to restrict students to a single paper-based representation. We would argue that what is necessary is for researchers and teachers to be aware of the effective realisations that any representation offers and work towards developing a small suite of representations that invite students into the plurality of multiplications while addressing the most significant properties, contexts and behaviours of multiplications.

From many to one to a few

We are sympathetic to Foster's argument that mashing a bunch of representations together and hoping students will figure out multiplication is problematic. We would argue that a small set of well-chosen representations might provide a good balance that acknowledges the multiplicity of the very idea of any concept (including multiplication), that supports the noticing of relations that pertain across contexts, that recognise the materiality of concepts and that therefore remains flexible on the digital potential of well-designed tools. We suggest that more research attention be paid to creating coherence across representations. Maffia and Maracci (2019) have started doing this by exploring the semiotic interferences that arise when students move between different artefacts (often physical and virtual).

Regarding the axiological dimension of Foster's argument, we note that in gravitating towards the single representation of the number line, there is an obvious appeal of simplicity and unity, which may well facilitate the development of curriculum materials and tasks. Our own predilection for multiplicity carries different aesthetic tendencies, which might tend towards eclecticism and variety. However, it is the associated ethical implications we think need to be clearly articulated. If we imagine that learners come with diverse prior experiences, capacities for learning and aesthetic preferences, then should we not be meeting them with diverse representations? We would not suggest that the number line would be intimidating for students, but it may well be less intuitive, persuasive and relevant to them.

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Glocaliszation in mathematics education

ARINDAM BOSE, DANYAL FARSANI [1]

We were waiting for a colleague in a hotel lobby in Barcelona soon after the PME conference in Alicante. There was light music in the background and the following conversation took place:

Arindam	Oh wow! I know this song, I have heard this in India in Hindi and also in Brazil in Portuguese with exactly the same beats and melody.
Danyal	Yes, it is everywhere! Even in conservative places.
Arindam	Is it that Macarena was initially a local song which became popular and turned global and spread to other places? And thereafter many other places then customised this song in their languages and also gave a local flavour of their practices to it? Like global turning into a local and yet retaining its 'global' form?
Danyal	Yes, precisely so!
Danyal & *Arindam*	[*together and spontaneous*] That's Glocalisation!

This was how we got to the word 'glocalisation' [2]. When a dominant global voice gets the flavour of a local practice, that is when 'glocalisation' occurs. Much like how the Macarena tune has been glocalised, many other things, objects and practices have been glocalised as well, not only products or goods and but also languages such as the one which we are using to write this short communication—it is one such glocalised form of the English language. English is a language of power, and in many academic contexts it is the lingua-franca [3] through which we write, publish and interact. One or two centuries ago French was the language of power, and the lingua franca in which diplomacy and science was carried out. Recall that the editors of the first journal in mathematics education chose French as the sole language of publication (Reid, 2023). After the Second World War, the United States became the dominant military power in the West, as well as giving rise to new forms of popular music like jazz and rock that influenced the world over and yet acquire local flavours. Gangnam Style, for example, is a pop song glocalised to Korea.

The term 'glocalisation' has previously been used while referring to the combination of local (emic) and global (etc) approaches in ethnomodelling research contributing to a holistic understanding of mathematics (Orey & Rosa, 2021). We argue here that the modern trends of standardisation, for example, standardisation of the representation of mathematical ideas, concepts and symbolisation, processes of foreign economic trading, modern outlets of fashion and design, home décor, food outlets, modern cultural forms

(including rock and pop music) and so on, are testimony to the growing attempts to standardise human (also read: mathematical) processes of interaction—*etic* colour over *emic* practices. These standardised processes have become so normative that there is no other process that can be recognised as valuable and legitimate. Colonisation practices in the last two centuries have given shape to such normative discourse. To add to this, there exists an interplay between hegemony of economic progress and development, and hegemony of power relations and language in which these global actions are carried out. All these processes are driven by the principles of globalisation (see Fasheh, 1983; Skovsmose, 2008). Interestingly however, there seem to be underlying counter-attempts to carve out, preserve and protect local identities and local resources which can be seen through the efforts to customise global practices in the local flavour. Arguably therefore, there exists a growing propensity of localising the 'global' (rather than globalising the 'local') and mathematics is not untouched by this. It is this process which we have termed 'glocalisation'. Here we present one of many possible examples of glocalisation of mathematical processes and practice.

Our example comes from measurement in work processes of micro-enterprises in low-income settlements in developing world contexts, such as Mumbai, India. Significant use of different measurement modes and units is the hallmark of the informal work-contexts interspersed in Mumbai. In such practices, glocalisation can be seen in the interplay between old British units (inch, dozen, yard, ounce, gross, ream, *etc.*), standard international units (metre, centimetre, kilogram, etc.), non-standard units (cubit, finger-band), indigenous units (*desi, waar, kattha, bigha)* and informal units (*mutthi* = 'fistful'). Some of the standardised and global measurement units such as 'gross' and 'ream' have been customised for convenient computation [4]. Our work has shown how children from low-income urban homes who participate in work related activity acquire familiarity with different measurement units, measurement processes, ability to estimate quantities, or knowledge of the costs of different kinds of materials or goods (Bose, 2015). In contrast, what is observed in formal classroom practices is school learning, where children largely encounter standard units that are pre-given in the form of measuring instruments (tapes, weights, *etc.*). While the choice of a unit and the construction of a convenient unit are the first steps towards quantification of an attribute, and are important aspects of the concept of measurement in classrooms, these steps are rarely emphasised. In many classrooms, these may at best be explained verbally. However, there are several out-of-school contexts of glocalised practices where children encounter a variety of ways of constructing a unit.

Story of 's' to 'z': a form of asserting national identity?

There are many different dialects of spoken English that vary from region to region. Despite these widespread oral dialects, there are only a few written variations in English orthography, with the two most dominant being US and UK. One of the most notable differences in spelling between US and UK English concerns words that end in 'ize' and 'ise'. For example, globalisation versus globalization or, colonisation versus colonization. There has been a trend of US forms becoming global forms, starting with the rise of the US as a dominant military power in the twentieth century and accelerated by the dominance of US software companies as human communication became more technologised. This global form enforces itself on local forms or swallows them up.

Glocalisation as a theoretical construct

The process in which local traditions, art-forms and practices meet with the dominant and emergent global phenomena or practices and new local forms emerge from the global is glocalisation. This process creates a space where the global and the local forms flow together but the global form is dominant over the local and therefore, it is 'glocal' and not 'lglobal'. Glocalised mathematical practices can be seen in the examples of diverse cultural practices involved in, say, numeration systems and clock reading (subtractive counting techniques), probability reasoning (Farsani, 2022), and estimation strategies (Bose & Subramaniam, 2015). Glocalisation can also be seen through an ethnomathematical lens in linguistic landscapes that offer the possibility of learning from and empowering local forms of doing mathematics. The glocalisation lens helps us to understand the effects of global hegemonic attempts on mathematics pedagogy. Glocal examples in mathematics textbooks in many developing countries bear testimony to this. One example is the use of pizza and cakes in elementary mathematics word problems on proportional reasoning, particularly in books brought out by private publishers aiming at the highly aspiring and competitive middle class Indian or Iranian societies. These glocal terminologies are perceived as more modern, representative of higher social class and often seen as gateways to higher rungs on the social ladder. Local breads and chapati/paratha are made to resemble pizza through the ever-increasing hegemonic practice of glocalisation.

Another form of colonisation?

Glocalisation takes over all other smaller forms and practices. Globalisation is hegemonic, powerful and pretends to be valuable and therefore it is able to usurp traditional practices which have smaller voices. This process of taking over happens through the process of global turning into glocal which paints over every other form in the same colour. Local identities, value-systems and even methodologies become identical and turn into one singular, standardised form. This is the face and the form of modern colonisation. This form of colonisation influences schools particularly in the developing and under-developed countries, and among Indigenous groups, for whom glocalisation appears to be the only way to educational reforms. It appears that the mathematical practices that we have become habituated to in the last century are heavily influenced by a set of 'correct' norms drawing from the western notion of school mathematics. "How far are we willing to go" (Abtahi, 2022, p.15) away from a "white space" of mathematics education research (Miller & Hunter, 2021, p. 2)? This question invites us to realise that the overarching value system that has emerged

strongly seeks to paint everything in one global colo~~u~~r. This single-colo~~u~~r knowledge-system symboli*s*zes and is celebrated as the marker of globali*s*zation. This form of one-ness leads to a sense of belonging within only one type of knowledge, thus creating not just a stereotype but also taking pride in following the processes of glocali*s*zation.

Notes

[1] Both authors share the first authorship.
[2] 'Glocalisation' with an 's' in it is the term in UK English. We explain later the story of 's' turning to a 'z'.
[3] 'Lingua franca' means 'Frankish language' in Late Latin, and was originally the language of commerce in the Eastern Mediterranean. 'Frankish' referred to any Western European group, and the language was a pidgin of northern Italian dialects, the language of the Mediterranean coast of France (the *lenga d'oc,* distinct from the *langues d'oïl* spoken in northern France, from which modern French is descended) and Catalan. The later dominance of French as the lingua franca of Europe has led to mistaken etymologies.
[4] 'Gross' refers to 12 dozen or 144 units but for convenient computation, one gross is considered to be 140 units. This is a regular practice having spread since the times of colonisation.

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Problem-posing tasks and the inclusion principle

FARZAD RADMEHR

Problem-posing (PP) tasks, if well designed and implemented, can meet several principles of effective task design, such as being accessible to students with different prior knowledge and experiences. In this short communication, I discuss an approach to design and implement PP tasks in mathematics, illustrated with a set of PP tasks designed for the Norwegian context. This discussion is a reflection on the inclusion principle, by which I mean considering students' diverse prior knowledge and experiences *in* and *outside* mathematics. These reflections are based on my past research on designing PP tasks with my colleagues in Iran and New Zealand.

While problem situations have been discussed in relation to designing PP tasks, and a distinction has been made between real-life and purely mathematical contexts (*e.g.,* Cai & Hwang, 2023), this short communication offers new insights on how the inclusion principle could be addressed in designing and implementing PP tasks in mathematics, particularly when task designers focus on real-life contexts. Such conceptualisation and operationalisation of inclusion in designing PP task could be considered when designing other task types.

Christou et al.'s taxonomy of the PP process

I am very much inspired by Christou, Mousoulides, Pittalis, Pitta-Pantazi and Sriraman's (2005) taxonomy for designing PP tasks. Based on their taxonomy, one could design four types of PP tasks: 1) *Editing quantitative information:* Tasks where students pose a problem without any restriction based on the given information; 2) *Selecting quantitative information:* Tasks where students pose a problem that matches the given answer; 3) *Comprehending quantitative information:* Tasks that students pose a problem based on the given equations or calculations; and 4) *Translating quantitative information:* Tasks that requires students to pose a problem based on the given graphs, tables, or diagrams (Christou *et al.,* 2005). Using this framework with this simple description is a significant step toward designing PP tasks that elicit students' mathematical thinking (see Nedaei, Radmehr & Drake, 2022). For instance, let us look at an example of *selecting quantitative information* task administered to Iranian and New Zealand students.

> Task 1. Please can you pose a problem about the area enclosed between a curve and a line with any two arbitrary bounds that will give an answer of 1 (*i.e.,* the enclosed area will be equal to one) (Radmehr & Drake 2019, p. 91).

One could argue that this task has some value. It could be considered a task that has 'wide walls'; students could consider different pairs of curves and lines that have an enclosed area of one, including pairing lines with trigonometric and cubic functions. This task also activates higher-order thinking; I observed that several students in New Zealand were challenged by it and experienced productive struggle (Radmehr & Drake, 2019). However, reflecting on the PP tasks we developed over the years and several important task design principles, I believe our previous design could be improved, as I discuss below.

Quantitative information in PP tasks

When deciding what type of quantitative information could be given to students in PP tasks, one could think about *inclusion* as one of the main task design principles (see Radmehr, 2023, for further details). Here task designers could move from *dress-up tasks* to *tasks with authentic context* (see Vos, 2020). But this is not all. When thinking about inclusion in terms of students' prior knowledge and experience *outside* mathematics, having a meaningful, authentic context is not enough. We also could consider designing tasks in a cultur-