The "Grand Entertainment": Dramatising the Birth and Development of Mathematical Concepts

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There is no theory which has not passed through a period of growth; moreover this period is the most exciting from the historical point of view and should be the most important from the teaching point of view.

Imre Lakatos

1. Introduction

As mathematics teachers, we are naturally eager to attract gifted young people to study our subject. Yet the most creative minds seem to be repelled early in the education process and go on to do other things. The problem is particularly acute in developing countries, as is the need to convince politicians and administrators of the importance of mathematics and its requirements as a creative human endeavour.

The effect of the education machine is frequently to leave the impression that mathematics is an unimaginative jiggling around of formulas and grinding out of algorithms, and that mathematicians are a peculiar hybrid of pedant and prophet: when we are not laboriously proving obvious things we are conjuring up obscure formulae from nowhere, to be learnt and cast like spells over our problems. Where is the excitement and challenge of creativity in all this?

There has been much discussion recently about how we can best teach our students to approach problems creatively, to "play with the data", so that they see the point of the theoretical tool that is best for the job. We would like our students to encounter new theories in such a way that the new impinges constructively upon their present understanding and comes into explicit, creative conflict with "naive" ideas. Yet the accepted style of exposition, especially in mathematics lectures and texts at tertiary level, but also in the classrooms of poorly motivated and poorly trained teachers, is often imbued with a contrary spirit. the student is sickened by a monotonous succession of barely-introduced definitions and theorems—mathematical incantations to call up a procession of apparently dull concepts shuffling, unheralded and unwelcome, into consciousness, perching themselves precariously on the very edges of the armchairs in the shadowy periphery of the mind. Who are these strangers? What is their ancestry? What adventures, what exploits, have they seen?

The challenge is to allow the student (and the lay person) to share something of the conflicts, creative tensions, and intellectual excitement experienced by the human mathematics-makers in their historical problem-situation when these venerable concepts were just beginning to make their names, to communicate the exhilaration of the hunt, not just the "state of the art": the adventures of the journey, not just the landmarks, monuments, and trophies. Mathematics is, and always has been, a "grand entertainment" (Kepler's phrase), in contrast to the static sterility of its public image.

The relevance of the history of mathematics for restoring the image of mathematics, and for enlivening and illuminating the teaching of mathematics, is widely accepted—even if only to supply a sprinkling of apt quotations. Arcavi, Bruckheimer and Ben-Zvi [*FLM* Vol. 3, No. 1] have advocated using a sequence of extracts from original authors as a basis for discussion. This technique certainly helps to recover the original motivations and to order the ideas in something like their original priorities. But much of the drama of creation (including the false trails and the human emotions) remains forever unreported or unrecorded; the puzzle is fundamentally incomplete.

In this article we explore the use of dialogue and theatre to reconstruct (or reincarnate) the flow and ancestry of ideas in time. We would thus employ the art of the dramatist to weave together the fragments and complete the puzzle—to capture with the more natural flow of a good storyline, or dialogue the complex historical dialectic behind the reported fragments—and also to spice the story with human foibles and emotions. Our goal is the response "so *that's* how it must have felt!"

Dramatisation of the history of mathematics can take many forms. If designed for up-front classroom performance, as are the plays of Rolf Grunseit [14] (evaluated in [15]), then importance may be attached to staging, action, and up-dated language, valid within the local culture. Some plays may be effective for reading aloud in class. Others (including the genre specifically discussed in the latter half of this article) which attempt a more authentic evocation of the history, may well work best as texts to be read, thoughtfully and at leisure, by the more gifted students we are seeking to win over to the real joy of mathematics-making.

The purposes of such a "mathematical-historical" play may vary enormously depending on the degrees to which it incorporates artistic, didactic, and historiographic motives, and it has to be admitted that the literary demands are not easy to meet. What is called for is a special breed of playwright-mathematician. For the real *dramatis personae* are the *ideas*, while the human beings in their historical context constitute the theatrical clothing. We must tell the concept-story, dressed up in the human story: how concepts do not spring ready made into mathematics but have a long

For the Learning of Mathematics **12**, 1 (February 1992)
FLM Publishing Association, White Rock, British Columbia, Canada

gestation period followed by a chequered formative career For some time they may lead a clandestine, nomadic, obscure existence awaiting official recognition Once recognised and granted formal status as named free citizens of the world of mathematical constructs they may make demands of their own—the mathematics comes under cognitive pressure from his own creation: he feels like an observer, a mere garment! The drama mounts as concepts interact, and progeny appear...

2. Telling the real story

It is always a shock to meet, yet again, that parody of mathematics represented by the question (posed even within university communities), "But how can you do research in pure mathematics?" The formal mask (that chilling, austere facade, which we ourselves have created and perpetuated in our mathematical communication) is a deceit, hiding not only the human face of mathematics-making—the struggles, victories, mistakes, disputes, the competition and the comradeship—but also the process of discovery, the "rational reconstruction" or dialectic of the story. There is an enormous gulf between the original creative thinking—often woolly, tortuous and tentative, but always exciting—and the routine reasoning which we spin out for the journals, for public consumption, or for self-justification. Where do we look for the real story? Peter Medawar's comments of some years ago have not lost their force, and apply to mathematics as much as any other science:

> "What scientists *do* has never been the subject of a scientific, that is, an ethnological inquiry. It is no use looking to scientific "papers", for they not merely conceal, but actively misrepresent the reasoning that goes into the work they describe [...] Nor is it much use listening to accounts of what scientists *say* they do [...] only unstudied evidence will do—and that means listening at the keyhole". [9]

This task of "listening" might mean taking account of everyday conversation, correspondence, etc , and certainly must take seriously the whole environment of the scientist: the intellectual climate and the historical heritage. The following extract from the book by Broad and Wade is couched in strong language, but deserves to be taken seriously here as much of what they say about science in general is very evident in mathematics:

> "Considered as a literary form, a scientific paper is as stylized as a sonnet: if it fails to obey rules of composition it will simply not be published. In essence, the rules require that an experiment be reported as if every aspect of the procedure had been performed according to the philosopher's prescriptions. The conventions of scientific reporting require the writer to be totally impersonal, so as to give the appearance of objectivity. Thus a scientist cannot describe the excitement of discovery, the false leads, the hopes and disappointments, or even the path of thinking that may have led him through the various steps of his experiment. Only in the most formalized way, usually by describing the current state of research in the field, may a scientist allude to his reasons for undertaking an investigation [...] The very nature of the scientific paper is profoundly anti-historical because the guiding principle of reporting demands that the historian's basic principles—who, what, why and when—be jettisoned from the start. Because science aspires to be a universal truth, linked to neither space, time nor person, the iron dictates of scientific style demand that all reference to the particular be omitted. In the name of objectivity, all purpose and motive must be suppressed. In the name of logic, the historical path to understanding must pass unmentioned. The literary framework of a scientific paper, in other words, is a fiction designed to perpetuate a myth. Scientific textbooks are equally anti-historical, although in a different way. To the extent that they refer to the past, they do so only to present it as reflecting the views or concerns of the present All the false leads, fallible theories and failed experiments which form so great a part of scientific endeavour are resolutely ignored: the textbooks portray the history of science as a straight line pointing inexorably forward." [1]

There is also substance, when applied to the dogmatist, deductivist, mathematical tradition, in the allegations of Thomas Kuhn:

> "[...] both students and professionals came to feel like participants in a long-standing historical tradition [...] Partly by selection and partly by distortion, the scientists of earlier ages are implicitly represented as having worked upon the same set of fixed problems and in accordance with the same set of fixed canons that the most recent revolution in scientific theory and method has made seem scientific. [...] The depreciation of historical fact is deeply, and functionally, ingrained in the ideology of the scientific profession". [7]

It may be true that the terse impersonal character of scientific reporting and recording derives from wholly admirable motives, epitomised in the vow of the members of the fledgling Royal Society (the first society of its kind) to eschew rhetoric, to "reject all amplifications, digressions and swellings of style", and to prefer straightforward language to that of "wits and scholars". But it was not always so dry. Here is Kepler:

> "The roads that lead man to knowledge are as wondrous as that knowledge itself [...] What matters to me, is not merely to impart to the reader what I have to say, but above all to convey to him the reasons, subterfuges and lucky hazard which led me to my discoveries. When Christopher Columbus, Magelhaen and the Portuguese relate how they went astray on their journeys, we not only forgive them, but would regret to miss their narration because without it the whole grand entertainment would be lost. Hence I shall not be blamed if, prompted by the same affection for the reader, I follow the same method."[4]

Kepler was an unusually selfless man. The purpose of his rhetoric was not to convince people of the truth of his results (that was for demonstration alone), still less to persuade them of his own worth; it was to share as fully as possibly his adventures of discovery and to communicate as effectively as possible the desirability of those tantalizing far shores lest the "fairest Cape in all the World" should be degraded to a mere name on the map, a technical Lemma on the way to a distant dull Theorem. Kepler's profound humility before the facts, expressing itself in years of devoted, painstaking labour and labyrinthine intellectual pilgrimage, was entirely compatible with his speculative mysticism and intensely religious personal involvement. He shared with the founders of the Royal Society a religious heritage which generated a revolutionary attitude toward Nature: that combination of respectful obedience and bold trust that is characteristic of the (ideal!) growing, learning child. This attitude has proved so fruitful that the modern scientific method may be aptly summed up as a successful mimicry of the behaviour proper to it, including the suppression of the subjective, and the bland, simple presentation of fact as the basis for cumulative knowledge. *Not what I want, but what Nature says!* But healthy children also manifest other striking qualities essential to growth: that primal conviction of the worth of the struggle, a zest and eagerness, a delight in learning and a desire to please: qualities which played their role, too, in the birth of the Royal Society For it was effectively founded at a meeting which John Evelyn was moved to call, as he read and wept over the manuscript of Robert Boyle's "Seraphick Love"—an eloquent celebration of scientific research as an act of worship and humble adoration of the Creator.

The "scientific paper" was inaugurated in an atmosphere of respect for objective truth, but also of vigorous enjoyment in the challenge and exultation in the communal adventure of discovery; there is none of our unhealthy modern reticence in the entertaining pages of pioneer journals such as the *Acta Eruditorum*. Objective scientific detachment is fully consonant with imaginative and emotional commitment whenever the ultimate object of our study is to make it possible for others to encounter the same reality as objectively and as delightedly as we have. Without this "affection for the readers", coupled with a sense of the "whole grand entertainment" of our intellectual journey, we are reduced to poor witnesses indeed.

The misrepresentation of mathematics may have serious consequences in the number and the quality of students and researchers who are attracted. By a process of natural selection certain kinds of minds (often the most creative) are repelled. Perhaps they are bored by the routine drudgery of following steps that seem to demand no more than a slave-mentality; tired of being hustled along the heavily signposted trail like a package tourist, losing all sense of the excitement of the trail-blazers, or the secret lives of the denizens of the hinterland. Perhaps they become dismayed by the sheer complexity of the machine, having little notion of the guiding principles that motivated the construction of each component part, and the primitive forms through which they developed; or else nauseated by what appears to be the brutal imposition of grotesque alien symbolic devices, never to understand how each is the grand finale of much experimentation, exquisitely tailored to meet specific needs: the symbols made for (and by) man, not man for the symbols. Perhaps they are simply bewildered by upstart solutions to problems hardly grasped The history itself can be seriously misrepresented in ways other than described by Kuhn. Nigel Calder writes:

> "Their heroes' mistakes are often passed over by the chroniclers of science, who thereby hide the human face of science and make discovery seem automatic and easier than it really is, in ways discouraging to non-geniuses."[2]

Many who read Kepler's story (see e.g. [6]) will not envy him his struggle, but none will hear of Kepler's Laws again without a surge of excitement, tinged with reverence; and not a few will feel that they might, in Kepler's shoes and with Kepler's luck, have discovered them too. The hardwon Laws are charged with splendour and high significance, yet the road to them is an earthly road. Kepler does not come across as a superman, but (with all his eccentricities) as one of us, and his discoveries as part of our natural heritage.

The challenge to tell the real story of mathematics comes at two levels: there is the human story, and there is the concept story According to him, informal, quasi-empirical mathematics does not grow by a monotonous increase in the number of indubitably established theorems, but through the incessant improvement of guesses by speculation and criticism, by the "logic of proofs and refutations" [8]. In contrast to what he calls the dogmatist, deductivist, authoritarian tradition, we should place emphasis on the problem-situation, the groping for definition, the flawed proofs and half-formed concepts, the provocative or embarrassing counter-examples. The call is to attend to that strange logic (and poetry) of discovery which gives birth to new concepts and results, some of which we may describe as proof-generated, some pathology-generated, and some of which defeat our best attempts to track into the past which but nevertheless had their ancestry, their gestation in the dark womb of mathematics, and their unsung childhood.

3. Communication by dialogue and theatre

In most cultures at most times, communication of profound ideas has been achieved most vividly and effectively by means of those dramatic devices we associate with the art of theatre. Plato's dialogues were not intended to substitute for the sort of systematic exposition he would give orally to his elite students, but as powerful artistic statements they vastly widened and enlivened his appeal. Galileo's grand polemical aim could not have been better served than by his three-cornered discourses, which entertained while they persuaded, subtly recasting the engrossed reader's view of the world. Knuth, in his "Surreal Numbers" [5] has given a striking demonstration of how the vitality and excitement of profound mathematics-in-the-making (in this case Conway's approach to the numbers) can be communicated effectively by dialogue. (Equally effective, incidentally, is the very existence of J.W.H. Conway himself!)

There are many forms in which dialogue can be used. That canvassed here, and represented in the appendix, is unashamedly historical: we attempt to reconstruct the formation and interaction of concepts in time, using people in their historical setting as the carriers, the theatrical dress. This poses a variety of problems Many people will prefer a "modern" dialogue set within the contemporary cultural context of a particular target audience. May such be written! The historical approach may not be the only natural one and is, for many mathematical ideas, far from being the most direct, logical, or elegant approach. However, it *is* usually a natural one, in the sense that the intellectual gradient and obstacles are unlikely to be overwhelming; moreover, it has its own intrinsic fascination for having actually happened, and for meshing in with the way everything else actually happened, constituting an unbroken theme in the endless fabric of the "real plot". Mathematical creativity is not to be isolated from the wider cultural climate

In this form of dialogue, the people involved should be (like any theatrical props) represented as authentically as possible, without obstructing or obscuring the expression and movement of the ideas. The ideas are primary—the props are there to serve the ideas. Thus, for example, converse would normally take place between near contemporaries whose ideas were in interaction. They may never have met in the flesh, and may never have corresponded directly. Their lives may even fail to have significant overlap They are subpoenaed, as it were, to appear on behalf of their ideas, which interacted historically through their works on their followers. It is, of course, pleasing to be able to weave together in the reconstruction authentic conversations, speeches or phrases, drawn from primary sources. But much of the content must be imaginative extrapolation carried out with a combination of artistic, historical, and didactic ends in view. Dramatic licence may be experienced in various ways and in varying degrees, according to how the primary purpose is conceived: if evocation of history, rather than teaching of ideas, then greater dependence on primary sources and careful referencing would be called for. In any case, the well-disciplined imagination may reach down into the vast substrata of unrecorded human and debate, to the benefit of student or scholar. Examples are Plato's Socratic dialogues, Renyi's "Dialogues in mathematics" [10], and Lakatos' classroom discourses in "Proofs and refutations" [8] Lakatos deliberately dissociates his characters from any historical counterparts, naming them "Alpha", "Beta", etc., but he indicates in the footnotes the historical counterpoint to the classroom dialectic. With characters shorn of all colour and associations except those arising out of their arguments, his drama has "a beauty cold and austere like that of sculpture, without appeal to any part of our weaker nature" (to use Bertrand Russell's words about mathematics itself)—a form fit for philosophers but not ideal for those less accustomed to consorting with naked ideas.

Of course, the history of mathematics is rich in human anecdote, providing a colourful backdrop to the intellectual drama Virtually guaranteed to evoke interest and command attention are the universal human motifs: fierce competition, the tantalising prize, the gambler's lust, pride in priority, touching incredulity, stubborn intransigence, humility, ambition, self-delusion, fear of the unknown, courage and endurance, the exultant cry of victory, the paean of praise, the grand old man, the youthful prodigy, the great-hearted teacher, the fellowship of minds... The sex motif may be a little difficult to find in the saga of mathematics, but death wields its inexorable fascination: Archimedes' murder, and the diagram on his tomb; Bernoulli's epitaph about the "spira mirabilis" (logarithmic spiral); Galois' last night; all are powerful symbols of the enduring vitality of mathematics and its compelling grip on the human mind. It is extraordinary that such heaven-sent aids in the task of enlivening the exposition of mathematics and unmasking its grim parody should ever be despised and neglected, in the gruelling service of the pale, earnest god of deductivist rigour.

The stage is easy to set. But the real *dramatis personae*—the mathematical concepts themselves—are often very difficult to portray authentically. And yet the fundamental aim, in presenting a drama of the kind suggested in this article, is that the concepts (like characters in any good novel) should take on a life of their own in their gradual development from inchoate form to vague adumbration and, finally, to precise formulation and fruitful incorporation in mathematical theories. The creative strategies (and the less dignified antics) of the mathematicians in their endeavours to pluck these elusive sprites out of the intellectual air will often culminate, with dramatic irony, in the concepts themselves getting a firm grasp of the mathematicians! "Things are more or less discovered" (Jaques Hadamard) until, in the full glare of conscious definition within some theoretical environment, they can appear as inevitable as a phrase in a Beethoven symphony.

4. Getting into ancient (and modern) heads

One of the outstanding themes in the history of mathematics is the way in which a cultural climate becomes ripe for the crystallization of certain new ideas, with the associated phenomena of simultaneous and independent discovery and multiple rediscovery. "Mathematical discoveries", exclaimed Janos Bolyai in typically poetic hyperbole, "like springtime violets in the woods, have their season which no human being can hasten or retard". It is hard to put on the old spectacles—to get inside the old heads (or the heads of one's pupils), but any reasonable success in doing so will be rewarded with a deep appreciation of the new. Therefore, the conversations of our drama should represent as widely as possible all relevant aspects of the climate of thought of the time, and bring to light as vividly as possible the conflict between old and new—explicitly recognising the role and the inadequacy of the old, and hailing the new as the victor in the conflict. (An analogous challenge is to get into the heads of children, and write dramatizations of their struggles in coming to explicit recognition of their naive theories seen in conflict with the theories proposed in their science education.)

Various dramatic devices may be used to effect in depicting the conflict of old and new. The incommensurability of different paradigms may be brought our in humorous cross-purpose and confusion (imagine a modern person in converse with Pythagoras). In "A comedy of unrealities" (see Appendix), the impact of the pioneering spirit of quantitative experimental science upon a still rationalistic Cartesianism is depicted in the dialogue in Scene 1 between Galileo and Descartes It is, alas, only too true that individuals often hang on to their ideas to the bitter end, and that scientific theories and philosophical prejudices have seldom died before their founders and archdefenders Thus, in presenting the victory of one set of ideas over another, each represented by its foremost proponent, the apparent discomfiture or conversion of the loser may be felt to be an oversimplification of historical complexities and a misrepresentation of that individual's intellectual pilgrimage. But from the (quite defensible) standpoint of the drama of ideas, he may be seen as living in his ideas and defending them through his writings, followers, and successors, until he capitulates in the final passing or transmutation of the ideas.

It may be objected that exposing students to such conflicts—bringing them to share the struggles of the discoverers—may reinforce the notion that the content is difficult and inaccessible. But it can be a liberating experience to discover that others—even the greats—have had to grapple with the same difficulties; and there is psychological gain in seeing ideas in their formation as part of the flow of discovery. Participating in the tenacity of the pursuit and the joy of discovering, the student may come to appreciate the ideas in perspective and value them as worthy of mastery, while the teacher may be enabled to see them afresh and perhaps infer the ripeness of the moment in teaching them. At any rate, terror is less likely to be inspired by the dragon you have watched growing up from babyhood!

5. The message and the medium

The major problem in expressing dramatically the flow and conflict of ideas is that the concepts are incarnated, as it were, in human conversation—they are clothed with language and symbolism, and translation cannot be undertaken lightly. The critical function of *naming* is often strikingly illustrated in teaching/learning experiences when the use of the right word, with useful logical, cultural, and historical associations, preferably drawn from community vernacular, can clothe a concept so as to bring it in from the cold of the unfamiliar and threatening and into the inner circle of guests with good family connections. A mere sound or written symbol is capable of thus evoking, from the rich sweep of human experience preserved in the collective consciousness of the community, this tangy, connotation-laden cluster of sensations, which moves one to admit the named object, granting it a place, a status, an audience—a listed subscriber number in the telecommunications network of the brain.

In the story of mathematical ideas, the essence of a conflict is often inexplicable apart from the medium of discourse; re-expression in more modern translation or symbolism may implicitly resolve the conflict and enshrine the very discovery whose dawning we are attempting to re-enact. Notation has frequently been integral to major theoretical advance, as in the development of algebra climaxing with Viète and Descartes, and in the wonderful symbolism of Leibniz' calculus which "embodied Leibniz' discoveries so successfully that later historians, deceived by the simplicity of the notation, have failed to notice some of the discoveries". [Weil, 13] Even when translation of the dialectic of ideas is feasible, there are grounds for caution; there is a place for old-fashioned dialogue, for archaisms. It is well, sometimes, to be transported to a remote world where our mental habits and preconceptions are more easily suspended, and the origin and essence of a concept can be observed in the strange but revealing light of the environment in which it emerged. Wrenching ideas out of the environment of discourse in which they have their roots may do serious violence to them. Translating Shakespeare, or the Hebrew scriptures, or Homer, etc., poses immense problems in effectively communicating the power, beauty, or spirit of the original for its audience when it is divorced from cultural and historical context. Conversely, the universality of great works of human imagination or reason is proven in what survives the translation process, and communication can be achieved across enormous gulfs of time, language, and culture For example, certain beautiful cadences of the King James version of the Bible may have no effective cognitive meaning for a modern Englishman, who is then startled by the compelling force and immediacy of contemporary translation It is true that mathematics aspires to a special level of universality, but I have argued above that the wider cultural climate is important in appreciating discoveries and concepts in mathematics; when the form of expression is very much a part of the fabric of intellectual discourse and part of the historical flavour of a given time, it would be desirable, for our form of dramatic representation, to preserve as much of it as possible, without sacrificing too much intelligibility and clarity The tensions here are not to be underestimated. I have also argued the case for historical perspective, and that, in mathematics, must have a linguistic and symbolic dimension. Max Horkheimer's words cannot (after Lakatos) be dismissed as inappropriate for mathematics:

> "Definitions acquire their full meanings in the course of a historical process. They cannot be used intelligently unless we humbly concede that their penumbrae are not easily penetrated by linguistic shortcuts If, through fear of possible misunderstandings, we agree to eliminate the historical element and to offer supposedly atemporal sentences as definitions, we deny ourselves the intellectual heritage bequeathed to philosophy from the beginning of thought and experience". [3]

This impoverishing self-denial is experienced acutely in mathematics, where major efforts are directed towards the consolidation and systematization of existing structures. In the words of Leo Rogers [11], "mathematics is a subject which defines away its past". The consequent loss of the temporal perspective and the driving dynamic of the dialectic, is an affront to the real nature of mathematical activity. (See also the quote from Jacques Barzun on page 30 of *For the Learning of Mathematics, 6, 1.*)

6. Conclusion

Coming in on a conversation in the middle can be fraught with misconceptions. Towards developing a deeper appreciation of the conceptual and notational strategies of mathematics, as well as celebrating the "whole grand entertainment", I propose dramatic replays of the mathematical journeys of the past as a tool and an art form worth exploring. As an exhibit, I have appended to this article the synopsis of a six-scene play, "A comedy of unrealities", which is my own contribution to the genre. It uses the rise of the negative numbers as a vehicle to portray some of the profound changes which have taken place in our understanding of mathematics and its relation to the physical world over the past few hundred years.

This is the first of three plays, forming a trilogy on the development of number concepts, currently in preparation.

There will be many other approaches deemed appropriate for different needs. The choices to be faced are similar to those of the historical novelist: "Do you carry the contemporary style back with you, and all the mental luggage that involves, or do you travel light and dress like the natives?" [12] Or, if the requisite historical sense and intellectual sympathy with past eras be lacking in your audience, do you dress the natives up in contemporary garb and seek to help your audience identify with them in their own mathematical pilgrimage? The scope is vast, and the challenge is issued herewith.

References

[1] W. Broad and N. Wade, *Betrayers of the truth* Simon & Schuster, 1982, p. 128

[2] N. Calder, *The comet is coming,* p. 42

[3] M. Horkheimer, *The eclipse of reason.* O.U.P., 1947; Seabury Press 1974, Chap. V, p. 165

[4] J. Kepler, *Preface to 'New astronomy'.* 1609

[5] D.E. Knuth, *Surreal numbers.* Addison-Wesley, 1974

[6] A. Koestler, *The sleepwalkers* (Part 4, The watershed) Macmillan, 1959; reprinted as *The watershed, a biography of Johannes Kepler.* Doubleday-Anchor Books, 1960

[7] T. Kuhn, *The structure of scientific revolutions.* Chicago University Press, 2nd edn., 1970

[8] I. Lakatos, *Proofs and refutations: the logic of mathematical discovery.* C.U.P., 1976

[9] P. Medawar, *The art of the soluble* Pelican, 1969, p. 169

[10] A. Renyi, *Dialogues in mathematics* Holden-Day, 1967

[11] L. Rogers, The mathematics curriculum and the history of mathematics. *Proc. of the Fourth International Congress on Mathematical Education,* Berkeley, 1980, p. 401, Birkhäuser, 1983

[12] Tom Sutcliffe, *The Literary Review,* May 1986

[13] A. Weil, History of mathematics: why and how. *Proc of the International Congress of Mathematics,* Helsinki, 1978, vol. 1, p.235, Academia Scientiarum Fennica, 1980

[14] R. Grunseit, *Thales of Miletus* and *The genesis of conics.* Available from the author, Vaucluse Boys High School, Vaucluse, New South Wales, Australia 2030

[15] B.J. Fraser and A.J. Koop, *An evaluation of some motivational materials related to the history of mathematics.* Research Report School of Education, Macquarie University 1977

A COMEDY OF UNREALITIES

A Dramatization of
The Rise of the Negative Numbers

A G HITCHCOCK

Synopsis

PART I

SCENE 1 : STRANGER THAN FICTION *The Provocation of Nature*

False and fictitious roots—the grand Book of Nature—the motion of a falling body—quantitative vs. qualitative description—reduction to essentials—mathematics as the language of Nature—solving equations—the authority of Nature—experimental method—empiricism—geometrical and physical interpretation of negative roots—analytic geometry—the algebraic symbolism—algebraic technique and geometric representation—the age of measurement—algebra gives back more than you put into it—negative numbers work!

[Galileo Galilei, René Descartes, c. 1641, in Galileo's house]

PRELUDE TO SCENE 2: *The Presumption of Man*

[Blaise Pascal, c. 1658, in a monastery garden]

SCENE 2 : LEARNING TO LIVE WITH THEM *Confusion and Paradox*

Negative numbers as directed numbers—confusion about ratios—most unruly sequences—outrageous behavior of geometric series—pens are wiser than people?—Metaphysics and mathematics—limits on domains of variable—the fallen idol of mathematical reasoning—various responses to paradox and crisis: renunciation, retreat or reflection.

[John Wallis, Antoine Arnauld, Gotfried Leibniz, c. 1690, and (frontstage commentary) Leonhard Euler, c. 1730]

PRELUDE TO SCENE 3 : *The Courage of the Quest*

[Blaise Pascal, c. 1694, in the afterlife]

SCENE 3 : INTRODUCING THE STRANGER *18th Century Didactics*

Saunderson, Euler and Laplace and the demonstration of the law of signs—rigour, heuristic, example and intuition in teaching new concepts.

[Nicholas Saunderson, c. 1730, in a Cambridge lecture-room; Leonhard Euler, c. 1770, dictating to an insecure scribe in his Petersburg house; Pierre-Simon Laplace, c. 1796 at the Ecole Normale in post-Revolution Paris]

EPILOGUE : *Forward in Faith!*

[Jean D'Alembert, c. 1783, at his desk, reviewing one of his entries in the great Encyclopédie]

PART II

PRELUDE : *Forward in Faith! (Reprise)*

[Jean D'Alembert, c. 1783]

SCENE 4 : STATUS SYMBOLS *The Coming of Age of Algebra*

The dialectic of abstraction and application—arithmetical vs. symbolical algebra—the principle of the permanence of equivalent forms—the law of signs—logical demonstration vs. expeditious calculation—empirical truth and necessary truth—abstractions conditioned by experience—convenience as criterion—arbitrariness of postulates—the liberation of algebra—doubt and fallibility in mathematics teaching and research.

[William Frend, Augustus De Morgan, Robert Woodhouse, George Peacock, Charles Babbage, c. 1827, over port in De Morgan's rooms at Cambridge]

PRELUDE TO SCENE 5 : *Back from the Brink!*

[William Frend, 1796, in frontstage outburst to audience]

SCENE 5 : HISTORICAL ROOTS *Solving Equations through the Centuries*

Viète's "Analytic Art"—symbols for numbers—Bhaskara and Brahmagupta and the partial acceptance of the negative—Diophantus and the classification of equations—general formulas: the abolition of slavery to the particular—symbolism as catalyst in the formation of new concepts—Frend, Masères, and the rejection of negative and multiple roots—physical problems can have more than one solution—incongruity and prejudice—problems as mediators of new concepts

[William Frend, Augustus De Morgan, George Peacock, c. 1827, at Cambridge, and (on a stage-within-a-stage) François Viète, Bhaskara, Diophantus, Francis Masères]

PRELUDE TO SCENE 6 : *Reminiscences*

[Augustus De Morgan, 1871, musing in his armchair in the last year of his life]

SCENE 6 : BOSOM FRIENDS *A Hundred Years of Didactics*

[Various classroom situations:]

- Part I : Charles Smith (1888) on multiplication
- Part II : Alfred North Whitehead (c. 1918) on extending the concept of number through the idea of "operations".
- Part III : Edmund Landau (1930) on the constructive definition of real numbers.
- Part IV : Tom M. Apostol (1957) on the axioms and properties of real numbers.
- Part V : American schoolteacher (SMSG text, 1961) on the operation of multiplication of real numbers.
- Part VI : English schoolteacher (SMP text, 1966) on directed numbers.

EPILOGUE : *Retrospection on the Long Journey*

[Felix Klein, 1908, delivering summer-school talk to German schoolteachers]