Teacher-Student Interactions in the Mathematics Classroom: a Review*

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Models of teacher-student interaction have most frequently sprung from general questions of educational psychology, and thus will hardly deal with the specific features of individual subjects in school [e.g. Brophy and Good, 1974; Dunkin and Biddle, 1974; Ulich, 1976]. This is why the following will be a closer look at those approaches which have either confirmed their relevance for mathematics by means of empirical studies, or which have proved to be promising after preliminary thought.

Some general features equally applicable to all school subjects are outlined in the first section. Teacher-student interaction is regarded from a cognitive point of view in the sense that teachers and students are mutually influencing subjects in an interaction process which they interpret and anticipate while acting within a classroom. Certain constructs are considered which may clarify this interpretative act of teachers and students and illuminate the mechanisms within the "black box" of the instructional setting, with teacher behaviour as "input" and student performance as "output". The second section deals with those aspects that typically arise in mathematics classrooms. Some problems concerning the influence of the structure of mathematics on teacher-student interaction are outlined. The importance of the language in mathematics classes (not mathematical terminology) is pointed out and its relevance for the learning of mathematics.

General educational aspects
The concepts to be considered in this section are mainly constructs which clarify certain aspects of social perception and the structuring of the surrounding world. These are: expectation of performance outcomes (one's own or others), causal attributions to outcomes, higher cognitive schemes such as reference norm-orientation and their different influences on classroom communication. A tentative model of teacher-student interaction is given.

The so-called "Pygmalion effect" has been a starting point for a number of studies concerning teacher-student interaction [Rosenthal and Jacobson, 1968]. In their investigation, the authors told teachers that 20 percent of students in an elementary school were "late bloomers", i.e. that they would advance faster in the course of the school year than their classmates. Indeed, these randomly selected students showed, at posttreatment eight months later, significantly greater rates of IQ increase than their peers. Obviously, teacher expectations as to the intellectual performance potential of these students had led to an actual increase of performance in the direction expected.

What are the prerequisites for this teacher expectation effect to occur, and what is the mode of its functioning? For an explanation, Heckhausen [1974] proposes the following process model:

- An increase in the student's performance presupposes that his present results are below his performance potential;
- This poor performance, in spite of greater ability, is due to insufficient effort;
- The student makes little effort in coping with the task as he has a low ability self-concept, i.e. he believes that even increased effort will not lead to success;
- To himself, therefore, the student explains failure in meeting academic requirements as due to his own lack of ability. Success, however, he attributes to chance or good luck;
- The teacher, however, trusts the student to perform better than he presently does, and thus sees failure as caused by insufficient effort; success, however, is attributed to the student's ability and skill;
- While interacting with the teacher, the student becomes aware of the fact that his ability image in the teacher's eyes is incongruent with his own, and that the teacher attributes differently in case of success and failure;
- As the student gradually adopts the teacher's image of his own ability, his causal explanations of his own performance level shift in the direction of those of his teacher, and the student becomes more confident with regard to success, and more willing to make efforts as he thinks this makes sense;
- This, again, serves to improve his performance in the long run, and every improvement reinforces the new performance expectation, and the changed causal attribution.

This explanatory approach toward the "Pygmalion effect" thus links several variables to one another: the student's self-concept with regard to his own academic abilities, his causal explanation of his own performance, the explanation for this performance supplied by the teacher, the teacher's image of the student's ability, and, finally, the classroom interaction behaviour.

This hypothetical situation is illustrated in Figure 1. In this diagram, the teacher-student interaction is seen from a cognitive point of view. Teachers and students are conceived of as subjects interpreting the classroom situation,

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the interactions of whom can be described in their cognitive antecedents and consequences.

The causal explanation of achievement results has been integrated into this approach as an important parameter of the effort invested. According to the attribution theory developed by Heider, four factors are considered potentially causative for achievement and classified according to the dimensions of ‘stability’ and ‘person-dependency’: ability (stable-internal), effort (variable-internal), task difficulty (stable-external), and chance (variable-external).

The conditions leading to the identification of one of these factors as a causative source can be specified. Thus it is assumed that, in the case of a repeatedly observable activity, a factor will be considered a causative source for the result if it covaries with this result. If the teacher, for instance, observes that one student invariably solves the mathematics problems whereas his peers frequently fail, he explains this success by the student’s ability in this subject. Conversely, the teacher sees the persistent failure of another student (as compared to the success of his peers) as being due to this student’s low ability. Consistency of the achievement results over time together with low consensus (i.e. little congruence with peer results) will have the teacher look for the causes in the student’s personality.

If, however, a mathematics problem is solved or not solved by all students, i.e. if there is a high degree of agreement by all students, i.e. if there is a high degree of agreement:

- If the teacher attributes the success a student attains in solving a mathematics problem to the latter’s simplicity, this result will not provide him with any information on the student’s ability, as a low ability on the part of the student would be sufficient to solve the problem. This means that, if several causes can be made responsible for the activity’s result, there will be a tendency to play down the impact of each. This will happen especially in cases where there is no information as to consistency or consensus, or where this information does not yield a clear picture. For instance, a mathematics problem may have been solved by a student not known for success on similar tasks in the past. In addition, some of his peers may have solved the problem and some not. On the basis of the class result, the teacher will start from the assumption that the problem was one belonging to a moderate level of difficulty. Now what is responsible for the student’s result? If nothing indicates an influence of special circumstances, e.g. luck, the cause may be effort and/or ability. From the teacher’s perspective, moderate effort in combination with average ability should tend to be sufficient in order to achieve that result, and the same is true for a combination of high effort and low ability, or for one of low effort and above average ability. If both factors, i.e. effort and ability, can be considered causative for success, and if any one of them is sufficient to explain the result, the teacher will be little inclined to assign a high value to one of them, but will rather consider both as being average.

For extraordinary events to occur, such as success in an extremely difficult task, two causal factors will have to be present simultaneously. High effort, if combined with low or average ability, would not yield success, and, conversely, even above average ability would require additional high effort. In order to evaluate student performance, however, teachers will single out various data from classroom events. Marx [1976] was able to identify several classroom variables (activity of the class, participation in the lesson, voiced student opinion, stock of knowledge, general working behaviour, persistence, etc.), which twelve teachers used, to a different degree, in order to form their judgments. A similar result was obtained by Kehle, Bramble and Mason [1974], who asked 96 teachers of the fifth form to state their performance expectations. Obviously, the use of the above data varied, expectations being based not only on performance result, but also on the variables of sex, intelligence, and looks. The results of these studies suggest that a habitual orientation arrived at by processing the available information may be assumed in teachers. This means that intra-individually stable covariance relationships may be assumed between student performance and the causative factors as perceived by teachers. Rheinberg [1975, 1976] has studied the cognitive intermediate processes in the as-
essment of students under the specific aspects of the assessment standard used by the teachers. He distinguishes two perspectives.

The question whether a student’s attempt to solve a mathematics problem was successful, for instance, could be decided. Rheinberg says, either by comparing it with the results of a peer group, i.e. the class, or just as well by comparing the student’s result to his former mathematics performance, success being defined as “better than before” in this case. While in the first case an interindividual comparison is effected, and the activity’s result is integrated into the performance distribution of the peer group, an intraindividual comparison is effected in the latter case, and the result is evaluated within a longitudinal study of the individual student’s performance. Whether a result is assessed as success or failure will essentially depend on the qualitative standard of comparison.

The method of comparison has far-reaching consequences for the interpretation of causes. When a result is rated successful in intraindividual comparison, the rater tends to hold variable factors responsible, like increased effort, external features, and particular situational circumstances. In interindividual comparison, classification of success is rather based on the assumption of causative factors which remain constant over time, such as special skill for this type of problem, general ability, strength of will, work discipline, ability to cope with stress, etc.

In the classroom situation, the sanctions occasioned by any result will depend on the rater’s standard of comparison. The students’ results will hardly be evenly distributed over the entire class; there will be students who frequently have above average results, and those who will have mostly below average results. In the case of inter individual comparison, praise and criticism will thus be unevenly distributed among the students of a class, the first group getting more positive sanctions, the second group getting more negative sanctions. In the case of intraindividual comparison, however, praise and criticism will have the same probability for all students, as performance stagnation may occur in “good” students just as performance progress may occur in “poor” students.

Another important variable, the organizational form of teaching chosen by the teacher, will also have its effects. Thus, an orientation towards social reference norms at all times results in a levelling of the subject matter offered and the degree of difficulty across the students in the class, enabling the teacher to rate performances as good or poor according to his standards of comparison. This requires a medium level of task difficulty, which, in turn, leads to requirements too undemanding for good students, and too stiff for poor students.

Individual reference-norm orientation, however, observes the “principle of adequacy” [Prinzip der Passung, Heckhausen, 1974] which must necessarily fail in case of social rating standards as this tool is not sensitive enough to individual rates of growth in learning. The teacher who has chosen an individual perspective may look for problems and problem types adequate for each individual student. This means that the degree of task difficulty and the methodological approach to concepts will vary within one class.

The causal explanation of previous performance results will determine subsequent behaviour in two ways. First, the anticipation of success and the level of requirement depends on whether stable or variable factors are held responsible for performance. Student skill and low degree of difficulty in the task presented will lead the teacher to expect similar good results for future problem-solving, and to moderately raise the level of requirement. Lack of student skills and high degree of difficulty in the task presented also leads to a stable expectation, and the level of requirement will be lowered appropriately. If success or failure, however, are attributed to variable factors (mood, chance), expectations with regard to future performance might just as well change, as luck will change, and the same effort may not be required a second time.

On the other hand, different causal explanations of failure, and differences in the subjectively anticipated probability of success resulting from these interpretations, help to explain the differences in persistence shown by individuals in achievement-oriented activities. It has been shown that highly achievement-motivated persons, in cases of failure to solve problems of medium difficulty, show more persistence in selected situations than subjects with low achievement motivation. Highly motivated students attribute failure to lack of effort. Their success expectancy thus remains high; they think that they must merely invest more effort to attain the goal. If failure, however, is attributed to lack of skill, as is typically the case with students having low achievement motivation, the prospects for success are weakened. Investing more effort seems to make no sense, and the activity is soon abandoned. This result was observed by Meise [1974] in his study of mathematical education. He was able to demonstrate the effects of attributing a performance on the subsequent results in students in the upper division of elementary school and in students in schools for the mentally retarded. If failure was attributed to lack of effort, subsequent performance was better; if it was attributed to low ability, however, subsequent performance was worse.

Andrews and Debus [1978] report a similar result. They collected data on success/failure cause attribution subsequent to performance-oriented tasks of sixth-formers, and after that measured student persistence in the attempt to solve an unsolvable problem. The results show a significantly positive correlation between previous failure attributed to lack of effort and the amount of time spent trying to solve this problem, and a significantly positive correlation between the attribution of ability and persistence.

The opposite, however, is true in the presence, not of failure, but of success. Attributing success to high ability will lead to a cessation of achievement-oriented behaviour, whereas attributing success to high effort will have the subjects persist further. Obviously, success in cases of self-perceived high ability is not experienced as being informative of one’s own performance potential, and there is no tendency to continue the achievement-oriented behaviour. If subjects, however, have a lower self-perception of their own skills, and if success, therefore, will require still more effort, further work on these problems may provide more information as to one’s ability. This is why the objective-
oriented activity is maintained.

The constructs of self-concept of ability, of reference-norm orientation, and their attributions, however, are not sufficient to explain the teacher expectancy effect. This effect presupposes that the classroom situation contains indicators enabling the student to draw conclusions about his image in the eyes of the teacher. It should be possible to determine these indicators objectively. Any attempt to do so must start from the assumption that the teacher does not explicitly verbalize his ratings and his expectations, as pedagogical beliefs tend to make him refrain from doing so. Rather, it must be presumed that the student perceives the teacher’s opinion in dyadic interactions with the latter, i.e. that the student interprets certain teacher behaviours as indicators of expectancy.

Indeed, it has been shown that teachers having a higher performance expectancy offer more subject matter, and in more varied ways, than teachers with lower performance expectancy. It seems warranted to interpret this result as an indication that teacher expectations will not only be expressed in subtle differences of verbal interaction, but will also modify teaching styles, thus influencing learning styles and, finally, learning results.

Dalton [1969] and Krank, Weber and Fishell [1970] found that the frequency of interaction was dependent on teacher expectation. They determined the latter by questioning the teachers about their placement of students within the performance hierarchy of the class. According to these author’s results, teachers tended to address more frequently those students whom they expected to perform better.

Further studies have yielded the result that differences in the teacher’s interactions with different students not only pertain to the frequency with which the teacher calls on them to answer questions, but also to specific variables. If the student, for instance, gives an incorrect answer to a question, or none at all, the teacher has several response options. He may repeat the question, rephrase it, or give prompts, i.e. maintain interaction with the student concerned. He may turn to another student whom he expects to furnish the correct answer. The action or behaviour chosen by the teacher will depend on his performance expectancy with regard to the student concerned. High expectancy will make prompts seem useful, as the student will probably be able to solve the problem with their help. In case of low performance expectancy, however, continuing the interaction is superfluous, as even additional prompts will probably not yield a correct answer.

Assessing the time interval the teacher allowed to elapse after having posed a question showed significant differences between the answering time allotted to high performance and low performance students. While the teacher, in dealing with those he perceives as low performance students, will wait only about 0.7 seconds for an answer, he will intervene, in the case of those he perceives as high performance students, not before 1.3 seconds have elapsed. This result, again, may be explained by teacher expectancy.

Further differences in teacher behaviours towards students assigned different performance predictions were shown in the feedback given for correct student responses (praise, positive or no feedback) and for incorrect responses (criticism, negative or no feedback). Voice and tone alterations on the part of the teacher correlating with his expectancy were also reported.

The ability rating and the performance expectancy with respect to a student, however, do not constitute the only determinants of teacher behaviour in dyadic contacts. This behaviour must be seen as determined by multiple factors. Besides students’ personal characteristics, like sex, attitude towards the subject taught, socio-economic status, outward appearance, etc., situative factors, like general noise level, content structure of the subject matter, and actual student behaviour will act to shape the teacher’s behaviour. The separation of roles between initiator-respondent does not adequately describe the teacher-student relationship; rather, it is the students who will “control”, by means of their willingness/unwillingness to communicate, the course taken by the lesson.

**Aspects specific to mathematics**

The limiting aspects of the structure of mathematics are considered. On the one hand the preceding model of teacher-student interaction can be applied to the mathematics lesson. On the other hand some additional restraining effects of the subject must be considered as mathematics seems to lead to idiosyncratic forms of language and communication in the classroom.

The aspects of student attribution, reference-norm orientation, and teacher-student interaction discussed in the preceding section hold true for almost all subjects in school.

From the viewpoint of the teachers, however, mathematics has a special position, so far as it is assumed that student performance in this subject will be much more determined by ability (presumably understood in the sense of general cognitive competence, closely connected to intelligence) than in any other subject [Hofer, 1975]. This assumed connection, as opposed to other school subjects, e.g. German, between a (relatively permanent) specific gift for mathematics and actual performance, and the assumed relatively low correlation between achievement and effort, tends to make the teacher conclude that his chances of influencing the student’s learning process are slim, and that those effects aimed beyond simple teaching of content, at attitude, investment of effort, and motivation, will be of little importance.

On the level of teacher-student interactions, this leads to incorrect answers to questions being attributed more to internal-stable student factors like lack of ability, learning deficits, etc., which, again, prevent a differentiated teacher strategy of offering assistance to attain the desired goal. This is expressed by the frequent failure to give prompts for finding the solution; merely repeating a question will not help produce the correct answer, as fundamental deficiencies in student strategies will not be uncovered. Teacher expectancy with regard to the solution is not fulfilled, his expectancy concerning the student’s lack of mathematical ability is reinforced, prompts will appear hardly promising, and individual assistance will appear too tedious. The teacher’s way out is to turn to another student. The explanatory connection between ability and performance typically established for mathematics thus limits the teacher’s
scope of action in a specific and pedagogically undesirable way. That these affective and/or social variables co-monitor the mathematical learning process has been shown in longitudinal surveys [Anttonen, 1969; Aiken, 1976; Suydam, 1975].

On the other hand, linking of ability and performance leads to a particular situation on the level of performance assessment in mathematical education. The two possible rating criteria of student performance, i.e., considering individual performance growth in a student while taking note of longitudinal variations on one hand, and cross-section comparison with the class norm defined by matched controls on the other, are pointedly abandoned in mathematics education in favour of the second perspective of social comparison [Bülts et al., 1978]. The attribution of stable personal determinants makes a class appear, in the eyes of the teacher, as grouped in a hierarchy according to the students' (assumed) potential performances, and partially blinds the teacher to the variabilities in the individual learning processes, which would be the very thing to enable him to intervene appropriately. This state of affairs is duly reflected in the marks given.

Specific to mathematics, in addition, is the discriminating behaviour of teachers towards boys and girls in elementary school, a behaviour reflecting a fundamental bias, as sex-specific differences in performance are absent in his age group [Fennema, 1974]. These differences, however, develop during the first stage of secondary school, a fact which may be explained by role assignments in the elementary school [Fox et al., 1977].

Besides the dyadic interactions already described, which supply the student with evidence of the teacher's perception of his own ability, there are general teacher behaviours, e.g., teaching style, class organization, and group management, which are addressed less to the individual student than to the students at large. Their effect, accordingly, can be globally shown, as for example, in a general increase or decrease of anxiety in connection with mathematics as a subject, which — as opposed to "manifest anxiety" as a personal characteristic — is operationalized as "test anxiety." The persistence of anxiety for performance in mathematics has been shown in a number of studies [a survey is given by Schell, 1972; Trimmer, 1974]. Anxiety is assumed to exert an influence on two levels: (a) on memory, by side-tracking attention from relevant information, and (b) as a similarly diversionary and interfering factor affecting the generation of hypotheses. The influence of anxiety on mathematics performance must thus be seen in a differentiated way, as it depends both on the complexity of the problem offered [Weinert et al., 1975; Simons et al., 1975], and on student characteristics like intelligence and socio-economic status [Lissmann, 1977], and not only affects the style of the response but the learning process as well. The link between anxiety and mathematics performance must not be assumed to be linear; strong and weak anxiety manifestations both lead to better mathematics performance than average anxiety [Friederich, 1976; this result, however, has not been replicated since and contradicts psychological models of the connection between anxiety and performance].

By means of an appropriate teaching style, the teacher may not only succeed in maintaining a low anxiety level in the students, but also in reducing the influence of anxiety on mathematics performance [Heymann, 1978]. This enhances the influence of other personal dispositions, e.g., intelligence and creativity, and furthers the development of these variables.

A further aspect of interaction behaviour specific to mathematics is the conditioning of students to the mathematical operation required in a problem by means of the numbers given in the description, a phenomenon which can be observed particularly in elementary school. If the student is unable to grasp the nature of the problem, it is mostly sufficient for him to take a look at the magnitude of these numbers: two small numbers will have to be multiplied, two medium numbers will have to be added, a large one will have to be divided by a small one, and a medium one will have to be subtracted from a large one. The error rate of this strategy is relatively small, and if the student guesses wrong as to the nature of the mathematical operation, the teacher will certainly give verbal or nonverbal prompts which enable the student to go on guessing. In most cases the teacher successively asks for the prerequisites of the individual thinking steps, naming, in the event, the necessary operations himself. Communication then is reduced to "ping-pong" teaching, to question and answer, and the teacher's effort is reduced to eliciting, while reducing autonomous thinking, a mere reproduction of expected partial answers to the problem posed. The interaction ends as soon as the correct answer is given, the student being able at any point to break off the communication pattern by giving the correct solution [Bauersfeld, 1978].

It is not in the main the funnel-like communication pattern, with its reducing perspective, which is typical of mathematics. This phenomenon may be observed, with slight modifications, in other subjects as well. As Bauersfeld says, however: "There seems to be no other subject where acknowledgement that a meaning has been understood is made so dependent on formulation, and in which content-related understanding is so much dependent on realization of content in the classroom" as in mathematics [Bauersfeld, 1978, S 163]. There is no other subject in which the teacher is so tempted to misinterpret a (numerically) correct student response as an insight into the underlying problem structure. And nowhere is the student more willing to accept overt or covert prompts in order to conceal his understanding problems. The rigidity of the language of the discipline and of its use in elementary school has been developed in no other subject to the degree that correct verbal reproduction of a term by the student is seen by the teacher as equal to its meaning, and that a real grasp of the concept, if formulated in everyday language, is rejected by the teacher [Winter, 1979]. This influences the channelling of communication as well, as the teacher's question is aimed at eliciting a fixed student response, and additional questions, hints and prompts, are intended to reduce the problem to one theorem, to one number. The low performance student who has no grasp of the mathematical facts can only move about in this code and willingly adapt to a pattern which to him seems to consist of mere empty words, but which is his sole crutch. It becomes a matter of routine.
for him to reproduce those theorems and part answers learned by heart which, by their temporal proximity to the "topic treated just before", seem to suggest a mathematical reference.

The channelling of communication in mathematical education depends on the teacher’s expert knowledge, i.e. on the mathematical approach and way of thinking. Mathematics is often conceived of as not being subject to discourse. Results are correct (true) or wrong, and can be attained, in elementary school, only by a single path. Different models merely serve to illustrate and to represent the variability of concepts, but do not point to different potential ways of solving a problem posed. According to Bruner, the student, in learning the specific subject matter (basic calculus, elementary geometry, set theory, etc.) always learns something about mathematics as well, mostly that there is some set of rules to which he must adhere. This system must be learned by heart and provides, when correctly applied, the characteristic feature of mathematical education depends on the teacher’s expert knowledge, in learning the specific subject matter (basic calculus, elementary geometry, set theory, etc.) always learns something about mathematics as well, mostly that there is some set of rules to which he must adhere. This system must be learned by heart and provides, when correctly applied, the characteristic feature of mathematical education.

This perspective on mathematics acquired by the students will thwart subsequent efforts in problem-solving behaviour, as insight will now be required where routine was sufficient before. Perhaps our approach may help to explain and correct the failures in this respect lamented by teachers, and thought to be due to lack of intelligence or ability in their students.

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