

Platonic Rhetoric in Distance Learning: How Robert Record Taught the Home Learner

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When we talk about learning mathematics we generally have an image of a face-to-face classroom situation. And of course the classroom is an arena for mathematics education where historical inputs can play several important roles. But face-to-face in a classroom is not the universal pattern. For many people the learning of mathematics is done at a distance from the source of instruction, far from any teacher, either through studying textbooks, or following planned courses using print and other media such as cassettes and videos. How the student learns to use and benefit from such material is one meta-question which arises, and another is how the teacher learns to prepare material suitable for distance learners in the first place. My aim in this paper is to see what can be learned from the past about communication at a distance in the teaching of mathematics, through studying the practice of one very skilful teacher, the sixteenth century textbook writer Robert Record.

When any of us sits down to write a textbook, or in other ways devise material suitable for teaching mathematics at a distance, there are plenty of models to follow—or break away from, as we choose. In the sixteen century things were rather different. Record was essentially the first English mathematics textbook writer, and had few precedents to guide him about what should or could be done. The result was a set of texts of astonishing freshness, vivacity, and great pedagogic insight, which put many textbooks of the subsequent 400 years to shame.

Robert Record

Record was a Welshman, born in Tenby about 1510, he graduated from the University of Oxford in 1531, and became a Fellow of All Souls; like others of the period he combined mathematics and medicine, taking a Cambridge MD degree in 1545; but his later public career was as a civil servant, controller of the Bristol mint in 1549 and from '51 to '53 surveyor of mines and monies in Ireland. But he did not manage to plot his way skilfully enough through those politically dangerous times, and a long-standing quarrel with the powerful Earl of Pembroke landed him in jail, in the King's Bench prison in Southwark, which is probably where he died in 1558, when only in his late forties.

So Record was a child of the generation of English humanists, brought up in the new spirit towards learning inculcated by such as Erasmus and Sir Thomas More.

Secular education had become enormously important, and Record was really the first person to think through what mathematics should be taught, and how to teach it, in the circumstances of increasing literacy, growing trade and prosperity, and new technological capabilities for peace or war. His strategy was an ordered series of textbooks, whose titles tell of the mathematical pilgrim's progress:

<i>The ground of arts</i>	1543
<i>The pathway to knowledge</i>	1551
<i>The gate of knowledge</i>	? not published
<i>The castle of knowledge</i>	1556
<i>The treasure of knowledge</i>	? not completed
<i>The whetstone of wit</i>	1557

Of the four which survive, *The ground of arts* and *The pathway to knowledge* concern elementary arithmetic and geometry, respectively, *The castle of knowledge* is an astronomy text, and *The whetstone of wit* is a more advanced arithmetic, leading on to algebra. The latter contains Record's celebrated introduction of the "equals" sign, and ends in a blaze of theatre with the author's being taken away to prison.

The ground of arts was Record's most popular work, with four or five editions during his lifetime, subsequently edited by the young John Dee, and regularly reissued for a century and a half, the last of fifty or so editions (if we don't count the modern facsimile [1]) coming out in 1699. The characteristic Record style was fully formed from the start; the *Ground* begins with a "*declaration of the profit of arithmetic*" which introduces Record's dialogue form, his humanist pedagogical policy, and not least his propensity to versify at the slightest opportunity. Here the two characters who converse through most of his textbooks, Master and Scholar, make their appearance, discussing why numbering is a good thing; the Scholar says he wants to learn, to which the master responds:

Master: I am very glad of your request, and will do it speedily, since that to learn it you be so ready.

Scholar: And I to your authority my wit do subdue; whatsoever you say, I take it for true.

Master: That is too much; and meet for no man to be believed in all things, without showing of reason. Though I might of my Scholar some credence require, yet except I show reason, I do not it desire.

We soon learn that the scholar is not a simple feed-man,

but has a strong personality of his own. This is no dull Victorian catechism. In a piece of dialogue which resonates for examiners down the ages, the master asks

Master: Tell me briefly, what call you the science that you desire so greatly?

Scholar: Why sir, you know

Master: That maketh no matter, I would hear whether you know, and therefore I ask you. For great rebuke it were, a science to have studied, and yet cannot tell how it is named

We may notice, here and in the previous extract, Record's care to explain *why* he asks particular questions, even at the risk of sounding slightly forced, in the context of his overall aim to get the scholar to think for himself.

Reason versus authority

A theme running through all the books is the balance between authority and reason as the proper grounding for knowledge. Record gives the most forthright formulation of his views in *The castle of knowledge*, when in discussing the work of Ptolemy the master remarks to the scholar that

yet must you and all men take heed, that both in [Ptolemy's] and in all mens' works, you be not abused by their authority, but evermore attend to their reasons, and examine them well, ever regarding more what is said, and how it is proved, than who saith it: for authority often times deceiveth many men.

Throughout the texts, indeed, remarks to this effect are dropped about balancing the sources of knowledge: reason and authority, reason and custom, reason and learning by rote, reason and observation. In this way Record gradually imparts a world-view [2] in which thinking for oneself, using one's reason aright, is a prime duty.

Mathematics, too, is something established on a basis of reason, rather than, say, an empirical craft (we are reminded that Record was trying to explain the nature and value of mathematics to a country that had no particular previous views on the matter). We see this, for example, in a snatch of dialogue from *The whetstone of wit*, where the master is explaining about cubic numbers, with a diagram:

Master: In the .4 figure 4. is the root, and is drawn agreeably to that form

Scholar: This is manifest enough to sight

Master: Yet reason ought to weigh it more exactly than sight can comprehend it. For as their triple multiplication doth resemble the nature of sound bodies,

But the emphasis on reason is not a call for providing immediately a full panoply of mathematical justification. On the contrary: Record's geometry text *The pathway to knowledge* explains the constructions and theorems of the first four books of Euclid's *Elements* without giving a single proof. This was part of a careful strategic judgement that the beginners he was writing for had to under-

stand what the results were about, in the first place, and only later see the demonstrations

Hereunto I say that although the cause do go before the effect in order of nature, yet in order of teaching the effect must be first declared, and then the cause thereof showed; for so shall men best understand things: first to learn that such things are to be wrought, and secondarily what they are, and what they do import, and then thirdly what is the cause thereof.

In short, we see in numerous aspects a most carefully considered pedagogy informing Record's texts. And he was very conscious that his particular audience was the growing band of learners at a distance, as he explains in the Preface to *The ground of arts*.

I doubt not but some will like this my book above any other English arithmetic hitherto written, and *namely such as shall lack instructors*, for whose sake I have so plainly set forth the examples, as no book (that I have seen) hath done hitherto, which thing shall be great ease to the rude reader

Platonic rhetoric

Crucial in Record's pedagogy was his choice of a dialogue form (for all but his geometry text *The pathway to knowledge*). He explains this in the Preface to *The ground of arts* in a simple and straightforward way:

I have written in the form of a dialogue, because I judge that to be the easiest way of instruction, when the scholar may ask every doubt orderly, and the master may answer to his question plainly.

But this rather underplays the enormous versatility the form takes on in Record's hands. It enables him to engage with subtle teaching points and convey the learner through difficulties which might be hard to handle in any other way. It has two global advantages as well: it forces the author to be constantly putting himself in the place of the reader, to raise and respond to any difficulties that the learner might have, so there is greater and more persistent reader-friendliness than in most texts of a more one-sidedly didactic form. Secondly, from the reader's perspective, the dialogue form gives a dramatic momentum to the text which encourages the reader to persevere through any hard passages, and generally makes the task of learning at a distance more productive as well as more agreeable.

Teaching in written dialogue form is somewhat rare nowadays, perhaps because we have been in an Aristotelian rather than a Platonic phase of textbook culture over the past couple of centuries; that is to say, one in which the abstract and theoretical is accorded greater value over the more literary and humanistic. At all events, the roots of Record's choice of form are fairly clear: as a well-educated Oxford graduate he was quite familiar with the works of Plato, then as now the paradigm for philosophical dialogue. Elsewhere [3] I have spoken of Record's text as using *Platonic rhetoric*, to draw attention to several

features of Record's dialogues showing a familiarity with Plato's style, most notably his dialectical use of error to move towards truth. The point is not just a dialogue *form*, of which there are other precursors of a more or less tedious nature, but the handling of that form to lead the reader towards greater skill and enlightenment.

A sign of Record's genius in handling the form is the character of the scholar. Far from the placid inquirer of the conventional catechism, the scholar is lively, ready to speak his mind, slightly over-self-confident, but emphatically committed to learning, quite bright, and unafraid of making mistakes—but making enough mistakes for all readers to be able to empathize. Indeed, the scholar's errors are a critical part of the teaching process, enabling ambiguities to be cleared up and common mistakes and misconceptions to be guarded against. What he does wrong is as instructive as what he does right. And that he can go wrong at all, and be led on the right path in a gentle friendly supportive way, carries a crucially important pedagogic message.

A simple example of learning through error is found early on in *The ground of arts*, where the scholar is getting to grips with addition. After the master explains what to do, through an example, the scholar is confident of being able to do likewise, and indeed of being able to devise his own example; but he stumbles on both counts, and has to be rescued by the master:

Master . . . So that now you see, that 160 and 136 do make in all 296

Scholar What? this is very easy to do, me thinketh I can do it even such. There came through Cheapside two droves of cattle; in the first was 848 sheep, and in the second was 186 other beasts. Those two sums I must write as you taught me, thus.

$$\begin{array}{r} 848 \\ 186 \end{array}$$

Then if I put the two first figures together, saying: 6 and 8, they make 14, that must I write under 6 and 8 thus,

$$\begin{array}{r} 848 \\ 186 \\ \hline 14 \end{array}$$

Master Not so, and here are you twice deceived. First, in going about to add together two sums of sundry things, which you ought not to do, except you seek only the number of them & care not for the things . . . [and secondly] in writing 14, which came of 6 and 8, under 6 and 8, which is impossible: for how came two figures of two places be written under one figure and one place?

The warnings that Record is trying to impart at this stage are on two levels: one a technical detail about carrying digits when the sum goes over 9, and the other the principle of not mixing numbers which refer to different things (though this seems a rare instance of "do as I say, not as I do", since the master's own example was a sum of Latin books and Greek books). Then as the sum progresses

there are other very plausible difficulties which the scholar discusses with the master, and from which we can see how experienced a teacher Record was. We learn from Antony à Wood, writing a century later, that Record's reputation as a teacher lived on in Oxford memories:

'Tis said that while he was of All Souls coll. and afterwards when he retired from Cambr. to this university, he publicly taught arithmetic, and the grounds of mathematics, with the art of true accompting. All which he rendred so clear and obvious to capacities, that none ever did the like before him in the memory of man. [4]

His books certainly confirm the impression of someone who had both a marvellous expository gift (as well as writing prose of exemplary clarity) and also a keen perception of just where learners at a distance might run into difficulties. In a sense it is somewhat like the famous Bourbaki double bend sign, with the difference that Record not only signals a difficulty but shows how to get out of it.

The scholar perseveres

We can get quite a good idea of Record's style, and see how its different aspects blend together, by looking at how he handles, in *The whetstone of wit*, the extraction of cube roots. This is quite a complicated recipe, consisting of a succession of verbal instructions, and it is not surprising that the scholar's first comment after the master enunciates it is "*This rule is very obscure in words*". So the master tries again, and summarizes the crucial stage of the rule more concisely "*And if you understand this*", he concludes, "*there resteth no more difficulty*". To which the scholar replies, "*I trust by example to understand it better*". Which the master proceeds to supply; it is part of Record's technique that principles are closely joined to examples, and indeed become understood, as the scholar points out, through application.

The example (the cube root of 26463592) takes some time, as there are slight differences between the two formulations of the rule, but eventually the result comes out (298), and the scholar seems to feel he's quite convinced: "*I need no more instruction for this. I think myself so cunning, by occasion of your examples.*" We can be sure that pride is going to lead to a fall, a double bend must be coming up, and indeed when the master challenges him to prove his understanding, with the number 47832147, sure enough he runs into difficulties. So he has to pick himself up, work out what's gone wrong ("*I see now my error. I must take a less quotient, which thing I might have perceived by the second number. . .*"), and carry it through to a successful conclusion (363).

What we should note is that the scholar's error is not a spurious interpolation for purely dramatic purposes, but actually points up a tricky detail in the procedure; when I tried to work the problem following the master's earlier example, I too came to grief in exactly the same way as the scholar. There is a very real difficulty in the instructions, which Record has recognised. In short, Record has

focussed on just where the home learner might go astray, and warns by the scholar's example

When that is sorted, the master takes the subject further, and shows how to find the nearest cube root for something which is not a perfect cube (he takes 694582951). This procedure leaves you with a remainder, so how should this be expressed most accurately? "*There be as many ways*"; says the master, "*as there be writers almost*"; and he goes on to detail two of these: Cardan's rule and Scheubell's rule. True to his pedagogic principles, Record gives the rules, together with one of his own, but leaves for later the question of which is the most accurate. After pretty hairy calculations, master and scholar get the answer down as far as

$$885 \frac{6076}{10000} \text{ and } \frac{1}{8} \text{ of } \frac{1}{10000}$$

by which time the scholar has grown somewhat restive, and breaks in with "*This is sufficient preciseness. And so I judge it sufficiently taught*" But he knows, of course, that he now has to be tested on his understanding of what he has learned. And this is the problem the master propounds, the classical *Delian problem*:

The Grecians given to idle banquetting, and such-like wantonness, did procure thereby such mortal sickness that the quick were scarce able to bury the dead. Wherefore consulting with their Gods for redress thereof, they received answer that when they would double the Altar, which was of cubic form, they should be delivered from that plague. Meaning, that learning is a due means to deliver realms from plagues and enormities. But to the question, what say you? If the side of a cube be 3 foot (as that altar might be), how many foot shall the side be of that cube which must be double unto it?

The scholar sees quickly that the problem is to determine the cube root of 54, and uses the problem in the first instance to check out the three rules for remainders given earlier, concluding that Scheubell's rule gives a closer result than Cardan's, but still it is "*not so good as he would it were*" To which the master, who sometimes teeters on the brink of being a character in Pirandello, replies "*You are leapt very suddenly from a scholar to a controller. And yet I cannot but praise your diligent observing of such things*". Then the scholar sets to calculate a more accurate value for the root, and finds

$$3 \text{ foot and } \frac{77}{100} \text{ and } \frac{1}{7} \text{ of } \frac{1}{100}$$

quite a good value really: about a 0.6% error.

Record's legacy

Thus Record conveyed his learners, provided only they could read, from innumeracy to quite a sophisticated handling of a range of mathematical practices. In fact we may judge from the numbers of editions that not many readers got beyond *The ground of arts*: but that was really the level that needed consolidation in Tudor England,

anyway. The very real grounding that Record supplied made possible the development of mathematical arts and sciences in Elizabeth's reign in such fields as navigation, and other activities of mathematical practitioners, and in constructing an appropriate mental attitude towards mathematics and its value for society.

How, then, do Record's texts work as teaching documents? Throughout, we get the impression of an author who carries in the forefront of his mind what the person studying the text is supposed to do with it, which can broadly be characterised as getting inside, empathising with, the learning situation represented on paper. Its success therefore depends on how well the author has anticipated the degree to which the mind of the reader and the scholar work similarly. Obviously not every misunderstanding of every reader can be foreseen, but Record makes a more creditable effort in this respect than most later textbook writers. Working in dialogue form helps the author here; it is harder to skim over, or indeed not notice, difficulties when there is an empty space on paper needing to be filled by the scholar/learner's response to the immediately previous material. On another level, a further way in which the texts work is that some characteristics of the scholar's personality may by moral example transfer to the reader, who absorbs an understanding of what it is to be an inquiring mathematician as well as the details of mathematical skills and techniques. This level of moral understanding gleaned from books is particularly important for distance learners, who may need guidance and reassurance in the very business of being a student, of inquiring into knowledge.

Of course, there are problems in learning from Record's texts also: the pacing of the text cannot be quite right for every reader, the crowded black Gothic type of early editions is rather hard to skip back or forwards through, and one may not realise just how complicated a "simple" arithmetical technique is until the master describes it in words!

Textbook writers and distance teachers today may gain inspiration and food for reflection from re-examining the pedagogic techniques employed in Robert Record's teaching texts. And perhaps not only distance teachers. In some cases it might be that his dramatic scenes could be transmuted into modern classroom activities; could children be encouraged to explore their growing understanding by constructing dramas around mathematical ideas, techniques and errors? Just as mathematicians still gain benefit and inspiration from studying the masters—Euler, Abel, Poincaré, and so forth—so teachers benefit from restudying the teaching practices of the great teachers of the past—Socrates, Robert Record, Steiner, and so on. Of course, teaching is a much more difficult activity than doing mathematics, and recovering past teaching practices and evaluating the lessons of the past needs care and subtlety too, but it may be that we could make a start. Certainly the role of dialogue in teaching at a distance is something that might fruitfully be explored more fully, though time will tell whether it demands a master craftsman such as Record to bring it off effectively.

Notes

- [1] The Bodleian Library's 1545 edition was reproduced in facsimile by Theatrum Orbis Terrarum, Amsterdam, and Da Capo Press, New York, in 1969. This is the edition I have used, modernising spelling and punctuation. On the dating of this edition and its relation to other early editions, see Joy B. Easton, "The early editions of Robert Recorde's *Ground of Artes*", *Isis* 58 (1967), 515-532.
- [2] The fullest and most valuable discussion of these aspects of Record remains the classic paper by Francis R. Johnson and Sanford V. Larkey, "Robert Recorde's mathematical teaching and the anti-Aristotelian movement", *Huntington Library Bulletin* 7 (1935) 59-87. On Record's world-view, an interesting paper is Samuel Lilley, "Robert Recorde and the idea of progress", *Renaissance and Modern Studies* 2 (1958), 3-37.
- [3] John Fauvel, "Cartesian and Euclidean rhetoric", *For the Learning of Mathematics* 8, 1. (1988), 25-29.
- [4] Antony à Wood, *Athenae Oxonienses*, (1692) 1721 edition, vol 1 col 255.

But one single person is never merely a magician and not one of us is impervious to the seductions of magic. When we contrast the figure of the philosopher with that of the magician, we must avoid assigning anyone wholly to one category or the other.

The philosopher knows what he is doing when he thinks and acts; the magician does not know what he is doing nor how he is doing it. Both of them hit and miss the truth, but the philosopher incessantly corrects the content of what he considers true and becomes the master of his ideas; the magician examines not so much the truth as his own gestures, modes of expression and ways of impressing others. The philosopher tests the truth by searching for counter-arguments and enemies. The magician refuses to have his truth put to the test; he seems to be blind to the difference between truth and untruth, reality and appearance. He cannot really converse with others; he cannot engage in a candid discussion. He goes through life a personification of the will to power, never seeing through his own motives.

The enemy lurks in all of us. To fight it, we have to fight ourselves. No philosophy becomes true without a conscious detachment from magic, from every form of magic, however sublime, whether scientifically encysted or poetically alluring.

Karl Jaspers
