# AUTHORITY RELATIONS AND THE TERTIARY-TO-SECONDARY (DIS)CONTINUITY

YVONNE LAI, ERIN E. BALDINGER

Reflecting on her experiences learning proof in university, a secondary teacher said, "It's very much playing the game of how your professor likes proof". Another secondary teacher, upon reviewing a sample proof, said, "I think it's valid. But a college professor would call it invalid". She then circled 'not valid' on the evaluation form. These two teachers, and many of their peers, may learn from their university mathematics courses that someone else is the arbiter of validity in proof. Yet, arguably, one goal of secondary mathematics teacher education is for teachers to graduate knowing that they can determine for themselves whether a mathematical argument holds. How can teachers reconcile their experiences with mathematical authority at the tertiary level with the idea that their secondary students ought to learn to wield mathematical authority?

Ostensibly, proof-based courses at the tertiary level prepare secondary teachers to teach their future students to prove. However, many (though not all) secondary teachers still find their tertiary coursework disconnected from secondary teaching. This tertiary-to-secondary discontinuity was described almost a century ago by Klein (1932) as the second half of a 'double discontinuity', the first half of which is the transition from secondary mathematics student to tertiary mathematics student. We suggest that the relations between teachers and students, including university instructors and teachers-to-be, can influence whether teachers experience discontinuity or continuity as they return to schools as teachers. We argue in this article that it is worth considering an interactional lens to explain perceived (dis)continuity, and in particular, a lens of authority relations.

Proof is a disciplinary practice, and practitioners' norms and expectations shape a discipline, including the extent to which an expert's authority should dictate validity. When a perceived expert is no longer present, that expert's norms may or may not transfer. Consider the secondary teacher-to-be who graduates university believing that *only* their professor can tell whether a proof is valid. This teacher-to-be may become a secondary teacher who rules by their personal perspective, different from that of tertiary professors, because the secondary teacher is now the expert, and expert rule is law. On the other hand, this teacher-to-be may become a secondary teacher who believes that proof has no place at the secondary level. After all, their professor is not there to say what could constitute proof. Moreover, their professor's standards for ways of doing mathematics may not be developmentally appropriate at the secondary level. On the other hand, the secondary teacher-to-be who learned to be an arbiter of proof may be better positioned to teach secondary students how to take up this role as well, and to find consistency in tertiary and secondary ways of doing mathematics. Authority relations may sow continuity or discontinuity.

Often, discussions in teacher education bestow upon 'discontinuity' an unambiguously negative valence, and upon 'continuity' an unambiguously positive one. Yet a continuity of expert authority is arguably undesirable for the purposes of nurturing mathematical community. In this essay, drawing on an analysis of interviews with secondary teachers, we suggest that secondary teachers' conception of authority shapes how they reconcile their positions as former mathematics students and current mathematics teachers. We differentiate between conceptions of authority that rely on expertise or on consensus. We illustrate these possibilities in the context of evaluating proofs, using episodes from interviews where secondary teachers first took on the role of teachers who were teaching proof and then took on the role of tertiary students who were asked to learn proof.

## Expert and shared authority

Amit and Fried (2005) proposed that authority relations in classroom communities can be modeled as *expert* or *shared*. Expert authority can take the form of teachers who expect to be treated by students as the final arbiter of what work is produced and whether it is correctly done. Expert authority can also take the form of students who look to teachers to be told what to do and believe. As Raz (1986) argued, expert authority means that directives issued by those in power replace the reasoning behind directives.

In contrast, shared authority leaves open the possibility that students can learn to be effective and legitimate arbiters of what mathematical work to take up and whether the reasoning holds. Amit and Fried further suggested that when students can and are encouraged to attribute authority to themselves, there may be a path to the highest expression of shared authority. This ultimate expression is 'anthropogogical' authority (Benne, 1970), where a community engages in a "continual attempt to discover rules and define their scope, as well as working within and finding the grounds of existing rules and knowledge" (Amit & Fried, 2005, p. 164).



*For the Learning of Mathematics* **41**, 2 (July, 2021)
FLM Publishing Association, New Westminster, BC, Canada

## Language of mathematical authority

Authority relations describe how people interact with one another, and such interactions are influenced by the particular social context. In this case, that context is the mathematics classroom, where community members often act from a position of teacher or student. As Herbel-Eisenmann and Wagner noted, “positioning is important because it recognizes that interpersonal relationships, especially relationships between teachers and students, necessarily involve issues of control, authority, and power” (2010, p. 45).

Authority relations become visible in the ways students and teachers talk with one another. Talk conveys relative status, and status can influence whose ideas are taken up in discussions. It follows that interactions related to authority can influence and reveal teachers’ and students’ identities as learners and doers of mathematics (Langer-Osuna, 2018). How teachers describe who is doing the mathematics, on whose judgment the mathematical quality relies, or whose contributions are taken up, can create storylines of expert or shared authority (Herbel-Eisenmann & Wagner, 2010). Opportunities for learners to be understood as capable depend on the agency and accountability with which they are positioned.

## A context of responses to parallel tasks

We draw on data collected as part of a study examining the role of context (secondary teaching, university student) in shaping responses of teachers to mathematical tasks (Baldinger & Lai, 2019). We examined interview responses from 17 practicing secondary mathematics teachers who had 1 to 14 years of experience in teaching, with most in their first year. For the interviews, participants completed parallel proof validation tasks. These tasks, shown in Figure 1, represent factors salient to teachers’ and students’ validations of proofs: the use of verbal representations, the presence of algebraic notation, and the use of examples. The tasks are based on TEDS-M released item #MFC709 (TEDS-M International Study Center, 2010).

We designed the study using the construct of priming, which has been used to reveal how context influences decisions (Förster, Liberman & Friedman, 2009). Before the first proof validation task, which asked participants to respond from a secondary mathematics teacher’s perspective, we asked participants about their personal experiences as a secondary mathematics teacher, such as asking them to list all the classes that they had taught at the secondary level. Before the second proof validation task, which asked participants to respond from a university mathematics student’s perspective, we asked participants about their personal

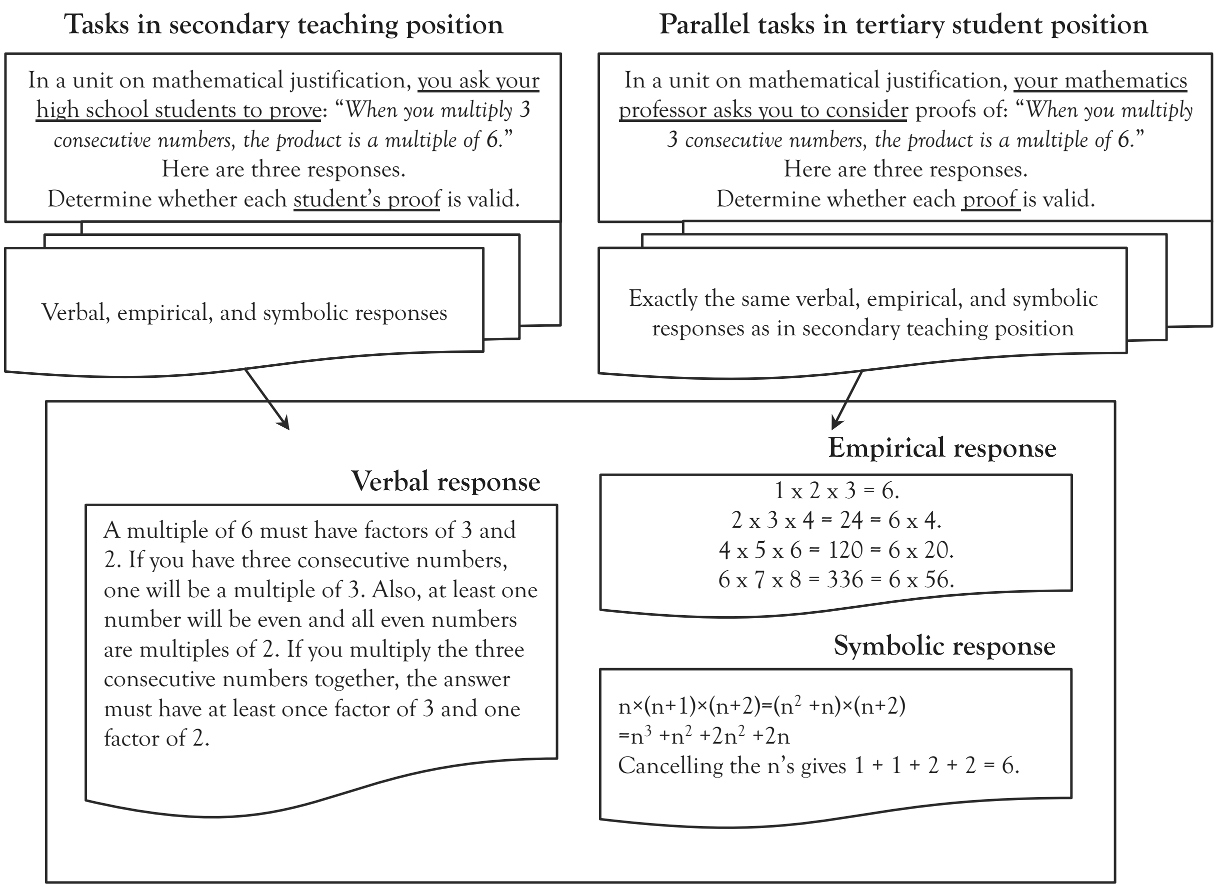



*Figure 1.* *Teachers were asked to validate the verbal, empirical and symbolic responses, from two positions. The responses and task statement were based on TEDS-M released item #MFC709 (TEDS-M International Study Center, 2010).*

experiences as a university mathematics student, such as asking them to list all the mathematics courses that they had taken. In view of participants' changes in validations across the parallel tasks, we theorized that change in position, as secondary teacher or university student, influenced how the participants validated the responses.

We note here a tension between the constructs of priming and position. Priming is theorized to activate non-conscious expression of behaviors consistent with the primed role (here, secondary teacher or university student) (Förster, Liberman & Friedman, 2009), in ways that may be more revealing than direct questioning about the behaviors. That is to say, priming, and any consequence of the priming on behavior, is done *to* a participant; the participant has no choice. However, positioning theory argues that positions are negotiated, not imposed. Moreover, no observer can know with certainty the storylines that guide a participant.

We will never know whether our participants chose to take on the position of secondary teacher or university student. Perhaps some participants took on the position of helpful research subject, or friendly former student (most but not all participants were former students of one of the authors), or whether they acted from another storyline. However, the differences in participants' responses across the tasks, the surprise we heard them express upon asking them to reflect on these differences, and their references to personal experiences as they worked through the tasks, all lead us to believe that at least some participants may have chosen to take on the positions suggested by the priming.

In iterative readings of all participants' responses, we focused on how participants appeared to attribute authority to themselves or others, as evidenced by their own language and written work. As we did so, we also considered the potential role the interviewer may have played in influencing language choices. In each reading, we considered how participant storylines aligned or misaligned with descriptions of expert and shared authority in the literature.

Here, we highlight authority relations that may be at play as people move between the positions of university mathematics students and secondary mathematics teachers. We present three vignettes (Xavier, Sharon, and Phyllis, all pseudonyms) to illustrate the possibility that particular conceptions of authority may help explain, at least in part, the (dis)continuities that teachers experience in tertiary-to-secondary transition. We use these vignettes to illustrate the major themes around authority relations we observed across our data. For clarity, we refer to participants as 'Student' when they were responding to interview questions in the context of being a university student and 'Teacher' when they were responding in the context of secondary teaching (*e.g.,* Student Xavier or Teacher Xavier). The cases that we present may not represent all possibilities of authority relations across context. However, they shed light on how teachers reconcile their positions as tertiary mathematics students and as secondary mathematics teacher.

### Expert authority can limit continuity across contexts

Xavier's case underscores the notion that when the validity of proof depends on someone else's expectations, it can be difficult to see proof at the tertiary level as relevant to secondary level teaching. Student Xavier's validations of the responses, overall, suggested a conception of expert authority that led to believing that proofs were valid only when expressed with formal notation. When asked to explain why he had evaluated all responses as not valid, Student Xavier first pointed to the verbal response and said, "The thinking is very valid, but the formal proof is not there, so that's what makes it not valid". When asked why a professor might assign to students to prove the statement about consecutive natural numbers, he said,

> I think learning how to prove is very [pause] I feel like it was taught to me in these college courses so I knew what was expected of me, and if those are the expectations across the board in college level math courses, I feel like the formality is important.

Consistent with Amit and Fried's description of expert authority, it was professors, not Student Xavier, who seemed to determine what counts as proof.

Student Xavier's statements also accorded with the sentiment among some students that proof is performative, with an audience of professor, rather than a piece of mathematical knowledge (Moore, 1994). When asked to identify the best proof for the context of a university math course, Student Xavier immediately responded, "I'm going to rate [the symbolic response] the best of these, as far as formal proof". Yet Student Xavier evaluated this response as invalid because he could not "distinguish what exactly they did there". Though he did not understand the logic, the response's use of symbols persuaded Student Xavier of its merit.

Turning to Teacher Xavier's validations, two patterns strike us. First, his evaluations followed a storyline of expert authority; his assessments of and comments to students (across the three responses) were generally concluded by a phrase such as 'telling me' or 'convincing me'. We interpret these statements to mean that his views are a *de facto* authority for students to satisfy. Consequently, we interpret these statements as consistent with Amit and Fried's classification of expert authority: Teacher Xavier, not a student, was the final arbiter of the validity of mathematical reasoning.

Second, throughout this part of the interview as well as when responding in the university context, Xavier rarely used the phrase 'proof' without saying 'formal proof'. It was as if formality was so inseparable from proof he could not conceive of proof without formality. The requirement for proof to be formal led to the possibility that proof (*qua* formal proof) may be developmentally inappropriate at the secondary level. When asked whether the verbal response should be considered less valid because it does not use symbols, Teacher Xavier noted,

> I would much rather see that you understand it than you can actually write down a formal proof and throw the variables. That's next. [...] I wouldn't say, 'Hey, let's see how good you are at formal proofs', and throw it [...] I just, I don't think that's appropriate for what I teach.

And so, instead, Teacher Xavier looked for 'understanding', where understanding meant that the statement had been shown in the generality needed, with reasonably complete deductions. Notably, his conception of understanding matched what others have described as proof (*e.g.,* Bass, 2015).

Raz (1986) argued that expert authority can result in following orders without understanding why those orders make sense, because knowing that the orders were given from an authority replaces the need to understand the orders. The conception of expert authority Xavier developed in the tertiary setting around proof as formal may have led to his seeing proof as having no place at the secondary level. When Xavier ceded authority to university professors, he learned the conception that proof must be formal. But if proof must be formal, then (in his view) it may be out of reach for many secondary students. He took on the mantle of expert authority in his teaching to navigate this apparent discontinuity, by declaring a distinction between understanding and formal proof, where neither was synonymous with proof.

## Shared authority can promote continuity across contexts

Teacher Sharon, when asked how she approached the task of validating the responses, said,

> I honestly thought back to my Number Theory in college […] I remember having a problem like this or similar to it. And I got excited by [the symbolic response] 'cause I do remember the whole $n$ and then $n + 1$ and $n + 2$ and using those to form your proofs.

Teacher Sharon gave a spontaneous—and positive—recollection of university experiences. We argue that Sharon's conception of shared authority, experienced at the university level, may have allowed her to view proof at the secondary and tertiary levels as compatible with each other.

In our interpretation, Teacher Sharon viewed mathematical proof as a form of communication that can be learned through community engagement. When asked to theorize why the statement about consecutive numbers might be assigned to secondary students to prove, Teacher Sharon said that she wanted to assign the task and share the verbal, empirical, and algebraic responses. She would "ask [the students] what they thought too, to help them realize what needs to be involved in their own process, in their own proofs and just talk through it with them". Furthermore, she wanted to make the point that, "So when you're writing a proof, you're actually communicating to someone else", and then discussed the principles of divisibility underlying the verbal response. Teacher Sharon talked about her students as persons who have their own processes for doing mathematics, who communicate mathematics to others, and who can learn mathematics from each other.

Student Sharon echoed Teacher Sharon's view of proof as a communal, communicative practice where reasoning is warranted by mathematical principles. When asked to explain why she selected the verbal response as the best for the university context, she first stated, "[the verbal response] is the only one that truly broadens to all numbers, and has very good logical thinking". She went on to explain that the verbal response connected divisibility by 6 to divisibility by 3 and divisibility by 2. She concluded, "[the verbal response] really explains that clearly to the reader". Moreover, when asked why a university professor would assign the statement to prove, Student Sharon said,

> It's a great introductory task [...] You can have a class discussion because in college, I guess that's for me was more of the environment where we're all gonna just discuss together and look at similarities and differences between all of our thought processes.

In both the university and secondary settings, Sharon viewed mathematics classes as a context where students share authority, in the sense that the mathematical agenda includes students' own processes, described in their own voices.

Amit and Fried suggested that shared authority can lead to community engagement in mathematics, particularly in negotiating the terms of mathematical practice. While Sharon did not explicitly discuss negotiation, she did raise the need to communicate so others understand, and she also suggested a desire for students to share personal "processes". Arguably, a prerequisite to negotiating mathematical practice is sharing individual practices.

While we do not know exactly how Sharon's conception of shared authority came to be, we do see remarkable parallels in her portrayal of college experiences and her classroom instincts. Across both contexts, when asked to explain her evaluation of validity or relative quality of responses, Sharon's language followed a similar storyline: the reasoning is sound and must be communicated to others in a way they can understand; further, communication about the proof, and how it came to be, should occur. Moreover, Sharon's responses gave parallel narratives while acknowledging contextual differences. As Teacher Sharon stated, when asked why a secondary teacher might assign the statement to prove, "We all can do numbers. You don't have to do variables. So it really allows all students to have a good understanding of how they need to write a proof". Thus, even as Teacher Sharon was "excited" to see variables in the algebraic response precisely because they reminded her of university experiences, she also recognized that the point was not variables; the point was engaging students in refining their own processes.

We use the case of Sharon to suggest—in contrast to Xavier's case—that shared authority around proof can afford coherence across secondary and tertiary concepts and norms. With shared authority, despite differences in how mathematics may be represented in the secondary and tertiary contexts, when a concept arises in both it can be taken up with similar norms: communicating clearly to others, reasoning with known mathematical principles, and sharing one's process of producing proof.

## Split authority can complicate storylines of continuity across contexts

The cases of Xavier and Sharon seem to show that expert authority impedes connections between tertiary and secondary teaching spaces, while shared authority supports such connections. However, most cases of authority relations are not nearly so straightforward. We conclude our cases with an account of Phyllis, who showed both shared and expert authority. We refer to this as split authority. Phyllis's case, like Sharon's, suggests how shared authority affords coherence between secondary and tertiary concepts. And, Phyllis's case, like Xavier's, indicates how expert authority can lead to idiosyncratic standards for proof. We argue that while Phyllis's case shows some evidence of continuity across contexts, it does not do so in a way that would foster mathematical community.

Two features exemplify Phyllis's case. First, Phyllis expressed an unmistakable conviction in the connectedness of secondary and tertiary mathematics. When Phyllis was asked to reflect on similarities in her evaluations across contexts (that the verbal response was valid, and others were not), she said, "It doesn't necessarily matter whether it's a high school student or college student; the validity of a statement should be roughly the same. I think that a college student should have higher expectations as far as their background knowledge". When asked to determine the validity of each response, both Student Phyllis and Teacher Phyllis discussed how various steps did or did not follow from each other, based on concepts of divisibility.

For Phyllis, mathematical validity was a connective thread between secondary and tertiary levels. Amit and Fried described this situation as 'mathematics as authority', where the validity of an argument is determined by the validity of each step, and this validity follows from known content of mathematics. Mathematics as authority enabled Phyllis to identify a clear continuity across contexts. However, while mathematics as authority has the potential to function as a form of shared authority, in that students can individually or collectively determine the validity of individual steps of a proof, it does not reach anthropogogical authority, the highest expression of shared authority.

To realize anthropogogical authority, a community must point "not only to the logic and content of mathematics"; the community must be part of the "procedural and regulatory practices" of the discipline of mathematics (Amit & Fried, 2005, p. 150). It is through these practices that "the activity of real human mathematicians has a central role in defining what mathematics is" (p. 166, footnote 6). The benefits of shared authority, then, center around the creation of a mathematical community. Phyllis's case illustrates Amit and Fried's distinction between *mathematics as authority* and *authority of the mathematical community.* Further aspects of her interview suggested that while Phyllis may have held shared authority for the sequence of steps included in a proof, she may have held expert authority when reasoning whether an entire text constituted proof.

For both Student Phyllis and Teacher Phyllis, determinations about whether a text constituted proof, as opposed to simply a mathematically valid sequence of steps, could vary from moment to moment. When asked which response was the best proof in the context of a university course, Student Phyllis responded immediately that it depended. For some courses, the algebraic response might be best, because "it's more generalized, with variables". For other courses, the verbal response might be best, because "in the last proof course I took at college level, everything had to be written out in paragraphs". Teacher Phyllis, when asked whether there was a proof that was best in the context of secondary teaching, said that there was not one. Although the verbal response was closest to "being at that great spot", Teacher Phyllis "would have liked to have seen more examples, like [in the empirical response]", or "generalization, like [in the algebraic response]". She concluded, "Yeah, I wouldn't say any of them are necessarily best; they're just different ways of representing it". Teacher Phyllis did not explain these statements further; her authority as teacher warranted them implicitly. In our view, Student Phyllis's expert authority for the communicative norms of proof, together with varied communicative expectations from college professors, may have led to Teacher Phyllis's 'anything goes' mentality for proofs as a whole despite her clear understanding of whether individual steps were valid.

Phyllis's case is more complex than Xavier's or Sharon's, and it represents more than half of our participants. Split authority around the communicative and deductive aspects of proof—holding both expert and shared views of authority—can complicate how tertiary learning shapes secondary teaching. Phyllis's case shows continuity with respect to deductive aspects of proof, stemming from her sense of mathematics as authority. However, the split around the communicative aspects of proof suggests a limitation on continuity. One possibility is that expert authority around the communicative aspects of proof, imported from her experiences as a tertiary student, limited the connections she saw around the forms of proof between tertiary and secondary contexts. A second possibility is that expert authority represents an undesirable continuity across contexts, in that Phyllis maintained a sense of 'it depends' regarding what counts as a proof, which could ultimately be confusing to students. This case of split authority complicates the storylines around authority and tertiary-to-secondary (dis)continuity.

## Authority relations and tertiary-to-secondary (dis)continuity

We entered this project with the mindset of trying to mend the double discontinuity, especially the tertiary-to-secondary discontinuity. However, using the interactional lens of authority to analyze our data problematized the notion of (dis)continuity. Not all continuity may be desirable. As the case of Phyllis shows, continuity of expert authority may counter a goal to nurture more teachers-to-be (and ultimately their secondary students-to-be) into a larger mathematical community.

To make sense of this problematization, we turn to Bass's (2015) characterization of the process of mathematical discovery. For proof to be accepted within a community, it must be certified. It is not enough for the proof to hold logically; the proof must also be communicated in some way that is acceptable to the community. For a community to be able to own the discovery process, they must share authority over both constructing and communicating proof. In our view, the process of certification is about "working within and finding the grounds of existing rules and knowledge" (Amit & Fried, 2005, p. 164) because it establishes what mathematicians take as given and what they need to show, in a way that is public to the community. As Amit and Fried argued, the ultimate expression of shared authority is developing authority as a mathematical community rather than simply taking on the authority of the mathematics.

Citing Esmonde (2017), Langer-Osuna (2018) observed, “Sociocultural theories of learning, such as the communities of practice perspective, have long posited, though insufficiently addressed, the role of power in organizing students’ classroom experiences” (p. 1085). Moreover, teachers at the secondary and tertiary levels may have different ways of doing mathematics (Corriveau & Bednarz, 2017), therefore amplifying the role of power across contexts. We concur with and extend this point: the role of power may not only organize students’ experiences but also teachers’ experiences both when teachers were students and when teachers are teachers. These experiences with power shape the way that teachers reconcile their positions as tertiary students and secondary students, and complicate aims to mend Klein’s (1932) double discontinuity. In moving forward, we must consider authority relations as part of mathematical practice, as well as part of teachers’ induction into mathematics teaching practice.

## References


Amit, M. & Fried, M.N. (2005) Authority and authority relations in mathematics education: a view from an 8th grade classroom. *Educational Studies in Mathematics* **58**, 145–168.

Baldinger, E.E. & Lai, Y. (2019) Pedagogical context and proof validation: the role of positioning as a teacher or student. *Journal of Mathematical Behavior* **55**(3), art. 100698.

Bass, H. (2015) Mathematics and teaching. In Cassazza, P., Krantz, S.G. & Ruden, R.D. (Eds.) *I, Mathematician,* pp. 129–139. Washington, DC: Mathematical Association of America.

Benne, K.D. (1970) Authority in education. *Harvard Educational Review* **40**(3), 385–410.

Corriveau, C. & Bednarz, N. (2017) The secondary-tertiary transition viewed as a change in mathematical cultures: an exploration concerning symbolism and its use. *Educational Studies in Mathematics* **95**(1), 1–19.

Esmonde, I. (2017) Power and sociocultural theories of learning. In Esmonde, I. & Booker, A.T. (Eds.) *Power and Privilege in the Learning Sciences: Critical and Sociocultural Theories of Learning,* pp. 6–27. New York: Routledge.

Förster, J., Liberman, N. & Friedman, R.S. (2009) What do we prime? On distinguishing between semantic priming, procedural priming, and goal priming. In Morsella, E., Bargh, J.A., Gollwitzer, P.M. (Eds.) *Oxford Handbook of Human Action,* pp. 173–192. Oxford: Oxford University Press.

Herbel-Eisenmann, B.A. & Wagner, D. (2010) Appraising lexical bundles in mathematics classroom discourse: obligation and choice. *Educational Studies in Mathematics* **75**(1), 43–63.

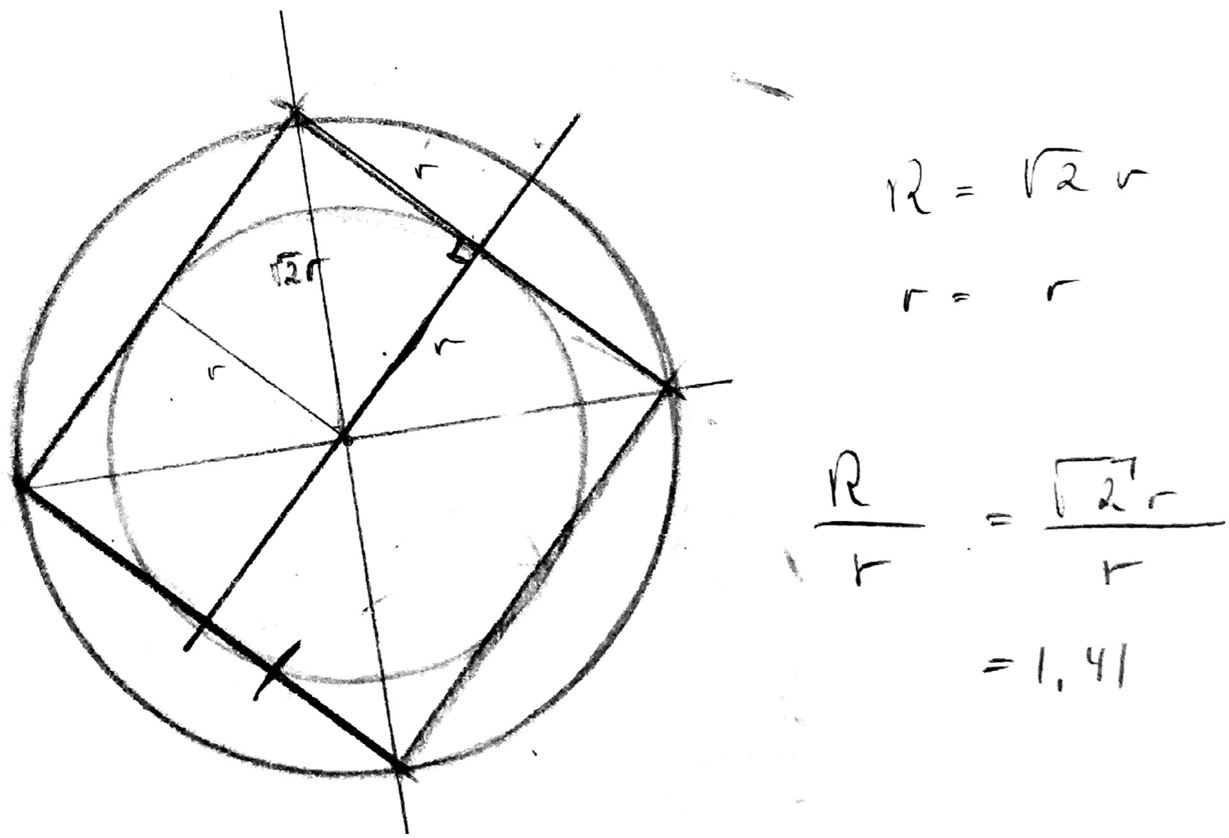
Klein, F. (1932) *Elementary mathematics from an Advanced Standpoint. Arithmetic, Algebra, Analysis* (Hedrick, E.R. & Noble, C.A., trans.). New York: Macmillan.

Langer-Osuna, J.M. (2018) Exploring the central role of student authority relations in collaborative mathematics. *ZDM* **50**(6), 1077–1087.

Moore, R.C. (1994) Making the transition to formal proof. *Educational Studies in Mathematics* **27**(3), 249–266.

Raz, J. (1986) *The Morality of Freedom.* Oxford, UK: Oxford University Press.

TEDS-M International Study Center (2010) *Released Items: Future Teacher Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge: Secondary.* Amsterdam: International Association for the Evaluation of Educational Achievement.


A geometric construction by Benjamin, age 14. A related problem is explored in Marion Walter’s article “Looking at a pizza with a mathematical eye’ in issue **23**(2).