Editorial

Although mathematics has a very long history it is only relatively recently that this history has become an object of attention. The first book about the history of mathematics was published in French about 250 years ago. The last 100 years, beginning, say, from Cantor's book, has seen the steady extension and increasing seriousness of the study of mathematical history.

The philosophy of mathematics is a younger field of study. I feel inclined to date its origin with Frege's attacks on Mill's empiricist views of arithmetic, though this could be argued. The choice makes it a little less than 100 years old, though it bloomed significantly and rapidly in the early years of this century when mathematicians became aware of an impending "crisis" in the foundations of the subject.

Both the history and philosophy of mathematics are now well established. Numerous associations and publications serve them. Universities offer courses in them. Increasing numbers of people work in both fields Various groups recommend that some history and philosophy of mathematics should be incorporated in graduate and undergraduate programmes of mathematics and in teacher training programmes. It is even suggested that school teaching should take some account of the history of mathematics.

(I am concerned here to mention the phenomena, not to support a case. If it is difficult to think oneself into a period before anyone attended to either the history or the philosophy of mathematics, it is even more difficult to think oneself out of a cultural context which attributes significance to historical and philosophical insights The effort may sometimes have to be made if only to avoid substituting some sort of historical or philosophical absolutism for the mathematical absolutism that these studies call into question It would be absurd to suppose that the historian or the philosopher can give us *more* of the truth about mathematics than the mathematician can.)

If one asks a mathematician what mathematics is, one usually gets an answer that falls into one of the "classical" philosophical categories They are part of the folklore now for all of us But if one asks what the mathematician does and thinks one gets quite different stories, as Hadamard discovered. These begin to define another field, the psychology of mathematics Although this field is now 60 years old, if we decide to date it from some of Poincaré's writings, it has not made such marked headway Most of us would be unable to state any hard results of studies in this field, and it has not developed anything like the institutionalised characters of the history and the philosophy of mathematics.

The most junior of these fields is the sociology of mathematics. The application of a sociological approach to science, and *inter alia* to scientists and scientific organisations, is now stimulating corresponding attention to mathematics and mathematicians. Against what earlier seemed to sociologists to be impossibly long odds, because of the special nature of mathematical activity, Wilder and Bloor, for instance, have shown that certain illuminations result from regarding mathematics as subject to social and cultural constraints in the way that other fields of knowledge are.

In descending order of longevity, then, we have

History of mathematics
Philosophy of mathematics
Psychology of mathematics
Sociology of mathematics

All four fields attest to the fact that mathematics is a human endeavour, but the progression suggests a gradual shift towards an absorption of mathematics into a relativistic framework that could fit around any substantial human intellectual achievement whatever.

Have we got something wrong? Has something been overlooked?

Hints exist in the work of Montessori, Piaget, Gattegno and others who have looked carefully at the ease and naturalness with which young children enter the world of mathematics, whatever difficulties they may experience later. The hints point to a privileged position for mathematical capability among the genetic endowments of children (akin to the privileged position of linguistic competence). They suggest that people are born with an ability to think mathematically just as they are born with an ability to acquire a language. What has been generally overlooked, therefore, is the possibility of what we may call a *biology of mathematics*

Writing these words, of course, achieves very little — except to indicate that the possibility of a biology of mathematics *can* be entertained. That's a start But giving substance to the words requires the development of a working theory that can be seen to produce results. Then research programmes could be developed that would relate the theory to the learning of mathematics and put effective techniques in the hands of teachers.

This doesn't mean beginning entirely from scratch as if to date we've found out nothing about teaching mathematics. Working on a biology of mathematics gives, first, an orientation to what we know and to what we still need to know and, more importantly, a tool for sieving the past and for designing a rather different pedagogical future.

This issue combines, I hope, usefulness with stimulus and provocation. The survey papers by Clements (continued from the previous issue) and by Robitaille and Dirks are substantial additions to the available resources. Sesay stretches the notions of symbol and social agreement further than usual, though keeping his eye firmly on the classroom; Cassidy and Hodgson raise some important issues about the simplicity and power of mathematical proofs. John Mason starts a hare and Joseph Agassi returns with some more fireworks. Can we have some good discussion, please?