Conditions and Strategies for Emancipatory Mathematics Education in Undeveloped Countries*

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Reflecting on the author's experiences in Mozambique, he suggests some conditions and strategies for mathematics education to become emancipatory in underdeveloped countries

Point of departure: mathematics education cannot be neutral

By its physical and intellectual labour, mankind is able to create an *ever more human* society Reflecting on its realisations, discovering the laws of nature and society, mankind creates its material and intellectual tools to transform reality, both nature and society. *Mathematics* constitutes an integrated body of those means to understand and transform reality. An ever more human society is the most rational direction. However, mankind has at its disposal, nowadays, the means to destroy itself Neither mathematics, nor mathematics educaiton nor mathematicians can be indifferent towards these diametrically opposed possibilities:

more human	more inhuman
peaceful	war-like
liberating	oppressing
creating	destructive
emancipating	exploiting

The history of Mozambique shows very clearly that mathematics education cannot be neutral [see Gerdes 1980, 1981 b] During the Portuguese domination, mathematics was taught, in the interest of colonial capitalism, only to a small minority of African children [see Mondlane 1969]. And those Mozambicans were taught mathematics to be able to calculate better the hut tax to be paid and the compulsory quota of cotton every family had to produce. They were taught mathematics to be more lucrative "boss-boys" in South African mines. After ten years of the people's war, led by the liberation movement FRELIMO, Portuguese colonialism was defeated and Mozambique achieved its Independence in 1975. Post-independence objectives of mathematics education are in the service of the construction of a socialist society [cf. Machel 1977, Ganhao 1978]. Mathematics is taught to "serve the liberation and peaceful progress of the people" Mathematics is taught to place its applications within the reach of the worker and peasant masses. Mathematics is taught to stimulate the broad masses to take an interest and delight in mathematical creation. These are general objectives But how are they to be achieved?

Mathematics education for emanicipation. How?

Independence and the option for socialism have implied an overall *democratisation* of mathematics education in Mozambique: the majority of children, and hundreds of thousands of adults, have already gained access to the science of mathematics; the mathematics teaching profession has been opened up for children of peasants and labourers; and the discussion on how to improve the quality of mathematics education is not any more reserved for an elite but is becoming the object of reflection of the mass the teachers, from adult education and primary school teachers to univesity professors [cf. Gerdes 1981 b]. Overall democratisation is a *necessary but not sufficient condition* for mathematics education to become really emancipatory — everyone mastering mathematics and capable of thinking mathematically, to the benefit of society as a whole. On the basis of the author's experience in the training of mathematics teachers, some other necessary conditions will be presented.

Problemising reality in classroom situations

Let us briefly listen to some dialogues occurring in our lessons:

> A student enters the classroom, late and noisy. Confrontation. Could your parents study in colonial times? No ... Who pays for your study? (With Independence, education became free of charge in Mozambique) The peasants and labourers .... How may days does a peasant have to work to pay for your being late? What do you mean? How much does it cost? What do we have to know to calculate it? (In a concrete case it was calculated, by the class, that one student coming one hour late corresponds to throwing away one day of a peasants' work.)

> Newspaper reading, a photograph: a truck crossing a bridge over the Changana River, the bridge collapsed. Why? Was the truck too heavy? The bridge ill-built?

> In a report by the Center of African Studies at our University it is told that a tractor driver drove his machine at full speed for two hours to fetch only three loaves of bread (from a state farm in Moamba). Why did he do so? Was it justifiable? Is it reasonable? Why? Why not? How can his behaviour be explained? How can the cost be evaluated?

> For a certain period, sugar production was going down. Why? Changes in the way of paying the labourers had been introduced: from payment in terms of the number of rows of sugarcane cut down to the number of kilograms of sugarcane. Why? How can we explain the economic consequences? How can production be raised? Is mathematics involved?

> Our country suffered floods in 1977 (Limpopo River) and in 1978 (Zambeze River). Why didn't we have any floods in the early eighties? Is mathematics involved? How?

> To combat speculation a food distribution system has been introduced in the city of Maputo (1981). How much rice for each person this month? How much rice can be eaten by a family of five persons each day?

These dialogues are examples of problemising reality (Freire's terminology). "Reality is thrown before our feet" (in the deep sense of the original *προ-βαλλειν*); one cannot fly from reality, one has to reflect. Experience shows that problemising reality leads to consciousness, to awareness of the *relevance* of mathematics as a tool to understand and transform reality. It leads to political awareness (as in the case of the student who came late); to physical awareness (as in the case of the bridge that collapsed); to economical awareness (as in the example of the tractor driver), etc.

Let us give some more examples of problemising reality before drawing a second conclusion — examples that give a meaningful, rich context to mathematics education.

> In Cabo Delgado province some students were told that the area of a cottonfield is so many metres. Area measured in m and not in m^2? Does it make sense? Why?

> Mozambique's soap production was 16300 t in 1980 and 23700 t in 1981. What will the national needs be in 1990? How can they be calculated? What will the soap production be in 1990? Regular growth of production capacities or not? Regular in what sense? Why?

> In order to win the battle against underdevelopment during the decade of the eighties, the People's Assembly approved a National State Plan for the decade. We need x schoolbuildings by 1990. In 1981 we already had y schoolbuildings and will construct z more schoolbuildings. How do we have to extend production capacities for the construction of schools? Regular growth? How regular? Linear? Parabolic? Exponential? (A possible entrance to the study of geometric or exponential progressions.)

> Women fetch water in their tins, cylindric cans, on their heads. Are they exploited? Development will lead to piped water in each house. And in the meantime? could the tin cans be less heavy? Could more cans be produced out of the same quantity of raw materials? How? What changes are implied? (A possible entrance to the study of differential calculus.)

Here it should be emphasized that a problemising reality approach leads to a *real understanding* of reality, it leads to an understanding of mathematics as a tool to transform reality.

Let us give one more example to reinforce this second conclusion [see Gerdes 1982].

> Rickettsiosis is a disease that kills a lot of cattle in our country. Veterinarians discovered a medicine to cure sick animals *(terramycine)*. How much medicine is needed for each animal? This quantity depends on what? On the colour of the animal? its height? How does this quantity depend on the weight of the cow? Linear dependence? Exponential? Why? How do we weigh a cow? Only on well-equipped farms will you find scales able to do so. What can be done in the less developed countryside? The weight of an animal is related to . . . ? Why? How? Is it possible to determine the weight of a cow in an indirect way? A cow knows how to swim? What does this imply? Relation weight-volume? How can the volume of a cow be determined? Let us take a very close look at the cow: approximation to a cylinder! (see Figure 1). And so on.

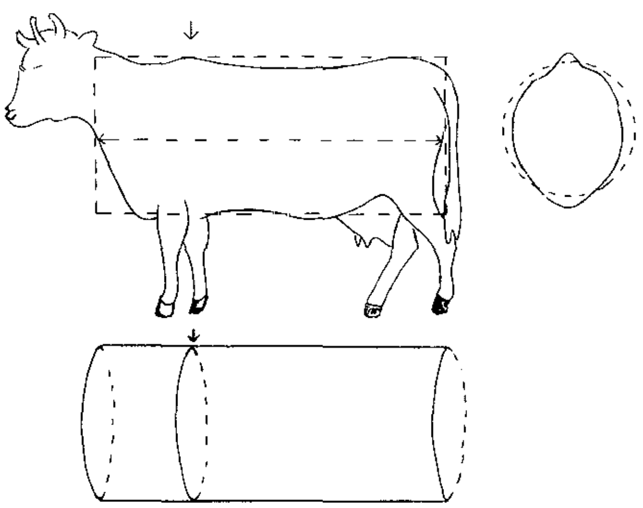


Figure 1

Reality can be changed More animals will survive. People will have more meat and milk. Improved calculations lead to a reduction in the quantity of medicine. And, as these medicines still have to be imported, well-applied arithmetic will save foreign currency.

Problemising reality *motivates* the pupils and gives an entrance to powerful mathematical methods.

Creating confidence

For mathematics education to become emancipatory, it is necessary to stimulate confidence in the creative powers of every person and of every people, confidence in their capacities to understand, develop and use mathematics. At least three types of strategies to produce such confidence have to be considered

Cultural strategies

Colonization has implied the underdevelopment of Mozambique as of most Third World countries. Underdevelopment is not only an economic process. Foreign domination also caused *mathematical underdevelopment*. African and American-Indian mathematics became ignored or despised; the mathematical capacities of certain peoples were negated or reduced to rote memorization; mathematics was presented as an exclusively white men's creation and ability Of the struggle against the mathematical underdevelopment and the combat against racial and colonial prejudice, a cultural reaffirmation makes a part. It is necessary to encourage an understanding that:

1) The people have been capable of and will be capable of developing mathematics

Examples:

a) In coastal zones of Mozambique fish is dried to be sold in the interior. How should the fish be dried? What if you place the fish in the following way? (see Figure 2) Some fish will be grilled, while others are kept wet. Through experience, the fishermen discovered that it is necessary to place the fish equidistant from the fire. They discovered a circle-concept, constructing a circle in sand using two sticks and a rope (see Figures 3 and 4).

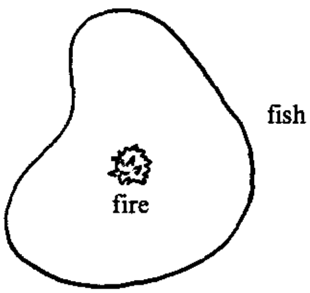


Figure 2

Figure 3

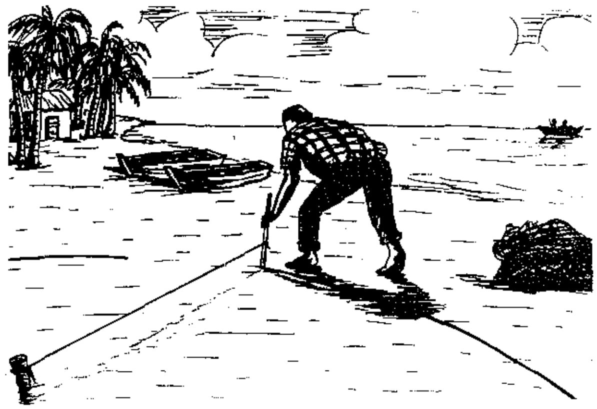


Figure 4

(drawings by Marcos Cherinda)

b) How can the rectangular base of a house be constructed? In some parts of Mozambique, peasants use the following technique One starts by laying down on the floor two long sticks of equal length (see Figure 5). Then the first two sticks are combined with two other sticks, also of equal length (Figure 6). Now one moves

Figure 5

Figure 6

the sticks until the closure of the quadrangle (one obtains a parallelogram: Figure 7). One further adjusts the figure until the diagonals are of equal length (so a rectangle is obtained: Figure 8). Now where the sticks are lying on the floor lines are drawn and the building of the house can start.

Figure 7

Figure 8

This empirical knowledge of the peasants has been propagated on a national scale, during the rabbit-raising campaign, for the construction of hutches.

2) The people's mathematics can and will enrich the understanding of mathematics, its education and its history
Examples:

a) In the north of Mozambique, traditional villages are structured in a "circular" way (Figure 9). This village structure and e.g. the structure of a bird's nest led to the formation of one circle-concept "to belong to" (already a high-level abstraction). The concept is extended by comparison of form or content ("belonging to") to e.g. the border of a basket (Figure 10)

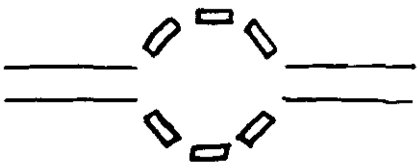
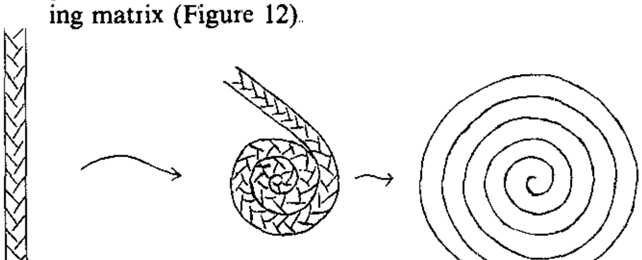


Figure 9

Figure 10

The fabrication of a sisal mat leads to the formation of another circle-concept ("spiral-circle": Figure 11). We have still a third circle-concept, that of the fishermen. In the official language of Mozambique, Portuguese, there are two "circle" concepts, one referring to the area and the other to the circumference of a circle. By interference between the mother tongue and Portuguese, the medium of instruction, we have the following matrix (Figure 12).



rolling up a string of sisal

Figure 11

mother tongue

Portuguese	M_1	M_2	M_3
P_1	$P_1 = M_1$	$P_1 = M_2$	$P_1 = M_3$
P_2	$P_2 = M_1$	$P_2 = M_2$	$P_2 = M_3$

matrix of language interference: circle-concepts

Figure 12

So six different, rather complicated, situations may be encountered: *now I have to use the third word* (= M_3) *in my mother language but the second word* (= P_2) *in Portuguese...*, etc. How can this be avoided? Comparing the Portuguese "circle" concepts with the square concept, we can see that there is no need to have more than one "circle" concept: it is possible to speak about the area of a square and about the circumference of a square using the same work "square". Paradoxically, by abolishing one of the two words in Portuguese, one enriches the scientific language: only one "circle" concept is retained and so one facilitates the learning processes of the students in this multilingual context.

b) In some regions of Mozambique, there exists a "cylinder" concept that can be described as a rolled-up rectangular mat (see Figure 13). This concept can be used as a way to discover a formula for the volume of a cylinder. It can be used not only in education, but also in formulating hypotheses on how long ago the first (approximating) formulas for the volume of cylinders could have been discovered [cf. Gerdes, 1985 b].

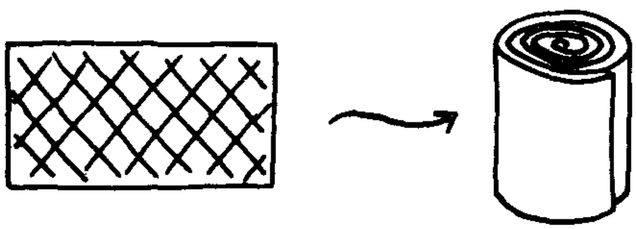


Figure 13

3) Every people is capable of developing mathematics
This can be shown by the *cultural history* of mathematics. By public lectures and by the publication of booklets on African, Indian, Chinese and Arabic mathematics, attention is drawn to the fact that many people have contributed to the development of mathematics. In this manner an attempt is made to combat an eurocentric and distorted vision of the history of mathematics. The booklets try to analyse, in a way accessible to a broad public, not only how, but also why and for whom, mathematics was developed in different societies at distinct times [see Gerdes 1981 a, 1984 a]

B. Social strategies

Within colonial society, as in every class society, prejudices about the mathematical talents of discriminated and exploited social strata and of women are widespread. For mathematics education to become emancipatory, it is necessary to encourage an understanding that:

> *children of all social classes and of both sexes have been capable and will be capable of mastering, developing and using mathematics.*

By means of *counterexamples*, ranging from Hypathia of Alexandria and Gauss to the winners of our National Mathematics Olympiads, sexist prejudices as well as those against children of peasants and factory workers are demystified. Special awards and scholarships are attributed to the best qualified girls in the Mathematics Olympiads [see Gerdes 1984 b]

C. Individual-collective strategies

All the forementioned strategies are already individual in the sense that they enhance personal self-confidence by stimulating the individual's relatedness to mathematics through the problemising reality approach, by supporting the comprehension of the relevance of mathematics, and by cultural and social self-confidence. More specific — say individual-collective strategies — are presented here by examples drawn from teaching experiences

a) Reflection on errors

Usually "solutions" of problems are collected and presented to the students, e.g., the following two solutions of the same problem (Figure 14).

$$(1)\quad \frac{\sqrt{5}+1}{\sqrt{5}-2}=\frac{\sqrt{5}+1\cdot\sqrt{5}-2}{\sqrt{5}-2\cdot\sqrt{5}-2}=\frac{\sqrt{5}+1\cdot\sqrt{5}+2}{(\sqrt{5}-2)^2}=$$

$$\frac{\sqrt{5}+1\cdot\sqrt{5}-2}{9+4\sqrt{5}}=\frac{(1-2)\sqrt{5}}{9+4\sqrt{5}}=\frac{-\sqrt{5}}{9+4\sqrt{5}}$$

$$(2)\quad \frac{\sqrt{5}+1}{\sqrt{5}-2}=\frac{(\sqrt{5}+1)(\sqrt{5}-2)}{(\sqrt{5}-2)^2}=\frac{3-\sqrt{5}}{9-4\sqrt{5}}=\frac{1-\sqrt{5}}{3-4\sqrt{5}}$$

$$=\frac{1-1}{3-4}=\frac{0}{-1}$$

$$=0$$

Figure 14

The students have to analyze these trial solutions, individually. Which transitions are right, which are wrong and why? In a second stage, they are asked to compare their analyses at group level and to try to achieve a common improved examination. The groups report their findings in a classroom session for a final analysis

It is interesting to see that the students like this type of "exercise", because it urges them to reflect, argue and rethink.

b) Reflection on concept building

The students know the following definition: When $a \in R_0^+$ and $n \in N$, we define ${}^n\sqrt{a}$ as the postive solution in R of the equation $x^n = a$.

We ask them to debate questions such as the following: Is it possible to define ${}^0\sqrt{a}$? For which numbers? Does it make any sense?

Is it possible to define ${}^1\sqrt{a}$? Is it necessary to do so?

Is it possible to define ${}^{-3}\sqrt{a}$? If so, calculate ${}^{-3}\sqrt{8}$. The questions are open-ended. E.g., in the last case, one finds answers by analogy: ${}^{-3}\sqrt{a}$ is the positive solution of the equation $x^{-3} = a$; or by a supposed property: ${}^{-3}\sqrt{a} = 1/{}^3\sqrt{a}$. This reflection stimulates further debate and so provides a more profound understanding of the original concept

c) Learning to discover by discovering together

What will be the derivative function of $f: R_0^+ \to R$, defined by $f(x) = \sqrt{x}$?

The students know: $\Delta y/\Delta x = (f(x+\Delta x) - f(x))/\Delta x = (\sqrt{(x+\Delta x)} - \sqrt{x})/\Delta x$

How do we calculate $\lim\limits_{\Delta x\to 0} \Delta y/\Delta x$?

How do we divide Δy by Δx? $\sqrt{(x+\Delta x)} - \sqrt{x} = \quad ?$

$\sqrt{(x+\Delta x)} = \sqrt{x} + \sqrt{\Delta x}$, Pedro suggests. Is it true?

Another suggestion by Lazaro: square all the terms:

$$(\sqrt{(x+\Delta x)} - \sqrt{x})^2 = \ldots$$

Not that way? Why?

Ferdinand: use another notation

$$\sqrt{(x+\Delta x)} - \sqrt{x} = (x+\Delta x)^{1/2} - x^{1/2}$$

How do we continue?

$$(x+\Delta x)^{1/2} - x^{1/2} = (x+\Delta x)^{2^{-1}} - x^{2^{-1}} = ((x+\Delta x)^2 - x^2)^{-1}\ ?$$

Why do you want to work with squares?

$$(\sqrt{(x+\Delta x)})^2 - (\sqrt{x})^2 - (x+\Delta x) - x = \Delta x$$

How do we arrive there?

In general $a^2 - b^2 = \ldots ?\ a^2 - b^2 = (a-b)\ldots ?\ a^2 - b^2 = (a-b)(a+b)$

And now every student advances:

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} =$$

$$\frac{(\sqrt{x+\Delta x} - \sqrt{x})(\sqrt{x+\Delta x} + \sqrt{x})}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} =$$

$$\frac{1}{\sqrt{x+\Delta x} - \sqrt{x}}$$

and obtains the end result $\lim\limits_{x\to 0} \dfrac{\Delta y}{\Delta x} = \dfrac{1}{2\sqrt{x}}$.

By discovering together and reflecting on their process of discovery, the students enlarge their creative powers and gain self confidence. They learn to understand the nontautological nature of mathematical knowledge [for a more profound analysis, see Gerdes, 1985 a]

Concluding remarks

A problemising reality approach as starting point is in itself already a confidence-creating activity. Problemising reality, reinforced by cultural, social and individual-collective confidence-stimulating activities will contribute substantially to an emancipatory mathematics education, to enable everyone and every people to understand, develop and use mathematics as an important tool in the process of understanding reality, the reality of nature and of society, an important tool to transform reality in the service of an ever more human world.

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Two Poems*

MIROSLAV HOLUB

THE LAST NIGHT BUS

The last night bus echoes away
into the depth
of the night's
spinal cord.

Stars shiver
unless they explode.

There are no other civilizations.
Only the gentle
galactic fear
based on methane.

* See page 11