# Aporism and Critical Mathematics Education [1]

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## A thought experiment

Imagine yourself back in the Middle Ages joining a conference on religious education In the dim light of the conference centre, we are listening to many interesting lectures One lecturer, who happens to come from France, makes an exegesis of the teaching of the Holy Trinity. He has investigated the sources of the Holy Writ and has arrived at new conclusions concerning the basic structure of the Trinity He suggests that the curriculum in religious education, including the teaching of how to pray, should be changed in accordance with his findings Instead of relating to a patchwork of information about the Trinity, the prayers can be categorised into simple structures, and a few basic prayers can serve as 'mother structures' in the development of all sorts of advanced praying. To change the curriculum in accordance with this insight would mean that children become better equipped for further education, which naturally means further praying.

Another lecturer has found it possible to interpret the basic concepts expressing the structure of the Trinity in a way comprehensible to every child, independent of the stage of the child's intellectual development This lecturer has obviously already been listening to the first one Other lecturers announce that they have produced textbooks in accordance with these new ideas about how to teach the Holy Trinity

However, critical opinions are voiced Why not listen to the way children already produce their own simple prayers? Children are already preoccupied with praying! The only thing needed is to push the children smoothly in the direction of praying for the real Trinity. In this way, the teaching of the Holy Trinity can be based on children's own prayers with which they are familiar

This suggestion is supported by scholars not living in the metropolis hosting the conference They explain that in their country they have observed many sorts of old and well-established prayers. Long before praying for the Holy Trinity was institutionalised people have been praying, and in many respects these old prayers anticipate praying for the Holy Trinity The foreign scholars suggest that the religious curriculum should take into consideration these ethnoprayers. Using these as the basis, learning can be directed towards the paradigmatic prayers for the Holy Trinity. But why, in fact, prefer some prayers to others? Why stick to the old paradigm? All prayers might be equal

During the happy hour of the conference somebody raises the following questions Why this concern for religious education? Why teach children to memorise prayers? What are the social and political functions of teaching everybody to pray? What is the purpose? Why not discuss the social function of religious education? These questions might show the voice of critical religious education. [2]

## Critical mathematics education

What, then, might be the voice of critical mathematics education? I do not see critical mathematics education as a particular 'topic', in the same sense that we talk about 'elementary mathematics', 'Cabri geometry' and 'advanced calculus'. Instead, I see critical mathematics education as an expression of some broader concerns about mathematics education. In our chapter, 'Critical Mathematics Education', in the *International Handbook of Mathematics Education*, Lene Nielsen and I (1996) tried to enumerate some of these concerns.

(1) Education cannot be discussed only in terms of preparation for further education or for the labour market Schooling also means preparation for citizenship and participation in social and political life What does this mean to mathematics education?

(2) Could mathematics serve as a tool for identifying and analysing critical features of social life?

(3) How could mathematics education consider the students' interests and competence for the development of knowledge and understanding?

(4) Mathematics education might provide 'cultural filters' by, for instance, being a gate-keeper for the technological society How could questions of equality, equity and justice be reflected in the mathematics classroom?

(5) Mathematics might become a problematic tool for solving a wide range of problems, as mathematics itself is part of the technological society Mathematics cannot only be a tool for critique; mathematics itself also has to be addressed by a critique, and in this sense become an 'object of critique' What does this mean for mathematics education?

(6) Every classroom becomes a micro-society and may represent democracy *in spe* (or otherwise). What does this mean for the interactions between students and teacher in the mathematics classroom?



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I do not think that responses to such concerns can be punctuated into particular claims about the organisation of the mathematics classroom. [3] Critical mathematics education cannot be translated into principles for, say, curriculum development. Nevertheless, I find that educational practices can be discussed in terms of such concerns.

In what follows, I shall concentrate on the mathematical aspects of those concerns. My aim is to search for a working philosophy *of mathematics* for critical mathematics education. In doing so, I shall naturally be keeping in mind that critical mathematics education needs other working philosophies - of classroom interaction, of democracy, etc.

## Absolutism and fallibilism

*Absolutism* refers to the basic assumption that it is possible, at least within certain areas of knowledge, to get access to absolute truths. The question is, however, how paths may be found that lead into this mesmerising landscape. Which faculty of the mind could provide human beings with some knowledge which they could share with an omniscient God? The most tried-out route has been through purified reason.

Different areas of understanding have been proposed as candidates for absolute knowledge, most often mathematics. Absolutism has been supported by the idea that certain kinds of knowledge can be established *a priori*, independently of empirical evidence. *Apriorism* has been manifested in many forms in the philosophy of mathematics: for instance, in trying to build the architecture of mathematics on a foundation of logic, Gottlob Frege wanted to ensure that no empirical elements could destabilise the foundation of mathematics.

It is not surprising, then, that Imre Lakatos chose to outflank apriorism. He advocated *aposteriorism*: the idea that even mathematical knowledge is ultimately based on empirical evidence. In 'A renaissance of empiricism in the recent philosophy of mathematics', Lakatos (1978) refers to a great number of philosophers who, in one way or another, have provided support for aposteriorism. Human senses have cheated the human mind too often; and in classical philosophy, it is a well-accepted idea that those commodities which our senses bring to the stock of knowledge easily become tainted. So, if mathematics becomes pervaded by aposteriorism, then *fallibilism* will follow and certainty will be tarnished by the mould of doubt.

## In search of a working philosophy for critical mathematics education

Fallibilism opens new doors in mathematics education. The constructivist idea, that knowledge cannot be imposed on students but must grow in a natural process of construction, deconstruction, rebuilding, repairing, is difficult to establish within an absolutist framework. Therefore, fallibilism has become a most welcome working philosophy of progressive mathematics education. (See, for instance, Ernest, 1991, 1998.)

However, can fallibilism be a working philosophy for critical mathematics education? My answer is 'no'. Why? Well, not because I find fallibilism to be a 'wrong' philosophy of mathematics. In fact, I agree that many studies demonstrate that Lakatos' (1976) analysis reveals interesting aspects of mathematics - see, for instance, Koetsier (1991). Nevertheless, the problem of fallibilism is its limited scope.

Lakatos' fallibilism is inspired by Karl Popper's (1963, 1965, 1972) critical rationalism and its internalist perspective on science. It is 'internalist' in the sense that the focus is on the development of scientific concepts, and this development is defined only with reference to science itself. 'The logic of scientific discovery' refers to a rational process taking place in what Popper calls the 'third world'; the first world being that of experiences and the second that of physical objects. The third world is inhabited by theoretical constructions. While Popper's philosophy of critical rationalism provides an overall description of the development of the third world, Lakatos provides a description of a particular province of this world, that of mathematics.

This grand theorising of scientific development provides a constructivist description of scientific progress which is linked with fallibilism; this I will label *rational constructivism*. This description does not represent actual history, but the rational history of the 'third world'. This description is without any reference to the social context. The Popperian third world is one which seems to exist and to develop without interaction with social reality.

Rational constructivism represents a conception of epistemic progress. Although Popper gives up on the possibility of ever coming to grasp absolute truths, as all prevailing theories are only not-yet-refuted guesses, he assumes the possibility of getting closer and closer to the truth. Rational constructivism presupposes an epistemic optimism: knowledge is progressing! Lakatos does not make any reference to the notion of mathematical truth, yet still his work shares a rational optimism: science makes progress! (See also Lakatos (1970) for a discussion of the methodology of scientific research programmes.)

The assumption of rational constructivism is that, by being rational, scientific development represents a benign growth. The logic of scientific development does not present us with ethical problems: it represents the ultimate progress intrinsic to science itself.

As already mentioned, Lakatos' work has provided much inspiration for mathematics educators, but it has, nevertheless, a huge limitation as a working philosophy of mathematics for critical mathematics education. Nowhere does fallibilism, as developed with reference to rational constructivism, accommodate a broader and more fundamental interpretation of the social role of mathematics. This fallibilism is deeply ingrained in the internalist perspective on scientific development.

The optimism of rational constructivism, which means a belief in the notion of scientific progress, also legitimises internalism. Therefore, the notion of critique, developed by critical constructivism, becomes intrinsic to scientific development, and is to promote the 'logic of scientific discovery', not to establish an extrinsic critique of this development. Therefore, I find rational constructivism (including fallibilism) inadequate as a working philosophy of mathematics for critical mathematics education.

## Aporism

The Greek word *aporia* can refer to places, questions or persons. With reference to places it means 'difficulty of passing'; with reference to questions it means 'difficulty'; and with reference to persons it means (among other things) 'want of means' or 'want of resources'. [4] The *paradox* which aporism acknowledges concerns the role of mathematics in society.

On the one hand, mathematics, the 'queen of science', seems to be 'pure' by only providing new extensions of human rationality. This must represent a worthwhile enterprise in itself. On the other, this extended rationality seems to play an essential role in social and technological development, manifested in the techno-nature which has come to contain devastating new risk structures

How can it be that the pure queen of science seems to play a contradictory role in social affairs? This paradox has been alluded to by several authors: see, in particular, D'Ambrosio (1994) Is it possible to make a clarifying reflection on the double role of mathematics?

By *aporism*, I refer to a certain perspective on mathematics which pays special attention to this paradox by grappling with the questions: what is mathematics doing to society? How can we conceptualise the social role of mathematics? These two questions will be discussed in what follows.

## Mathematics: a formatting power?

The thesis of the formatting power of mathematics states that, in many cases, social actions and decisions are prescribed by means of mathematics

What social circumstances provide the basis for a certain development in mathematics? This question is closely linked with the concern about the social role of mathematics: what impact does mathematics have on society? From an analytic point of view, it makes sense to separate these two questions, although, naturally, they are closely linked. The thesis of the formatting power concentrates on what mathematics might be doing to society, leaving aside discussion of the social construction of mathematics

It has to be acknowledged that this thesis is more complex than is indicated by its first simple formulation. It is not mathematics as such which is claimed to be doing anything: neither is it targetting mathematicians as a group. Rather, the thesis is about the interplay between mathematical techniques (systems produced in the process of mathematising) and resources for technological development

It makes sense to claim that modern warfare could not exist in any of its present forms without mathematics as a constituent part - see Tymozcko (1994), Højrup and Booss-Bavnbek (1994) and Emmer (1998) Mathematics is integrated in business and economic management in such a way that it does not make sense to try to separate this management from its mathematical expression. Mathematics is an integral part of all aspects of technology. These are just different manifestations of the thesis of the formatting power of mathematics But is the thesis in fact true?

In *A Mathematician's Apology*, first published in 1940, G. H. Hardy discusses the usefulness of mathematics. His general conclusion is:

> If useful knowledge is [...] knowledge which is likely, now or in the comparatively near future, to contribute to the material comfort of mankind, so that mere intellectual satisfaction is irrelevant, then the great bulk of higher mathematics is useless (p. 135)

Could mathematics, nevertheless, do any harm?

> So a real mathematician has his conscience clear; there is nothing to be set against any value his work may have; mathematics is [...] a 'harmless and innocent' occupation. (pp 140-141)

In the final pages of his *Apology*, Hardy writes this about his own particular work in mathematics:

> I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world (p. 150)

The thesis of the formatting power of mathematics contradicts this position directly: the Hardian claim of innocence is not tenable.

To emphasise the complexity of the role of mathematics within society, we can take a look at the notion of 'modelling'. This notion is often used in the discussion of mathematics and its application. 'Modelling' has become part of a well-elaborated 'metaphysics' about mathematics. The idea is simply that 'reality' is 'pictured' by means of mathematics. A model is constituted by a mathematical vocabulary and connections between certain parameters are established in terms of mathematical equations. The different elements in the mathematical model refer to entities in reality, and manipulations of the model provide information about reality. The model, naturally, has to be validated, and this concerns the degree to which the model actually 'pictures' reality. If this is an adequate interpretation of mathematical modelling, then Hardian innocence, most certainly, can be extrapolated to applied mathematics.

However, I find this metaphysics of 'modelling' to be just as problematic as the picture theory of language suggested by Ludwig Wittgenstein (1921/1992) in his *Tractatus Logico-Philosophicus*. I do not think that a picture theory of mathematics-in-use provides an adequate understanding of what mathematics is doing to society. [5] Instead of a picture theory of applications, we have to understand the deeper involvement of mathematics in social affairs.

By means of the notions of *modulation* and *constitution*, I have tried to express the idea that when part of reality becomes modelled and re-modelled, then this process also influences reality itself. Mathematics applied, for instance, in business does not consist of 'pictures' of a reality which exists prior to and independent of the modelling process. Mathematical models of advertising, marketing, investments, etc. become part of the economic reality themselves. They serve as a basis for decision making and for economic transactions. In this way, mathematics has become part of the economic reality. This not only applies to business but to economic policy-making in general: and not only to economy, but to categories like time, space, communication, transport, war.

Modelling becomes a continuous process of the modulation of reality, and the thesis of the formatting power of mathematics can now be formulated as:

> *Fundamental categories of techno-nature are continually modulated and eventually constituted by mathematics*

By talking about techno-nature, I refer to the fact that we no longer live in nature but in an environment reconstructed by means of technology.

The thesis might be true. It might be false. Well, it *might* be true, and this is the possibility which is considered by aporism. [6] The concept of the 'formatting power of mathematics' is at present best treated as a metaphor In order to develop further the thesis that mathematics has formatting power, the notions of 'formatting', 'power' and 'mathematics' are in need of a good deal of further investigation. As a first inspiration for this challenging analysis, I want to refer to a study by Keiko Yasukawa (1998).

In her paper, 'Looking at mathematics as technology: implications for numeracy', Yasukawa examines ways in which mathematics 'works' in society. She emphasises that mathematics must be interpreted as a technology:

> I would suggest that mathematics itself can be viewed as a technology, although it is clearly not associated with an artefact such as computers are with information technology, solar panels are with power generation technology, and ticket machines are with transport technology (p 352)

The technology of mathematics can then be characterised also as an 'invisible technology':

> Mathematics models, as experienced by the community at large, is an invisible technology It is invisible not only in the actual numbers that the models produce as 'solutions', but invisible in its derivation. [...] Risks, pay rises, new tax rates, cut-off scores for university entrance are all prescriptions which mathematical models generate. [...] What the general public sees is typically the answer only. Rarely do they even see the original question that drove the modelling process. Yet, they become co-opted as consumers of these model-generated solutions. (p 353)

Yasukawa also finds useful the characterisation used by Porter (1995) of mathematics as a technology of distance, in the sense that mathematics may provide distance between social groups. She writes:

> The authority of mathematics, based on the perceived objectivity of what 'truths' it can convey, is at one level a critical part of explaining how mathematics creates a distance (and in some cases gulfs) between the 'haves' and 'have nots' of mathematical knowledge (p 355)

The 'ideology of certainty' (see Borba and Skovsmose, 1997) might be an essential part of the explanation of why mathematics in fact is able to exercise a formatting power.

So, a first step into the analysis of the formatting power of mathematics could concern the invisibility of the technology of mathematics and the distancing which this technology may produce - see also Keitel (1993). Such a discussion would lead directly to the notion of power, and to a clarification of the nature of the power associated with mathematics.

## Critique of 'pure' reason?

The title of Immanuel Kant's work (1781/1953), *Critique of Pure Reason* (*Kritik der reinen Vernunft*), refers to a fundamental epistemic dilemma No doubt, reason (pure or not) needs to be criticised. In classical epistemology, the task took the following form: the foundation of knowledge has to be prepared by means of a fundamental critique. Rationalists, positivists and many others have embarked on such a once-and-for-all critique But from what source did they carry out such a critique? By means of pure reason itself!? This problem of self-reference is nicely illustrated by the title *Critique of Pure Reason*.

In the tradition of critical theory, a substantial amount of critical argument has been levelled at *instrumental reason*, which is certainly distinct from pure reason. Instrumental reason is related to Max Weber's concept of *Zweckrationalität* (goal rationality), which refers to a mode which would develop civilisation in bureaucratic forms. In order to discuss aspects and expressions of instrumental reason, critical theory has tried to incorporate a network of topics Two questions, however, remain: how to carry out a critique of the even more complex form of 'reason', which is produced in the process of mathematical development, and which expresses itself in mathematical formatting? How to provide a critique of the rationality of rational constructivism?

Let me try to express this problem in a terminology which has been suggested by Ulrich Beck (1992, 1994) In this way, I want to relate the discussion of aporism to a recent discussion in sociology. He puts forward the possibility:

> that the transition from one social epoch to another could take place unintended and unpolitical, by-passing all the forums for political decisions, the lines of conflict and the partisan controversies. (1994, p 3)

Beck has also introduced the notion of *risk society*:

> This concept designates a developmental phase of modern society in which the social, political, economic and individual risks increasingly tend to escape the institutions for monitoring and protection in industrial society. (p. 5)

Beck suggests *reflexivity* as an overall description of the automatic feedback system which has come to govern social development A risk society is produced by a forceful 'feedback' of industrial society:

> One can virtually say that the constellations of risk society are produced because the certitude of industrial society [...] dominate[s] the thought and action of people and institutions in industrial society Risk society is not an option that one can choose or reject in the course of political disputes. It arises in the continuity of autonomised processes which are blind and deaf to their own effects and threats. Cumulatively and latently, the latter produces threats which call into question and eventually destroy the foundations of

industrial society. (pp. 5-6)

It is produced because of the predominant 'certitude' of the industrial society, and Beck summarises:

> Let us call the autonomous, undesired and unseen, transitions from industrial to risk society reflexivity (p. 6)

Reflexivity, therefore, refers to self-confrontations. Industrial society will contain certain products of society itself, and the accumulation of these products re-directs society into new forms. The thesis of the formatting power of mathematics can now be stated in a different way:

> *Some basic elements of social reflexivity are modulated and eventually constituted by mathematics.*

In other words, mathematics takes part in the process leading us into risk society. This opens the possibility of a most interesting study: how could 'formalisation' become a vehicle for risk production?

Every society might try to develop an understanding of its social dynamics. We need *reflection* to grasp reflexivity. Beck again:

> In the risk society, the recognition of the unpredictability of the threats provoked by techno-industrial development necessitates self-reflection on the foundations of social cohesion and the examination of prevailing conventions and foundations of 'rationality' (p. 8)

The challenging problem is that (epistemic) reflection might be much weaker than (social) reflexivity. We might find ourselves trapped in a situation without sufficient means for grasping the social reality in which we are situated. In particular, we might be unable to grasp the role of mathematics in social affairs. It might be impossible to carry out an adequate critique of reason, in case this a reason which has expressed itself as mathematical formatting. Aporism acknowledges this possibility as a basic condition of humankind.

Naturally, a weakness of reflection would not constitute any problem, in the case that Hardy's analysis is adequate. If not, we face a big problem: We are not even able adequately to reflect upon and to understand what we ourselves have constructed, namely technological society. How is it possible that reason seems to express itself much stronger in social reflexivity than in epistemic reflection? The production of reason gets beyond the reach of reflection, and we are left with the devastating doubt: have we become cheated by reason itself?

Supported by Michel Foucault, we have come to realise the possibility that the whole rational enterprise brings humankind into a calamitous situation. This is the problem of the Enlightenment. The rational development of society, of technology and of the economy (supported by the most advanced elements of science) might bring us to disaster itself. The presumptions of the Enlightenment do not allow mathematical rationality to be identified as a critical epistemic challenge. Critical rationalism shares the same presumptions: scientific development means progress! And, certainly, mathematics does not cause any harm!

It is a well-known fact that it was for disobedience (human knowledge) that we were driven from the Garden of Eden. However, the further development of the rational enterprise might bring us into a much more heated situation.

## Aporism as a working philosophy of mathematics

Classical philosophies of mathematics do not face the possibility that a fundamental reflection of what is produced by mathematics is of social and ethical importance. Instead, they relate the discussion of mathematics to questions like: what is mathematical truth? What is mathematical reality? What is mathematical development?

Aporism suggests a different agenda in the philosophy of mathematics. Aporism tries to emphasise issues which are important to a 'politics of mathematics'. [7]

Several philosophers and mathematics educators have come to similar conclusions. As an illustration, I shall refer to the special issue of *Zentralblatt für Didaktik der Mathematik*: "Mathematics, ethics and peace", edited by Ubiratan D'Ambrosio and Marianne Marmé. Here, a wide range of concerns about the social role of mathematics is expressed: see D'Ambrosio (1998), Emmer (1998), Fasheh (1998), Rottoti (1998) and Scarafiotti (1998).

Mathematics is revealed as a cultural system, and not only in the sense that we can think of mathematics as representing a certain exotic and most fascinating culture which we can explore by, for instance, ethnographic means. Mathematics, including research mathematics and mathematics education, is seen as a fundamental aspect of our culture. In this way, the thesis of the formatting power of mathematics can be read as a thesis about conditions for the understanding of culture.

Thus, the thesis is indicative of our 'critique of culture' – critique of art, architecture, literature, etc. A critique of culture must also include a critique of technology, including mathematics; otherwise the term a 'critique of culture' will lose significance, especially in the context of our technologically-based society.

In 'Mathematics and peace: our responsibilities', this far-reaching cultural perspective on mathematics is expressed by D'Ambrosio (1998) in the following way:

> It is undeniable that mathematics is well integrated into the technological, industrial, military, economy and political systems of the present "Westernised" world. Indeed, mathematics has been relying on these systems for the material basis of its continuing progress. We may say that mathematics is intrinsic to today's culture. (p. 67)

Aporism involves taking this complexity on board and, in so doing, becomes a candidate for a working philosophy of mathematics for critical mathematics education.

In summary, aporism emphasises two basic elements of uncertainty about mathematics. The first concerns the possible roles of mathematics in social affairs; the second concerns our ability to come to identify and criticise these roles. These two elements are important for the development of critical mathematics education. What is more, aporism might be relevant outside the scope of mathematics education. If the claims of aporism are valid, they will be highly relevant to any projects in social theorising.

## What then does aporism mean for mathematics education?

I have suggested aporism as a working philosophy of mathematics for critical mathematics education. But what does this mean? As emphasised in the introduction, I do not see critical mathematics education as a particular topic. Instead, I have made an attempt to describe it in terms of concerns, referring to mathematics education in general Aporism might help to accentuate some of them

I discussed suggestions for practices which keep in mind a critical perspective on mathematics in my book (1994) *Towards a Philosophy of Critical Mathematics Education*. Here, I shall limit myself to a few comments about this theoretical perspective on mathematics education.

Aporism will challenge any working philosophy of mathematics education which does not provide a critique of scientific knowledge, including mathematical knowledge. A general implication of aporism is that a critique of knowing, and also of scientific knowing, becomes an educational task. Such a critique cannot be dismissed merely as a philosophical task or one for scientific methodology alone. The claim of any critical education is that all forms of knowing must be addressed by a critique as part of the educational process.

This has an implication for constructivism which has developed as a most popular working philosophy of mathematics education. I do not claim that constructivism is false Constructivism has helped to 'humanise' the mathematics classroom by embracing some of the assumptions of fallibilism which, first and foremost, concern the nature of mathematical knowledge However, fallibilism has not made any great inroads into a critical investigation of the role of mathematics in social affairs. Fallibilism has been developed in accordance with rational constructivism and constructivism as a perspective on education and learning, and has repeated the internalist perspective of knowing Nowhere in *Radical Constructivism in Mathematics Education*, edited by Ernst von Glasersfeld (1991), or in his own book, *Radical Constructivism: a Way of Knowing and Learning* (1995), do we find considerations related to 'a politics of knowing' Radical constructivism shares with rational constructivism the assumption that mathematics, as a scientific discipline, is in no need of an 'extrinsic' critique of rationality.

But what about social constructivism? First, a remark about Lev Vygotsky. It is clear that Vygotsky (1978, 1986) emphasised the importance of the social aspects of learning processes. At the same time, Vygotsky seems to have been influenced by the scientific paradigm which claims that science, once organised in a 'proper' way, does not need any further critical assistance from other faculties, in particular not from education. In fact, no critical education developed from within a Marxist or positivist paradigm. [8] The task of education, then, becomes the enculturation of students into the scientific area by supporting them in their constructions of knowledge and in challenging of students' knowledge from the position of the socially-accepted form of scientific knowledge. The basic aim of education is merely to provide a scaffolding, not a critique.

In *Social Constructivism as a Philosophy of Mathematics*, Paul Ernest (1998) suggests Lakatos' and Wittgenstein's philosophies of mathematics as the "two main sources for the social constructivist philosophy of mathematics" (p. 134). Both philosophies, however, ignore the social functions of mathematics and propose an 'internalist' perspective on mathematics. Therefore, a considerable amount of reworking seems necessary Ernest formulates a set of adequacy criteria for a philosophy of mathematics (pp 56-57). Such a philosophy must be able to account for questions related to the following fields: mathematical knowledge, mathematical theories, the objects of mathematics, the applications of mathematics, mathe-matical practice and the learning of mathematics. He confronts social constructivism with this set of criteria and manages to show that the adequacy criteria have in fact been met.

However, in his discussion, Ernest does not raise questions constituting a critique of mathematics. I do not see in Ernest's evaluation of social constructivism as a philosophy of mathematics a concern for topics and questions essential to aporism In his final chapter, Ernest discusses the important subject of values and social responsibility but adds the following remark:

> Treating these issues exceeds the primary objective of social constructivism in accounting for mathematics and mathematical knowledge naturalistically (p. 268)

Therefore, I find social constructivism, in this formulation at least, inadequate.

In its recent formulation, constructivism, radical or social, does not provide a framework which acknowledges the problematic nature of scientific rationality In this sense, constructivism is still situated within the paradigm of scientific optimism related to the internalism which characterises rational constructivism

Aporism attempts to consider mathematics in its social, cultural and political complexities and to express uncertainties about the possible role of this science as an educational concern. In this way, aporism might assist in articulating the voice of critical mathematics education


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## Notes

[1] This article is based on my talk 'Aporism and the Problem of Democracy in Mathematics Education' given at the *First International Mathematics Education and Society Conference* at Nottingham University UK, 6-11 September, 1998.

[2] A similar thought experiment was presented in Skovsmose (1998b)

[3] However, these concerns have certainly served as inspiration for much innovation in the classroom For instance, the notions of 'project work' and 'thematisations' have been suggested: see, for example, Skovsmose (1994) and Vithal, Christiansen and Skovsmose (1995). However, I find it important to distinguish between 'inspiration' and 'justification' I do not think that any conception of critical mathematics education can 'justify' a particular practice. Critical mathematics education expresses a set of concerns which might become a challenge to both practice and educational theory and, in this sense, it might become a source of inspiration

[4] This notion has been suggested to me by Irineu Bicudo from the State University of Sao Paulo, Rio Claro, Brazil. See also Derrida (1993) for a

discussion of *aporia*; see Skovsmose (1998a) for a first introduction to aporism.

[5] A different interpretation of mathematical modelling can be developed in terms of the speech act theory - see also Skovsmose (1994)

[6] If this is the case, then basic theoretical conceptions of social development and social nature appears to be inadequate How could social theorising, which proposes fundamental ideas for understanding the nature and the development of society, ignore the social role of mathematics? In this sense, the thesis of the formatting power of mathematics establishes a major challenge to social theorising.

[7] Hersh (1998) set up some 'criteria' for a philosophy of mathematics. In this way he establishes a critical stance in the philosophy of mathematics, breaking away from the tradition that the philosophy of mathematics first of all has to do with the foundations of mathematics Of the thirteen criteria Hersh does mention, he emphasises the following three as essential: (1) The philosophy of mathematics should recognise "the scope and variety of mathematics", it must fit "into general epistemology and philosophy of science" and it must be "compatible with mathematical practice - research, application, teaching, history, calculations, and mathematical intuition" (see p. 33). Considering this broad picture of mathematics, aporism represents a smaller element Nevertheless, I see aporism as essential in order to relate an understanding of mathematics to a broader understanding of society See also the criteria set up by Ernest (1998).

[8] Certainly, positivism suggests a critique of science to eliminate any kind of metaphysics from this field Marxism has also suggested a critique of science to eliminate any 'bourgeois elements which might have corrupted science. However, these suggestions have not been transformed into suggestions for a critical educational practice

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