REFRAMING THE DISCUSSION ON WORD PROBLEMS: A POLITICAL ECONOMY

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The practice of questioning the uses of word problems is nearly as old as modern schooling. The earliest attempts took the form of complaints about a perceived lack of connection between word problems and real life situations (see Verschaffel, Greer & De Corte, 2000). This has remained a fruitful line of inquiry: what makes mathematical word problems interesting for theory is an implicit promise that through their analysis we might arrive at an understanding of the relationship between school mathematics and the rest of the world. In the past three decades, research has arrived at considerably more sophisticated theorizations. For example, in a meticulously argued essay, Lave (1992) suggests that word problems do not merely disconnect from reality but also serve to impose mathematics on everyday tasks that either do not involve mathematics or involve a different type of mathematical thinking. Using literary theory, Gerofsky (2004) recasts word problems as more than simply fictitious, but as constituting a literary genre with its own requirements and traditions. Dowling (1998), by analyzing the way word problems function within the larger institution of schooling, concludes that beyond any pedagogic function, these problems serve to create various mythologies around the usefulness and relevance of mathematics and, by extension, schooling in society. Various empirical researchers have studied the way students navigate the relationship between problems and reality and concluded that students situate the problems within the confines of their experience of schooling (*e.g.,* Inoue, 2009; Schoenfeld, 1991).

To further the investigation of word problems, we must ask what foundations underlie the way in which the literature frames its questions. This line of inquiry is encouraged by a relatively recent book that brings together and summarizes the literature on word problems (Verschaffel, Greer, Van Dooren & Mukhopadhyay, 2009). The editors argue that this literature shares three stated assumptions:

1. that those involved in mathematics education should not use word problems or any other form of pedagogy "mindlessly, 'because that is how it has always been done'", but "rather need to reflect deeply on what they are doing and why";
2. that "attention should be given in teaching mathematics to make connections with children's lived experience";
3. and that "in contrast to typical […] teaching in schools […] an understanding of the very idea that mathematics can be used to model aspects of reality, and that this process is complex, and has many limitations and dangers, is essential to effective and responsible citizenship." (p. xxv)

In this article, I argue that each of these three stated assumptions envelops an essential and hidden assumption. The above statements, respectively, take for granted 1) a particular relationship between thought and history, 2) an unexamined description and even valorization of "lived experience" and everyday life, and, 3) a non-dialectical conception of "mathematics" wherein mathematics stands as a cloistered and coherent discipline, unaffected by the multifaceted reality whose aspects it models. Examining these hidden assumptions, I believe, can uncover new possibilities for research and theory.

In Verschaffel *et al.*'s first identified assumption, history is acknowledged as a factor impacting the present, and its impact is to be identified and critically understood by educators. However, here, and in the literature in general, the role of history seems to be limited to the force of tradition. It is of course true that educators need to "reflect deeply on what they are doing and why," instead of taking traditions for granted; but history does not only hand us what we do, it also forms the lens with which we reflect on what we do. Just when we think we are no longer acting with blind traditionalism, when we are at our most inventive, we may be most at the mercy of unrecognized, historically-formed assumptions. It is perhaps a lack of attention to this role of history that has allowed the literature on word problems, with the exception of Gerofsky's work, to be ahistorical: Lave (1992, pp. 74-75), for example, generally ignores historical evidence to incorrectly assume that the disconnect between word problems and real life activity are only recent products of modern schooling; and Dowling (1998) does not discuss whether extremely similar word problems throughout history also served to mythologize mathematics education. Why, for example, would Babylonian scribes need to pretend that their mathematics was applicable to all areas of life, as do modern mathematics textbooks?

Gerofsky's work, on the other hand, contains a detailed historical study of word problems and their functions throughout history. Drawing on evidence from Ancient Babylonia, Renaissance Europe and contemporary schooling, Gerofsky (2004) concludes that "the format of word problems has survived and thrived for 4,000 years while its purposes have changed from riddles to exemplars of mathematical generality to practical, applied problems" (p. 131). The statement is admittedly an over-simplification of Gerofsky's complex research; nonetheless it illustrates my main criticism of the literature's view of history as a linear process leading to the present. All three purposes that Gerofsky names for word problems are, in fact, present *simultaneously* in each of the historical eras she examines (see Harouni, 2015; Høyrup, 1994; Jackson, 1906). Each purpose, as I will show, is connected to a particular labor context and, therefore, to a particular type of economic institution. The question here is: why would certain institutions come to dominate an epoch or our current view of it? An explanation of such phenomena, I believe, is possible through an understanding of history that incorporates political economy: that is, the processes through which societal divisions interact with all aspects of culture.

The second stated assumption, which concerns the role of students' "lived experience," is connected to the above assumption about history. The literature on word problems tends to take concepts such as "practice", "everyday" and the "real world" without clarifying the tremendous complexity that these terms mask. In nearly all the literature I have mentioned, students' "everyday life" appears to be beyond criticism, a place of good, practical, and useful sensibility, in which the fictionalized settings of word problems can appear to be an irrelevant and even damaging outsider. But the everyday also produces waste, alienation, oppression, and organized ignorance (see Lefebvre, 1971). We can see the importance of this problem more clearly if we notice that even in Frankenstein's Freirian approach to teaching mathematics, in which word problems are reshaped to reflect socio-economic conflicts (Frankenstein, 2009), students seem always to reflect on an *external* reality. A major step in Freire's pedagogy, that is, reflecting on *one's own* complicity in relations of domination (Freire, 2000, p. 115), is not described.

By proposing a sociological investigation of mathematics education, Dowling (1998) and other researchers (for example, Gellert & Jablonka, 2009) have taken major steps toward remedying the above problem. By studying the institutional setting of mathematics education, Dowling reframes the school as a problematic part of students' lived experience, rather than a place isolated from reality. Dowling also acknowledges class divisions in society and how these divisions may shape the discourse within and surrounding mathematics education. In this way, he arrives at a conception of word problems as working beyond their immediate pedagogical purpose, as part of a discourse on the utility of mathematics, education, and schooling; a discourse that further enables schools to reproduce social inequalities. This analysis can be deepened to take into account the underlying structures of societal division, that is, the division of labor that occurs within specific modes of production. In this approach, everyday life is reestablished as a site of economic and political activities, the most important of which are production, alienation of labor, exchange and consumption, within which mathematics and mathematics education operate as flexible instruments.

The third stated assumption of the literature concerns mathematics as a discipline that can be used (not unproblematically) to model the world. The literature's overall conception of mathematics is undialectical. Dialectical thought, by not taking its own categories and relationships for granted, *i.e.* by acknowledging thought as always mediated by the systems (economical, political, cultural, *etc*.) that produce it, encourages a constant re-examination of reality and one's relationship to it. By assuming that "mathematics" refers to a unified, coherent body of knowledge, the literature creates the illusion that mathematics is shielded from all influences that do not come from within the (vaguely defined) discipline of mathematics itself. This is essentially a metaphysical (Hegel, 2010, pp. 64-78) conception of mathematics. My purpose here is not to offer a new, dictionary-ready definition for "mathematics": that would be to fall back on simple, metaphysical thought. Rather, I propose that we see mathematics as a vague category that refers to a host of practices mediated by historically situated activities ranging from accounting, to engineering, to social statistics, to philosophical work, and many others, last but not least of which is modern schooling. By extension, word problems also lose their simple definition as a single genre or a single pedagogical device and become part of a more complex conception.

Many elements of this type of thinking are already present in the literature. My proposal is to push these ideas toward their ultimate conclusion, so that they take into account the entirety of the socio-political systems that underlie them. This article contributes one step in that direction.

Categories of mathematics and their word problems

Høyrup has shown that the institutional setting in which mathematics is practiced and/or taught impacts its character (Høyrup, 1994). To Høyrup's thesis I add a second: that, in divergent contexts, similar relationships to labor and society lead to certain similarities in mathematical knowledge, practice and pedagogy (Harouni, 2015). Sixteenth century European mercantile mathematics, for example, shares many idiosyncrasies with Babylonian scribal mathematics, similarities that can be explained in terms of the comparable administrative stance of scribes and merchants toward the world. Word problems are not an exception to this double-sided thesis. Even when used in the isolated school context, these problems have to refer to a view of mathematics and the world, and these views are inevitably shaped by politics, culture and economy.

From a study of these similarities and differences, I have identified various *interrelated* categories of mathematics (Harouni, 2015). In this section, I examine the word problems that are associated with these categories of mathematics. The categories must not be taken taxonomically, as if they designate species of a larger genus. They may change, converge or disappear with context, culture, and scientific development. Furthermore, I have opted to include categories that can help us better understand our current situation. So, even if there may be enough evidence to speak of, for example, a category of architectural mathematics, doing so would not be helpful for understanding the dilemmas of modern schooling. I ask the reader to be aware of the subjective aspect of my descriptions. In my work with teachers and educators, I encourage them to reconfigure the categories and propose, using historical evidence, other categories that they feel relate more clearly to their own contexts. The strength of my own divisions is that they relate closely to divisions of labor, which at this point in history have universal relevance.

Commercial-administrative mathematics

The similarities between the mathematical texts recovered from Babylonian scribal schools and what was taught in Renaissance-era books on commercial arithmetic are startling (Harouni, 2015; Høyrup, 1994), and include an enthusiastic penchant for word problems. We find in both places the same emphasis on problems designed for ready-made algorithms, the same computational problems in which objects and people are reduced to numerical relationships, and even the same proclivity for allowing mathematics to overrun rather non-mathematical situations.

In practice, scribe and merchant share a characteristic that dominates all their other professional attitudes. They stand outside the process of creative labor, measuring the relationship between owners, producers, and products. The basic building block of this relationship is labor power, exerted by the producer, embodied in the product, appropriated by the owner. In modern economy, this abstract labor is often reified as money; in scribal schools we find it expressed both directly, as labor power itself (Friberg, 1996), and indirectly as wages. The reduction of all things to a value, to measures of abstract labor, is the root of what I refer to as *commercial-administrative mathematics.*

Word problems, particularly the computational type common in modern schooling, dominate mathematical learning most, if not exclusively, in situations where commercial and administrative sectors dictate the agenda. When researchers speak about the relationship between problems and reality, they generally leave out the subjective aspect that co-defines objective reality. Nesher (1980), for example, says that word problems are unlike "real world" problems, because a real problem does not hint at all the relevant data as well as the method required for solving it. In fact, "real" mercantile or administrative problems concern social interactions that are highly regulated, predictable, and self-contained. The primary way in which most word problems distort reality corresponds to the way in which an administrator tends to understand and act on reality.

The person posing the average computational word problem is uninterested in the particular fictional set-up of the problem, and yet he or she *is* interested once these particulars are turned into abstract, exchangeable values. When a problem asks how many oranges Sarah has after her mother gave her so many more oranges, we do not care about Sarah, her mother, or oranges. The movement of value from one side of an equation to another is all. The commercial-administrative mindset consistently turns its gaze on other sectors of production and consumption (including the family) and picks out new computational situations. Lave (1992), drawing a strictly commercial example (her only historical reference) from *Treviso Arithmetic*, a fifteenth century Italian text for merchants, argues that prior to the era of schooling, word problems referred to their immediate practical context. Her theory, however, would not explain why only a few pages later in *Treviso* we come across this example:

> I have bought 9 yards and 2/3 of cloth, 2 yards and 3/4 wide, wishing to make a garment. I wish to line it with cloth 1 yard and 1/8 wide. Required is the amount of lining needed. (Swetz, 1987, p. 133)

Which tailor, to ask a question after Lave's argument, would ever calculate the lining for a single garment in this way? The mathematical answer to the question, 23 and 17/27 yards, is, in practical terms, absurd: the tailor cannot and need not measure out 17/27 of a yard of anything.

Why should the author of *Treviso*, who need not prove the usefulness of mathematics to his audience of merchants, have felt the need to resort to such an example? The reason, I argue, has to do with where he stood: he did not think of the problem as a tailor, but as an *administrator* of tailors. The question might be absurd for one garment; it is quite sensible if we are dealing with two hundred garments and need to plan for materials. This explanation satisfies Dowling's discomfort with most of the examples he uses to argue that word problems are impractical and merely a function of the general tendency of mathematics to subsume real-world interactions.

One of Dowling's examples (1998, p. 8), involves a boy who wants to know how much tape he needs to buy in order to cover the top of a circular lampshade. In real life, of course, no one buys tape just to cover a single, small object. This seemingly impractical word problem, however, can be quite practical in the context of a factory making hundreds of lampshades. A type of mythology is, as Dowling correctly concludes, created in the problem's willingness to talk about handicraft as necessarily mathematical, but this mythologization is not a function of mathematics as such, or even of scholastacized mathematics. Its base is in an administrative attitude toward all forms of creative labor.

Both Lave and Dowling, among other researchers, express a concern that word problems "mathematize the everyday" and thus devalue everyday experiences. To demonstrate her point, Lave relies on studies that show shoppers facing problems that do not correspond to the mathematical attitude represented in word problems. In slightly higher spheres of a money economy than supermarket consumption, however, there is less discord between "school math" and the "everyday." The administrator uses this kind of calculation on an everyday basis; the average consumer, clerk, and many other sectors of society do not. The confusion in regard to most (though not all, as I will show) school word problems stems from the application of a commercial-administrative attitude to *consumer* situations, while denying or hiding the original administrative purpose entirely.

Philosophical mathematics

How mathematical activity turns into an ecclesiastical or philosophical tool is beyond the scope of this article. It will suffice to say that here the role of coordinated numerical and geometric systems is neither to tally up abstract labor nor to help create objects. This type of mathematics is interested in patterns, meaning, and magic, which are what its practitioners offer in order to justify their social positions. This mathematics is priestly, academic, or philosophical. I refer to it as *philosophical mathematics*, keeping in mind that it can contain all or some of these three attitudes.

While there can be no scholasticized teaching of commercial-administrative mathematics without some recourse to word problems (*i.e.*, to codified examples of exchange), philosophical mathematics can be entirely free of that need. Euclid presents his geometry without any reference to other areas of activity. Biruni's eleventh century *Instructions*, for centuries a primer for Iranians approaching mathematics as another aspect of their spiritual education, does not contain a single word problem. When a text with primarily philosophical concerns does use word problems, the questions are better characterized as puzzles. Books written in sixteenth century Europe for grammar schools, for example, contain far fewer word problems than those written for merchant schools by *reckonmasters* (Jackson, 1906). In grammar school texts, the word problems rely on artificial set-ups; but in most cases there is no attempt to pretend at practicality. Here is, for example, a problem from Frisius (c. 1540), meant for grammar school students:

> A man having a certain number of gold coins bought for each as many pounds of pepper as equaled half of the whole number of coins. Then upon selling the pepper he received for each 25 pounds as many gold coins as he had at the beginning. Finally, he had 20 times as many coins as he had at first. The number of coins and the quantity of pepper are required.

The problem, despite its mercantile set-up, has nothing to do with trade or any codified social interaction. Unlike the tailor problem from *Treviso,* you could not have it make sense by turning it into an administrative calculation. Puzzles of this sort are deployed in philosophical mathematics, not least because of their relationship to magic tricks: they give rise to a sensation of wonder and send the student searching for relationships.

This is not, however, to imply that this category and its problems, despite their relative intellectual autonomy, stand entirely outside the relationships of production. Høyrup has shown that the first of such puzzles appear in Babylonia as exceptions among much more abundant examples of scribal school administrative word problems. Their appearance corresponds to the rise of scribes as a semi-autonomous professional class. At this point, scribes began to seek meaning and signs of mastery within, but also beyond, their explicit economic function (Høyrup, 1994). It is not a coincidence, I believe, that the scribes' philosophical puzzles appear at the same time that magical and occult practices also become part of a young scribe's training (Høyrup, 1994, p. 6). The shared origins of these two categories of mathematics points to the possibility of their interconnectedness: a philosophical approach that owes its existence to a division of labor that places the approach entirely outside creative labor, also has a tendency to subsume the material identity of objects. The character of this subsumption, nonetheless, is in most cases distinct from its commercial-administrative cognate.

Artisanal mathematics

To call the abstracting systems of formal relationships that emerge *within* certain processes of creative labor "mathematics" is merely a theoretical device. In nearly all cases (engineering, architecture, carpentry, *etc*.), the mathematical systems are inseparable from the larger practice, tied to its tools, materials and products. It is more or less a modern academic misunderstanding to imagine that the engineer studies "mathematics", and that he studies precisely the same mathematics as the future accountant. In practice, the engineer needs a certain type of "mathematics". This type of mathematics does not annihilate the identity of materials, but places measurements of their relevant aspects in relationship to tools and the final product. Number, in this case, is primarily an expression of relative magnitude, mediated by instruments and the final product of the work, rather than exchange value or patterns. The idea taught in schools that measuring is basically the counting of units is the result of a commercial-administrative mindset applied to what is essentially (due to the importance of actual instruments and materials) an *artisanal* practice.

The impact of *artisanal mathematics* on modern schooling has been minimal (Harouni, 2015). It does, however, survive in academic programs where technological learning is still tied to some level of practice, such as in various forms of on-the-job training and apprenticeship. Despite the false opposition that much of the literature tends to draw between word problems and "practice," artisanal learning is not antagonistic to word problems. Rather, it produces a distinct kind of word problem that has not been considered in any of the literature. In my own engineering training, I was often given a problem (*e.g*., to draw the circuitry of an elevator that privileges a particular floor) and asked to *design* a solution. The fictional client with a fictional piece of land is a regular presence in architecture schools. The set up is imaginary, verbal; the solution is tangible, specific, and yet open-ended.

The absence of this type of problem in both school mathematics and the literature on word problems is a sign of the supremacy of commercial-administrative and philosophical socio-economic attitudes in modern times. One of the few efforts to go against this trend occurred in Soviet Russia, where Vygotskian psychology, under the influence of Marxist philosophy, emphasized the role of instruments and creative labor in shaping cognition. In the Davydov curriculum, for example (see Schmittau, 2010, p. 256), where measuring objects is the primary means of acquiring number sense, the teacher might ask the children to pretend to be members of an ancient people who do not know how to count. He then asks them to measure the length of an object and communicate it to another group. Students must invent objects and forms of measurement from what they can find in their environment (twigs, tokens, *etc*.). This is a word problem, but it refers to a creative activity, the product being an instrument for measuring and communicating length. Notice, however, how quickly the process has become self-referential: the product of measurement is an instrument of measurement. The Davydov curriculum could not offer true artisanal questions, because the Soviet school, like its Western counterparts, was isolated from actual creative labor.

Social-analytical mathematics

Under historical conditions where one or more social groups begin a critical examination of the commercial-administrative activities of itself or another group, a new type of mathematics emerges. This type, exemplified by social statistics and economics, is very closely tied to commercial-administrative mathematics; so closely that, once deprived of its critical agenda, it can itself become an administrative instrument. Used as a critical tool, however, it can reverse the reductive tendency of commercial-administrative activity: its aim becomes to take numbers and try to retrace them to the social experience to which they refer. In the process, *social-analytical mathematics* highlights imbalances that we could not have articulated without its aid. Rigor, which in commercial-administrative mathematics was defined by the ability to best predict an outcome, here refers to the doggedness with which one reconnects numbers to real-world situations.

Many educators have proposed that this type of mathematics, *i.e.*, mathematics that can help people "read the world," is the most appropriate for school education (*e.g.*, Frankenstein, 1989; Gutstein & Peterson, 2013). A large array of curricula have been proposed according to this ideal. Frankenstein (2009) offers one of the most sophisticated conceptions of this mathematical attitude, particularly in relationship to word problems. Here numbers are seen as "describing the world", which, as she points out, also includes "hiding" certain relationships, thus mystifying them. The role of calculation, instead of to predict results, becomes to discover, interpret and evaluate relationships. All these mathematical instruments, finally, are deployed with an understanding that they must constantly refer back to the world, forming and reevaluating connections.

This perspective leads to a new understanding of word problems as well. We can understand them as instances of Freirian "codifications": texts that contain a complex of relevant themes or contradictions, presented not in order to be solved, but in order to be decoded (Freire, 2000). For a reader, to decode a codification means to reformulate the relationships contained in it by applying his or her own consciousness, thereby generating a discussion that deepens his or her own, as well as a group's, understanding of a social theme. Nearly all of Frankenstein's (2009) examples fulfill these requirements, although they do not ask students to evaluate their own participation in the social problems that they are investigating.

Without a strong theory that distinguishes social-analytical mathematics from its commercial-administrative origins, however, the social-analytical word problem is in constant threat of returning to an uncritical stance. Gutstein (2006, pp. 238-240), for example, in a lesson on the impact of real-estate development on a low-income neighborhood, presents his students with a local newspaper article on the topic and then asks them a series of questions, which they are to answer and discuss. For instance:

> If a family needs [a yearly] income of $47,000 to buy a $125,000 house, how much is needed to buy a $350,000 house? (p. 239)

This problem, treated in separation or even in relationship to a newspaper article is still a commercial-administrative one. Notice how the ultimate aim is a calculation; how easily we can replace the situation with another; how the problem takes the business transactions that it refers to for granted (*i.e.*, that families must have an income, that they buy houses, that they borrow money to buy houses, *etc.*). It becomes a social-analytical problem only once it is used to engender discussion. Even then, the dialectic between the two types of mathematics is not at all simple. The moment students cease to go further in their critique, they risk falling back into a commercial-administrative mindset. In this sense, the above example is doomed from the beginning: its original relationship to the world is administrative, not analytical. Bankers do not calculate loans using simple proportion, as the question implies. Therefore, the students (unless they are trained to critically tear apart anything that the teacher offers them) cannot use the problem as an artifact to discuss loans and bankers. The question is not whether the word problem is real or not: the question is how it opens to reality.

Conclusions for school mathematics

How can we understand word problems as they are used in modern classrooms? The majority of school word problems are of the commercial-administrative type, applied to a vast array of situations; but this answer does not convey the complexity of the issue. Schools do not teach commerce or administration. The content of school mathematics has undergone a transformation, commensurate with changes in society and the aims of schooling. Overtly, today's school word problems do not refer to administration or production, but consumption. The chiefly domestic setting of these word problems, to which an administrative mindset is applied, is a result of the interactions between school and society, between consumption and administration. The everyday experience of students is one of consumption; the overall social attitude that oversees their everyday experience (including their schooling) is commercial-administrative. This attitude is so dominant that even the critics of traditional schooling usually fail to imagine an alternative that escapes its limitations.

Let me return to the hidden and stated assumptions of the literature. Gerofsky (2004) is correct to suggest that if word problems are to be studied at all, whether by students or researchers, the study must not treat the problems as neutral conveyors of mathematical knowledge, but rather must take them critically, to explore their ambiguities and limitations (p. 142). In their essence, these ambiguities and limits correspond to antagonisms and restraints within society itself; and society works hard to hide the true nature of its internal contradictions. Bar an awareness of the role of these social forces, the more minutely we analyze a word problem, the more we are in danger of being seduced into ignoring its larger implications. Even critical perspectives are not immune to this danger. Lave (1992) offers the astute observation that school word problems often turn "problems of sense" (*i.e.*, ones that can be addressed by people within their daily activities) into "problems of scale" (*i.e.*, ones where the sheer magnitude of the activity places the situation beyond the immediate reach of students). But in concrete terms, what are these activities and what defines their scale and people's access to them? Are they anything other than relationships within the system of production, consumption and administration? Only a properly historical perspective can force a word problem to, as Hegel might say, speak its name; that is, to reveal its larger purpose.

We cannot expect educators and students to arrive at a critical understanding of word problems by relating mathematical modeling to their own "lived experience." The commercial-administrative attitude that sees the world as an endless series of calculations is the result of life within a money economy. Those who worry that school word problems trivialize and reduce the complexity of everyday life have put the cart before the horse: everyday consumer life and its progenitor, alienated and administrated labor, are the *primary* trivializers and reducers of life. Dreams of inexhaustible wealth, of ever-greater production and consumption, operate within the average student as within the simple, average word problem. The main task of a critical mathematics education may be to constantly struggle against its own mathematical logic, which on the one hand is encouraged by society to dominate all aspects of life, and on the other hand can reveal the mechanisms of social domination. In this light, word problems that (perhaps by being overtly absurd) force students to come up against the material, psychological and moral limits of commercial-administrative logic may be more to the point than word problems that unwittingly encourage students to increasingly channel their own moral and emotional experience through that logic.

Finally, as mathematics, too, is demythologized so that it is understood as changing with context, at least three possibilities open up for a reframing of word problems. The first is the possibility to examine the content and meaning of word problems in relationship to their larger purpose, a step I have somewhat demonstrated in this article. Second, we can begin to pose word problems from relatively coherent mathematical vantage points that could not be clearly imagined if we were to remain stuck in common sense definitions of mathematics. Not only can we think of artisanal word problems, we can keep an eye on our own socially-acquired tendency to let artisanal activity be subsumed by other categories of mathematics. Third, the possibility of interaction between categories, in turn, presents us with entirely new possibilities for word problems, where types of mathematics that until now had remained within separate sectors of the economy can mix to form new categories. Word problems that, like artisanal problems, take into account the limitations posed by material conditions, if combined with the speculative aspect of commercial-administrative problems, can help set a proper foundation for a reworking of both categories within an environmentalist approach to administration: an approach wherein the administrator must constantly reconsider the impact of each act of exchange on the social and physical environment in which the act takes place.

All these possibilities, however, are at this point purely theoretical. There is no way to predict how researchers, educators and students, within particular situations, will re-examine or reinforce their existing relationships to mathematics, education and society. The structural elements, whose power I have outlined in this essay, by necessity will constantly re-introduce themselves into any attempt at new forms of practice. It is therefore only in practice that the extent and limitations of these possibilities can be determined.

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