviting students to enter further into the culture of mathematics, and I try to share my enthusiasm and perplexities with that culture. It is difficult to characterize my own metaphors for classroom time and space, but they are different from those of the hard sell.

Up till now, I have used genre analysis only as a topic of teacher and student talk. Perhaps this could be greatly expanded by engaging in activity and research around genre analysis – for example, when students research a mathematical topic, I could ask them to present their findings using several different genres; or students could work on genre analysis of mathematics textbooks, television shows or lectures. The focus would be on the different intended and perceived purposes embedded in different genres, and on a mastery and enjoyment of genre-switching.

In my research on initial calculus lectures, I contrasted the discourse of first-year calculus classes with that of a senior undergraduate mathematics class. There was a marked difference in discourse style, and the tone of the senior class was in many ways parallel to the tone I try to establish in my high school classes.

Professor Black’s fourth-year undergraduate math classes did not operate in the ‘math-as-hard-sell’ genre. For example, his lectures contained many fewer examples of the non-inclusive use of ‘we’ than did the first-year course lectures. Black used the personal ‘I’ extensively instead: “What I’d like to do today...”, “My solution...”, “I’ve got compatible states...”, “I’ve got closure”. Black based his discourse on a master-apprentice rather than salesperson-prospect imagery. The more intimate tone was that of a seasoned mathematician doing math with a group of junior colleagues, and the ‘master’ was willing to admit he sometimes made mistakes.

Black frequently stepped out of the frame of the lecture to make parenthetical asides to his listeners. Most of these asides were ironic or even self-denigrating comments on his own teaching:

The example I came up with was wrong. I don’t know why I gave it. It was garbage (Black)

I haven’t done this one before. I’m looking forward to checking my solution against yours (Black)

This is being done just as a teaching technique (Black)

I think that that point is worth the extra minutes it’s taken me. (Black)

Black’s tone conveyed an impression of modesty, of being on a more egalitarian footing with the listeners and an air of consultation with the audience, even within the undergraduate math lecture genre. It stood in marked contrast to the persuasion techniques used in the first-year classes, where students were treated as ‘low-interest sales prospects’ rather than as junior colleagues.

References
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