Pointing with Pronouns

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> *"What is implied in the proper use of pronouns? Do children recognise them early and integrate them in their own speech with ease and total comprehension?"* [Gattegno, 1981]

In Chapter 3 of his book *Speaking mathematically*, David Pimm discusses the use of the pronoun "we" in adult social practice, in particular in mathematical pedagogy. Within a classroom excerpt, he cites the teacher who says, "What do we take from the tens column? We take a ten, don't we?" It is a ritual intoning of a procedure (in this case, for subtraction by decomposition) which has been imposed on the audience, the child. Pimm poses the question, "Who is "we"?" He argues that the teacher, by using the plural pronominal form, is appealing to an un-named "expert" community to provide authority for the imposition of a certain kind of classroom practice. Pimm refers to a remark of Dorothy David Wills that "we" seems to have the greatest imprecision of referent of all English pronouns, and therefore is the most exploited for strategic ends," [Wills, 1977]

This point was well-illustrated by prime minister John Major in a recent speech (12 June 1991) to the right-wing Centre for Policy Studies. His assurance of his intention to see through (indeed, to stiffen) educational "reform" was couched in sentences which included:

> "We must be able to measure our children's progress in an objective and regular manner."

> "We have been engaged in the struggle to resist insidious attacks on literature and history in our schools."

I, for my part, am trying to understand better the use by children (and adults too) of another pronoun in mathematical discourse. In a nutshell, *my* question is "What is "it"?". I hope to show how a quick-thinking, articulate child is able to deploy the pronoun "it" as a pointer, as a kind of conceptual variable.

Susie is nine years old; she and I have engaged in mathematical conversation. We talked together for about two and a half hours in total, on four occasions over a period of five weeks. The transcript runs to about five and half thousand words.

I had spent most of the previous term with her class at her school. Her teacher favours children working and learning together, but Susie is a child who does not seem to thrive in a co-operative group situation; she rarely seems to be impressed by the ideas of her peers. Conversely, *her* proposals are usually ignored by them, partly (I think) because her insights are frequently inaccessible, or are elaborated at a length beyond the attention-span of her audience.

However I was frequently fascinated by her contributions to teacher-managed class discussions when she showed herself articulate and willing to expose her thinking to external scrutiny: for example, in a debate to determine the cost of one ruler, ten of which cost £3.50, Susie volunteered, "It's 35p, 'cos you cross off a nought". This was immediately followed by other estimates and proposals which suggested that very few of the children had listened to Susie's contribution or had regarded it as being especially significant. My own instinctive reaction was that she had chosen an appropriate operation but that she was "merely" rehearsing a rule she had learned somewhere to execute the calculation. Perhaps with the same thought in mind, her teacher returned to Susie, and invited her to say more to the class about her method.

> "You cross off a nought", repeated Susie, continuing:

> "If you have ten, and you take away nine ones, you have just the one left... it's because you take away a ninth... no, nine-tenths. So there's one-tenth left".

I was, and am, fascinated by the fact that Susie "explains" division by ten by talking about subtracting nine-tenths. Take away nine-tenths of ten (she seems to be saying) and you're left with one, just like crossing off the nought.

On this and other occasions Susie evinced confidence, efficiency, and an unusual self-monitoring capability in her mathematical thinking. As it happens, her reading and writing of English were at that time significantly behind her mathematical, scientific, and indeed her artistic attainments.

She has another quality which was invaluable for my research purposes: a quiet but determined intellectual independence (not to put too fine a point on it, stubbornness), coupled with a direct kind of honesty which can manifest itself as rudeness as measured by conventional social norms. Consequently I never felt, in our conversations, that Susie was saying what she thought might please me in preference to what she believed. She frequently interrupted me when it suited her to do so. I initiated our conversations, but I didn't feel that I controlled them. I originally transcribed our conversations in search of data about her imagery in relation to number concepts and operations. What I got turned out to be disappointing in that respect. However, on close inspection of the text, I uncovered an equally fascinating linguistic phenomenon, one which I now suspect is not peculiar to Susie. Specifically, I was struck by her use of a pronoun, the little word "it", and the fact that it appeared so frequently in our dialogue. By way of illustration, I offer the following extract from our third session.

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Tim: What about this one you did; two hundred and sixty divided by ten is what?
Susie: Twenty-six.
Tim: Right. And what's twenty-six times ten?
Susie: Twenty-six times ten.. twenty-six lots of ten ..ten lots of twenty-six...oh, **it's** with forty **it** doesn't work. With forty I don't think **it**...except with ones and tens and . ones and tens. **It** wouldn't. .and twenties, sometimes twenties,. ..um, sometimes thirties, sometimes forties, sometimes fifties, sometimes sixties, sometimes seventies,. ..
Tim: .. you mean...
Susie: If you do ten ..and I think **it** would be ten if **it**,...I don't, I'm not sure. suppose you had two hundred and sixty six.. I'm not sure about this, I'll just find out.
Tim: Yes, you experiment and find out. (Susie writes 266 ÷ 10 = 26.6)
Susie: That's twenty-six point six. And 26.6 lots of ten .so you'd in a way put a nought on the end, but you'd end up like that (she has written 26.6 × 10 = 266)
Tim: When you.. (interrupted)
Susie: So any tens with any other, with any number, **it** would end up like that.
Tim: With tens ..(interrupted)
Susie: [forte].. and the same with ones, but not with something like sevens, or whatever. And sometimes with twenties and thirties and forties and fifties and sixties and so on
Tim: What's another number like seven that you think **it** wouldn't work with?
Susie: **It** wouldn't work with .(writes 30 ÷ 7 =)
Tim: You're doing seven again?
Susie: Yep, I thought you asked me for seven
Tim: I said, **it** doesn't work with seven; is there another number like seven that **it** wouldn't work with?
Susie: Oh, with forty, keeping the forty?
Tim: I don't mind, you can change **it** if you want. I mean, let **it** be the number you're dividing by that makes **it** work, or the number you're dividing into .
Susie: I'm not sure, I'm not sure if five does do **it** or doesn't do **it**. So could I find out if five does do **it**?
Tim: Of course you can.
Susie: But I'm not saying that this one will not work, OK? (writes 30 ÷ 5) I want to know whether **it's** doing **it** or not. (pause) Six. and then six lots of five; five lots of six (writes 6 × 5 = 30). **It** does work.
Tim: So **it** works with five?
Susie: With tens...ones, fives and tens **it** probably always works. And sometimes fifteens, twenties, twenty-fives, . .(very fast).. eighties, eightyfives, nineties
Tim: **It** works with all those numbers, d'you think?
Susie: Sometimes **it** works. Sometimes

Of course I selected the passage above in order to make a point; but is it misleading? We ought to look at the full two and a half hour corpus. But in any case, how shall we judge whether the occurrence we then find of the neuter third person pronoun is in any way unexpected, or untypical? To make a start on this question, we need a list of the words which children use frequently, such as that compiled by Rinsland in the USA, from children's writing and conversation. More recent studies of the vocabulary of English children include those of Burroughs [1957] from speech data, and Edwards & Gibbon [1964, 1973] from children's spontaneous writing. Despite their differing methodologies there is a high level of agreement between these two studies about the ranking of very common words. The speech/writing data for both of these studies were from children up to two years younger than Susie. However at age 7+ the nine words used most frequently are (in decreasing order of popularity):

and, the, a, I, to, **it**, is, my, go

In fact the first five are way out in front on the basis of a "popularity index" used by Edwards and Gibbon, followed by "it" and the other three above, which are about equal to each other.

So I compared the incidence of "it" with that of these other eight words in my transcripts, for each of my meetings with Susie, and aggregated over the full five and a half thousand word corpus. It comes out like this — the bottom of each column shows the total word-length of the transcript for each session.

	22 April	29 April	13 May	20 May	All	Per 1000
and	32	38	42	26	138	25
the	39	68	39	25	171	31
a	21	48	26	28	123	22
I	47	38	16	25	126	23
to	31	47	11	30	119	22
it	26	52	48	28	154	28
it's	13	10	11	2	36	7
is	17	15	20	6	58	11
my	0	0	0	0	0	0
go	0	0	1	4	5	1
Length	1050	1940	1380	1150	5520	1000

The frequency-trend for the first five words is much as expected; extensive use of the definite article in our second session effects a take-over at the head of the ranking. The seventh-ranking (according to Edwards and Gibbon) "is" occurs significantly less frequently than those first five, as expected. I find it unsurprising that appearances of "my" and "go" are rare in our maths talk. The observation that Susie makes extensive use of "I" in the first session, and to a lesser extent the second, may be accounted for by the fact that she was at that time acquainting me with some of her personal approaches to arithmetic; perhaps Susie also feels the need to assert herself most in our initial intellectual manoevres.

The pronoun "it" (ranked sixth by Edwards and Gibbon) is consistently used more often, in my conversation with Susie, that the available studies of natural child-language would lead us to expect. If we include occurrences of "it's", then it seems to be out in front in the corpus as a whole. Indeed it (by which I mean "it") is a linguistic/statistical hallmark of our conversation. Starting from this observation, I was led to look at the purposes for which "it" is being deployed, and will suggest that "it" may be a distinctive feature of maths talk, to the extent that it acts as a linguistic pointer, invariant at the surface level. A pronoun, but on behalf of which noun(s)? To investigate this, I ask (for each occurrence) what "it" is referring to. What is it being used to mean? The variable character of the referent of "it" is illustrated in the examples which follow.

Our first conversation began as follows:

Tim: Imagine a very thin square The square.. floats off the table, spins around, slows down, and drops back onto the table

Susie: **It's** all floppy over. .**it** bounces and sort of flops down

I believe that Susie is making reference to a square (*the* square?) or, rather, to her mental image of a square. There are questions that we could pursue about this geometric image (where is it? what is it like? can I get access to it? of what class of squares is it a representative?). Note that "the table" is a viable (if unlikely) alternative referent on the basis of the interchange above; at the time I did not even consider that possibility since my own attention was on the square.

Towards the end of the second session I say:

Tim: You remember last week we were doing multiplying by five, and you said "that's easy" What you did, you multiplied by ten, you added a nought .

Susie: . .and then you halved **it**.

Here the referent is a number, ostensibly the original number multiplied by ten. Or is it the original number with a zero tacked on the end (which may be the same thing as far as its form goes; but recall my tale of Susie and the Problem of the Ten Rulers)? Or does economy suggest that it is precisely the original number which Susie halves, leaving the zero to hold the end place? Her use of the pronoun "you" — meaning "I" — is an interesting instance of *participant deixis* [Wills, 1977]. (In fact it turns out — as I discovered much later — that "you" occurs more frequently than any other word in the corpus as a whole, although there was no clue from the Edwards and Gibbon data [1973] that I ought to look out for that pronoun Further consideration of "you", however, will have to wait for another time, to be the subject of another story).

The dialogue continues:

Tim: Now why, do you think, when you multiply by ten, you just add a nought?

Susie: Because.. ten lots of.. if you count up in twos, suppose, the tenth will be twenty. And fours forty, fives 50, and so on. Sixes 60, sevens 70, eights 80, nines 90, tens 100 And so on. Ten lots of, that's just a nought on the end

Tim: That's extraordinary isn't it? Does that surprise you?

Susie: Not to me.

Tim: Why does multiplying by *ten* add a nought on the end?

Susie: Twenty would add two noughts, for instance.

Tim: So if I multiplied, say, 3 by twenty that would give 3 with two noughts?

Susie: No. No, no, no, no, no Silly me Silly me. No **It's** only with ten For twenty you would double the number at the beginning.

In this case ("It's only with ten") I submit that the referent is a symbolic procedure. She is noting a property which belongs to powers of ten but not generally to multiples of ten. The property in question is clear from the above context. Indeed I explicitly state it: Susie formulates and subsequently withdraws a generalisation of it.

I want, now, to return to the linguistic notion of *deixis*, which means the use of a word whose referent is determined by the context of its utterance. Deictic forms such as *you, now, here* are so commonplace that we automatically understand that their referents (respectively persons, times, places) are context-dependent variables. Divorced from the context of utterance their meaning may be ambiguous or obscure. As an example (of temporal deixis) observe that there is likely to be considerable variation in the intended immediacy of "now" in the two utterances:

a. "I'm going for lunch now." (context: workplace)
b. "I suggest you begin the next chapter now." (context: supervisor to student)

As another example (personal deixis), we have already noted Susie's use of "you" to mean "I" in her statement ". . and then you halved it". Whilst we recognise such a use of "you" as familiar in adults as well as children, we note that it would make perfect sense (but imperfect truth) if Susie had intended "you" to mean her audience (i e Tim). Our ability to interpret the deictic element (pronoun in this case) is dependent on our knowledge of the "coordinates" (time, place, speaker, topic, etc.) of the deictic context.

For our present purposes we properly associate "deixis" with its Greek root *deiknumi*, meaning "to show" or "to point". From this same root we derive the more familiar noun "paradigm" — meaning a pattern, an example which functions as a pointer to a possible way of living or behaving (or merely going about research!). I want to argue, from two further examples, that Susie sometimes employs "it" as a *conceptual deictic*; in particular, that Susie makes effective use of the pronoun to point to ideas of a general nature *which neither she nor I have named*, and whose nature I must be expected to infer. This is in contrast to the first example (the square) and to a lesser extent, perhaps the second (adding a nought) In each of those extracts I had already named the referent, and Susie's use of the pronoun is elliptic, i.e. she presumably uses "it" for reasons of linguistic economy. My next example is of quite different order, however. It comes from our third session.

Susie: No, no.. but times can do **it**, can't **it**, and add, and take. no, take aways can't do **it**.

The *second* "it" seems to have the earlier "times" (multiplication) as referent. The *first* (and third) "it" is more problematic. On the basis of these 18 words alone we can certainly surmise what the first/third referent — object of the verb "to do" — might be For example (given that Susie is nine), "which operations make bigger". Her syntax indicated that it is the *operations* themselves which can or can't do whatever "it" is. We really need some more context In fact a short lead-in gives us some help:

Tim: Why is it that 12 divided by 2 is equal to 6, then?

Susie: Well, what it is, is this number (12) and see how many times that (2) goes into there (12) How many times 2 goes into 12.

Tim: Ah...2 goes into 12 ..

Susie: Or 12 goes into 2 ..

Tim: Or 12 goes into 2

Susie: No, no...but times can do it, can't it, and add, and take...no, take aways can't do it.

I want to claim that Susie has introduced the concept of commutativity into our dialogue. Not only does she not name the concept, but she is *unable to give a name to it* (or, rather, it would be very surprising if she could). However she certainly knows when "it" holds. With the deictic "it" (used here as a provisional object) she can articulate aspects of what she knows, and she does so quite spontaneously and unexpectedly. I suspect that this kind of conceptual deixis is most unusual in adult social practice; to launch into a discussion of an object or idea which we have not taken the trouble to clarify or even to name would seem, by conventional norms, to derive from a self-centred and unacceptable preoccupation with private thoughts. The most charitable explanation of such behaviour would be to put it down to eccentricity.

I regard her use of deixis in my final example as even more fascinating. It comes from the 400-word extract already cited. To set the scene: Susie had divided 56 by ten and written 5.6. She explained that point 6 means "six of the number you're doing it by. .....six tenths of the real number". (Susie's use of "real" to mean "whole" is quite a discussion starter, but for now I stay with my theme, which is deixis.) I'm suspicious that she is using decimals as remainders and ask her for 56 divided by 17. Susie writes 3.4, and declares that the point five is "a half of the real number". It now seems that her remainder-is-decimal rule is getting some interference from her confident knowledge that point five is a half. So next I set up 40 divided by 7. Susie writes 5.5, and again affirms that the point five is a half. What's more, she volunteers that therefore "seven lots of five and a half is forty". So I go for cognitive conflict and ask her to work out seven lots of five and a half. She gets 38.5 (having worked with 0.5 for a half). Thinking that I have a checkmate situation, so to speak, I ask her

Tim: Isn't that a bit funny?
Susie: No, that isn't, because whatever number you put in there (indicates 7) you'd never reach forty, except for one. And you're not allowed one.

Susie is unperturbed. When 40 is divided by 7, and the quotient is then multiplied by 7, she has no problem in living with a product which differs from 40, or so it would appear. In fact, Susie subsequently enters into a lengthy and self-driven exploration into what she considers to be "special cases" in which divisor × quotient = dividend. She says, "Could I find out if five does do it?" and, "Suppose you had 266... I'm not sure about this, I'll just find out"

She makes and articulates generalisations freely:

Susie: Yes, but if five or ten you do **it** with, it always comes out the same number.
Tim: Yes, I was going to say that you said to me that sometimes... (interrupted)
Susie: ... and sometimes fifteen, twenty, twentyfive, thirty, thirtyfive, fifty, fiftyfive, sixty, sixtyfive, seventy, seventyfive, eighty, eightyfive, ninety.

But more fundamentally, from a single example with 40 and 7, Susie has abstracted the *connection between division and multiplication* which becomes the focus of so much of our subsequent conversation. She has no name for this relation, so she uses the neuter third person pronoun, frequently by saying that such-and-such a number (the divisor) will "do it", or in the phrase "it works".

Susie: I'm not sure, I'm not sure if five does do it or doesn't do it. So could I find out if five does do it?
Tim: Of course you can.
Susie: But I'm not saying that this one will not work, OK? (writes 30 ÷ 5) I want to know whether it's doing it or not. (pause) Six. And then six lots of five; five lots of six (writes 6 × 5 = 30). It does work.

Susie's use of deixis enables us to share and discuss a concept which Susie possesses as a meaningful abstraction, yet is unable to name. Rather, before you protest, I assert that there is no name (in the English language) for the identity $b \times (a \div b) = a$ which everyone is happy with. We might use words like inverse, reverse, or opposite. The Statutory Orders for the National Curriculum in England and Wales opt for "recognise that multiplication and division are inverse operations, and use this to check calculations". I have some difficulty with this statement — perhaps this is pedantry on my part? — because it is commonplace to conceive multiplication and division as being *binary operation* (give me a pair of numbers, and I'll tell you their product). Thus, for me, the word "inverse" implies, quite wrongly, that these ideas are set in the framework of a calculus of binary operations. But what we actually have (for every non-zero real number a) are two mappings (*unary* operations); multiplication *by a* and division *by a*; and now it does make sense to say that these are inverse mappings, under composition. I take it as read that the authors of our National Curriculum were in too much of a hurry to think about such matters.

The beauty of the deictic "it" is in its function as conceptual variable. It (i.e. "it") conveys the message, "I have something in mind. I know what I mean, and I think that you know what I mean". It can be a linguistic pointer to a shared idea, to an understood but un-named mathematical referent at the deep structure level. It can give both of us secure and economical access to an algebraic proposition whilst Susie sets about trying to put bounds on its generality.

The Greek work *deiknumi* has another meaning, namely "to prove". In fact a so-called *diknumi* proof [Fauvel, 1987] is one which is presented — typically by means of a diagram of some sort— in such a way that no explanation is necessary, for one can "see" the result and the argument. It is a natural, un-selfconscious style of exposition. It belongs to classical Greek thought around 500 BC, having been used to show (for example) that the sum of two odd numbers is even, by the arrangement of pebbles in pairs. In the *diknumi* proof a train of reasoning (like making a connection between this paragraph and the previous one) is *shared yet unspoken*.

Although she is not aware if it, Susie has caused me to notice and to reflect upon the deictic use of pronouns in maths talk. In the same journal issue in which Gattegno asks the question with which I began this paper, Stephen Brown observes, in another article [Brown, 1981]:

> "One incident with one child, seen in all its richness, frequently has more to convey to us than a thousand replications of an experiment conducted with hundreds of children".