Communications

Research Problems in Mathematics Education — III

More responses to an enquiry. Previous selections appeared in Volume 4, Numbers 1 and 2.

MERLYN BEHR, JOHN BERNARD, DIANE BRIARS, GEORGE BRIGHT, JUDY THREADGILL-SOWER, IPKE WACHSMUTH

1. The problem of dealing with mathematical symbolism appears to be a continuing obstacle to the access of "higher", formal mathematics for many students. A reason for this might be that many students never really learn to read mathematical symbol strings with comprehension and that their skills in reading written language with comprehension do not necessarily transfer to skills in reading mathematical symbol strings with comprehension. First, mathematical symbols are generally not written records of the sound of spoken language and thus cannot evoke meaning through the auditory system. Second, the construction of mathematical symbol strings often involves organizational principles different from those of natural language "Parsing" of a symbol string into meaningful subunits for comprehension requires certain knowledge about the organizational principles.

Question: Can an instruction in reading mathematical symbolism with comprehension improve students' success with formal mathematics?

2. In a discussion of scientific literacy, Hawkins identified a class of key concepts he called "critical barriers": seemingly elementary concepts that may be exceedingly unobvious and difficult for those who have not yet assimilated them, but that are essential for further science learning (e.g., mirror vision, size and scale, elementary mechanics). "Critical barriers" seem to exists in mathematics as well Understanding certain key ideas, such as ratio/proportion, linear order, equivalence, variable, and limit, may be essential for continued mathematics learning. Research is needed to identify such concepts, to characterize students' difficulties with them, and to design effective instruction for them

3 In educational research, we assess successful and unsuccessful students in order to find out about characteristics that give rise to or impede learning success. On the other hand, the part of the teacher is commonly considered as an important variable in the mathematics classroom which can even outweight the influence of carefully designed instructional materials.

Question: Can an assessment of successful/unsuccessful *teachers* (e.g., their skills and habits, major instructional paradigms, metaphors, etc.) contribute to instructional improvement, perhaps tackle the so-called teacher variable? (Individual differences could exist, different instructional approaches could be relevant for different teachers,...)

4 What are the characteristics of a good mathematics teacher?

5. There has always been considerable interest both in trying to determine the characteristics of a "good" teacher and in identifying effective *non-human* instruction (e.g., games, textbooks, computer assisted instruction, films) What are the characteristics of "good" non-human instruction?

6. What ways are there for measuring the effects of the instruction provided in mathematics? In particular, how can the processes that children learn for doing mathematics be attributed to the particular instruction provided? How can the affective notions of mathematics be attributed to particular aspects of instruction? How could one measure the involvement of students in the mathematics?

7 The reductionistic approach of mathematics seeks to construct chains of definitions in which each new thing depends only on other things that have been previously defined, and to construct mathematical knowledge in that "logical" fashion Current school curricula, especially at the secondary level, are more or less structured in accordance with such an approach of mathematics. On the other hand, while many (most?) children seem to enjoy mathematics in grade school, it seems to be the case that many children come to dislike mathematics under the exposure to the secondary curriculum.

For the Learning of Mathematics **4**, 3 (November 1984)
FLM Publishing Association, Montreal, Quebec, Canada

Question. Can a totally different approach to secondary mathematics teaching with increased emphasis on rich connections of mathematics to "imaginable" fields, like arts and music, physics (e.g., crystals), and decreased emphasis on reductionism be found that
(a) provides adequate mathematical instruction,
(b) attracts and emotionally involves students in mathematics,
(c) provides rich imagery components as a basis for a later reductionistic approach to mathematics for those who decide to go into mathematics or science?

8. The recent availability of microcomputers in schools has generated considerable interest in the uses of machines and other technologies in teaching mathematics. As these technologies develop, it will be possible to construct special-purpose devices for teaching specific skills or concepts. The availability of these devices may alter the environment in which children can learn the standard skills and concepts now taught in the schools.
Question: Which skills and concepts in mathematics would appropriately be taught by such devices?

9. The advent of computer technology raises concerns about what constitutes mathematics itself. For example, there is apparently now a program for the Apple which does algebraic "computations" such as solving linear or quadratic equations. The famous four-color problem was solved with the assistance of a computer.
Questions:
a. How will the definition of "computation" change to accommodate this new type of mechanical computability?
b. How will the nature of proof change with the use of computer and information technology?
c. Will we need to communicate a different kind of proof technique as part of the high school curriculum?

10. The availability of microcomputers and calculators has important implications for mathematics curricula on all levels. At the present time, in many texts there are supplementary activities which require micros or calculators. There has been no major revamping of the curricula. This revamping is in danger of being done in a haphazard manner, unless there is a coordinated effort by mathematics educators to undertake revision of mathematics curricula.
Question: How do we proceed in revamping mathematics curricula in the light of the availability of microcomputers?

11. What cognitive differences influence mathematics ability? What learning style differences affect the learning of mathematics? Is the ATI paradigm the best way to find these differences?

12. Through methods courses, in-service programs, and professional journals teachers are shown ways to teach mathematics for understanding (e.g., how to use popsicle sticks and multibase arithmetic blocks to teach place value). Yet, few of these methods appear to be used in elementary classrooms. Instead, teachers tend to teach mathematics the way they were taught--as an abstract set of rules. It has been reported that even teachers who helped created innovative curricula revert back to traditional methods in the everyday classroom.
Question: How can we get teachers to deviate from their prototypes of mathematics instruction and to implement new methods and curricula?

12b. (In the context of Problem 12) Would it be promising to implement instructional findings in teaching programs (that cannot revert back to old habits)?

13. Given the pool of interested teacher education candidates that make their way through teacher training programs in mathematics education, what adjustments might be made in such programs and/or in the certification procedures to fully develop and utilize their talents with assurance of professional level competence in the fundamental areas of ***communication*** skills, ***mathematical knowledge,*** and ***class-room management*** skills?

14. Current developmental and learning theories support the realities of readiness, partial understanding, and forgetting/decay of knowledge and skills, and call into question lock-step, logical, and technical aspects of the mathematics curriculum. What alternative presentation models might be developed compatible with these theories to teach the key ideas of mathematics to a wider range of students in a way that lays a firm, meaningful foundation for application and further technical and theoretical learning?

15. How is mathematics learning affected by what children think mathematics *is?* If children believe that mathematics is a collection of rules, for example, then is their learning influenced by their search for rules to memorize and attempt to apply?

16. What changes in children's ability to deal with ***proportion*** can be expected if decimals are taught (in the context of calculators) before fractions have been taught?

17. The *"frame"*-paradigm entertained in cognitive science seems to be a very useful concept in explaining phenomena encountered in the learning of mathematics; for example, it provides an explanation for the lack of interconnectedness of areas of mathematical knowledge that is often found in students, or for the phenomenon of "backsliding", that is, reverting to faulty procedures that were acquired in the process of learning but were already "replaced" by correct ones. These examples give rise to the general *Question*: What paradigms of cognitive research seem to be fruitful for adoption in mathematics education?

Department of Mathematical Sciences
Northern Illinois University
DeKalb, Il 60115, U.S.A.

JOHN MASON

Here are some pieces from my bottom drawer which bear on the question you ask.

How can perpetual reconstrueing be fostered and sustained?
The standard paradigm for mathematical instruction is

> Exposition: telling people what is true and, occasionally, what is false.

At the same time it is generally recognized that we only begin to understand something when we make it our own. So we go away and pore over, even reconstruct our notes.

Discovery learning was a catch-phrase intended to offer an alternative paradigm, but it rapidly fell into disfavour because of the associations with totally unstructured chaotic classrooms. As opponents said "How can everyone reconstruct human knowledge for themselves? There must be faster ways."

Mathematical education is currently bedeviled by a culture which focuses on facts, knowledge, and a demand for statistically "proven" efficient methods for doing things. In fact, mathematical education belongs to the domain of personal growth and interaction—growth of both the teachers appointed by society to accompany the young through childhood and adolescence, and of the students. Both are engaged in a constant process of re-constructing, re-searching and re-articulating for themselves, or else they are inwardly dead. Teachers must *Be Mathematical* with and in front of their students. Not only that, they must be able to *participate* in and *share* the *perceptions* of their students, the building up, the tearing down and rebuilding, the assimilating and accommodating. The curriculum must in the same way also undergo a continual re-organization, re-structuring and re-naming of critical aspects. If teachers fail to re-discover, then it is no wonder that students fail to detect life in the mathematical corpus. Because of the need for continual re-construeing there can be no *answers* in mathematics education. Yet there must be action, so that the necessary growth is possible both in teachers and students. What form can that take? The response that there are no answers, only questions, has too much of a culturally static ring to it. We may *say* there are no answers, but we go on reading and writing for journals, and attending conferences. What sort of picture can be sketched of a development in the asking of questions, in prompting perspectives to alter, in staying alive?

How can an appropriate balance between social and personal mathematics be recognized and supported?

Primary mathematical thinking tends to be small-group-collective. Workcards and own-pacing are often the order of the day. Language acquisition (meaning-making) is recognized as fundamental, and this demands interaction because we make things our own by expressing them to others.

As pupils get older, the groups get larger. Whole-class "discussion" becomes the norm just when personal responsibility, self-discipline and individual work become important. At the same time examinations stress the importance of individual work and virtually discount the virtues of collective work. Clarity and rationality of these forces seem essential, and lead to interesting questions about the ebb and flow of energy.

Groups can work collectively when exploration begins. Eventually people want to pursue particular directions. A group-sense puts pressure on group-decisions to follow one path at a time, for the quick-insightful person to stay with the careful checker. Much can be learned from social mathematical thinking. Excessive group-awareness can deny an equally strong aspect of mathematics as a distinctly personal activity with expressing results to others and attempting to justify work convincingly as an important feature. A group that is truly functioning as a group can afford to disband, confident that it will reform and benefit from a period of peace and quiet.

Much more needs to be understood about the nature of insight-energy which activates the recipient, of the force to express results to others, and the need to practice skills to mastery. So often classroom demands are out of phase with individuals' experiences of energies. On the other hand, it is sometimes the case that an authoritative intervention demanding certain behaviour can release a blockage, can force awareness of possibility. A key feature in this study will be the role of images and metaphors, of those experiences which represent the cohesion which are called meaning.

What's the real contribution of mathematics to culture? How can societal and personal images of mathematics be modified and updated?

The prevalent cultural picture of mathematics is totally impoverished—and has been for a long time. "Glorified arithmetic" and "harder sums" carries no sense of the essence of mathematical thinking. Yet mathematics is seen as an essential ingredient of everyone's education, and so there are tremendous forces acting on teachers.

Is mathematics really essential? What is this "mathematics"? The Cockcroft Report gives one of the most articulate justifications for the present role of mathematics in society and in eduation, yet even it is threadbare in places.

Calculators and computers demonstrate that the content of mathematics is not what is universally essential. Cockcroft's sub-reports on the mathematical skills required by industry demonstrate that there are very few skills actually needed by the bulk of the population, and the ones that are can be coped with by folk-maths and by the ubiquitous chip. Are we moving to a two-tier society of the numerate and the non-numerate? The widespread failure to be able to cope with percentages, even amongst businessmen and our highly educated civil service, suggests that something is not right.

A more comprehensive, more convincing image is needed of mathematics. To this end we need a renewed articulation of the nature and role of mathematics in society. We cannot permit mathematics to go the Latin route by falling back on "it teaches you to think", because current practice patently fails in this respect. Perhaps it *would* be better, as some have argued, to release mathematics from the bondage of core-curricular importance and permit mathematical thinking to take place.

A cultural image of mathematics cannot be divorced from personal images. How is it that a creative, exploratory world can be dominated by association with absolute and right-wrong answers? Struggles in mathematics lessons are often seen as attempts at high-jumps. Either the bar falls off, or it doesn't. Why can't mathematics be more often seen as a long-jump. No-jumps happen, but with care you can always get somewhere. Personal images need sharing. In the process we might discover that our own pictures change—but that is the concomitant of growth!

What interactions are there between adolescent experience and mathematics (particularly, but not exclusively, in women)?
During puberty human beings experience considerable changes to their hormone balance, and hence to their way of perceiving themselves and the world around them. At the same time they come across algebra, one of the intellectual watersheds of our society. The notions of infinity, of unknowns (Mary Boole's "as yet unknown"), find ready resonance in adolescent questions and perspectives.

It is not only mathematical content that interacts with changing awareness of self and peer-relations. Adolescence is a time for testing boundaries, for "tickling" authority by the state of the bedroom, for assembling own-constraints and finding out why norms and customs are as they are. At the same time, classroom experience tightens down in the authoritative stance of formal examinations, where previously teachers would have felt free to explore something that captures class attention or dwell on some notion that caused difficulty. Now "we haven't got time" and "we do it this way because *they* say so" become increasingly frequent. Where previously classrooms had a homely, resource-packed feeling with notice boards, book-racks and apparatus, they become places where particular subjects are studied. Just when questions of identity surface, the physical environment becomes less personal.

Some adolescents respond to the intellectual challenge offered, but cannot relate to parental gravy-train arguments for performance. "I can do O-level, why should I show it? I can't be bothered to slow down and check things". Adolescent energy can be rapidly focus-shifting, or it can concentrate on demands for precision, and neatness. What is an appropriate response from a mathematical teacher?

What are the foundations of children's experience of arithmetic?
There are several ways of placing arithmetic on a firm axiomatic foundation, but these are abstractly mathematical, and depend on considerable prior mathematic sophistication and experience. That experience includes, even depends on, early experience with numbers and arithmetic in childhood.

Even the briefest of observations of young children will suggest that we do not teach children everything they learn about numbers. On the contrary, they construct and construe for themselves independently of any teacher. It follows that teachers need a perspective of numbers and arithmetic which is much more than factual and computationally competent. It must be consonant with children's experience and construeing, and yet consistent with the more formal approaches of set theory.

Caleb Gattegno is one of the few people to recognize the need for, and the possibility of a foundation of children's arithmetic. His deep insights into the world of childhood-construeing need articulating and re-articulating so that they can be understood by others, and developed into a fully fledged mathematical-philosophical-psychological foundation.

What does it mean to "know" something in mathematics education? How can that "knowing" be passed from person to person?

What alternatives are there to the accretion model of research? Efficiency, facts and replicable experiments are all artifacts of the machine metaphor spawned by the industrial revolution. They are no longer appropriate in a world which recognizes feelings and perceptions as well as behaviour.

What are the energies involved in transitions between states such as not-knowing; having-a-sense-of; able-to-articulate; able-to-record-in-pictures, in words, in symbols. How do such transitions come about?

How can processes, understanding and mathematical sophistication be assessed?

How can teachers at all levels be encouraged to admit their doubts and uncertainties in the face of the disparities between teacher intentions and examination performance, as a preliminary to the possibility of changing perspectives?

How can a common vocabulary be developed to assist teachers to report, reflect on, and discuss learning and teaching in real depth, without being deflected by mathematical content?

How can the distinction between computer as apparatus controlled by the user and computer as a controlling machine be made sufficiently clear in educationable computing at least, to avoid the fiasco of educational television?

How much of the modern syllabus is as outdated as algorithms for computing square roots by paper and pencil —eg calculus?

How can I discern the extent of generality perceived by someone else when looking at a particular case of what I see as a generic example?

Faculty of Mathematics
The Open University
Walton Hall, Milton Keynes, MK7 6AA, U.K.

DAVID TALL
One of our main preoccupations as maths educators over the next quarter of a century will be a diversion to explore man's interaction with the computer. The contrast between carbon-based life and silicon will hone our ideas in maths education! I still believe the central questions are,

how do we do mathematics?
how do we develop new mathematical ideas?

Since we describe things in a manner dependent on our previous experience, we model things using known concepts as analogies.

Is the computer a good model for the brain? It is a beguiling idea, but there seem to be clear differences. Are these differences real or apparent?

At the moment we seem to teach "by osmosis", a little of our ideas as teachers filters through the membrane of the next generation but we still do not know the mechanism by which we are successful Until we can postulate reasonable models for the process of learning we cannot claim to be true scientists only alchemists

Teaching for a year in school made me fully aware of the difficulties of organising a classroom. If we knew how the individual learns (which we don't) perhaps we should address ourselves to the problems of how to organise the learning of the group. I found this aspect, in a word, difficult. In fact I came to the conclusion that I (at least) could not *teach* mathematics, perhaps it is impossible.

We have a lot to learn from the immediate past, the last two decades. What are the results of "the new math"? What are the implications of the new social order? "Comprehensives", it was claimed in the Sunday Times "get better results". The *average* is better but the very best children do worse. We must look back on our experiences to see how we can improve the future. It is the "experts" who, by and large, failed with "new math", who remain to try to do a better job for the future.

The advent of new technology brings with it different technological demands. In the next quarter of a century man will change more than he did in the previous 4 million years.

It is a daunting prospect, so daunting I must go and lie down and rest my aching back.

Department of Science Education
University of Warwick
Westwood, Coventry CV7AL, U.K

DAVID ROBITAILLE

I turned your request for me to formulate a number of problems in mathematics education into an excuse for a departmental seminar. Eight members of the department and four doctoral students met for a couple of hours earlier this week and we had a very stimulating discussion Thank you for suggesting the topic. I have attached a list of six questions which were suggested by one or more people at the meeting

Problem:

How can we evaluate and capitalize on the mathematical strengths of each individual?

We know little about assessing students' understanding of concepts. In our testing we concentrate on those things we feel we know how to test: knowledge of facts and mastery of skills. The recent interest in research in the area of identification and treatment of learning difficulties in mathematics has made us conscious of our lack of good techniques for evaluating children's understanding and of the tremendous waste of mathematical talent that is the result not only of pupils' misunderstanding (or lack of understanding) of mathematical concepts but equally, if not more important, of our own misunderstanding (or lack of understanding) of the concept of understanding and, particularly our lack of knowledge concerning how to determine whether, and to what degree, understanding is present in the mind of person X with respect to mathematical concept Y

Problem:

How can we least accommodate the teaching and learning of mathematics to the use of calculators and computers?

What is the point of spending years developing students' computational skills when calculators are so readily available and efficient? On the other hand, if we de-emphasize the importance of algorithms for computation what will we put in the curriculum in their place and who will teach this new material?

Problem:

How should changes in the mathematics curriculum be implemented?

There is a considerable body of evidence to the effect that much of the material which was introduced into textbooks under the guise of the "new math" was in fact, not taught by teachers. The implementation process would appear to have failed or, as Ernie House has suggested, to have been subverted by teachers. What is the optimal method for implementing change in the curriculum?

Problem:

How can we best enhance students' ability to apply the mathematics they have learned in new circumstances (a) within mathematics itself? (b) to situations in which mathematics is a tool?

Problem:

The concept of symmetry occurs with respect to our interaction with our surroundings; in particular, it occurs in many forms in mathematics Often application of the ideas that form the concept of symmetry can be used to help solve problems within mathematics as well as outside of mathematics. Can we use symmetry as a vehicle to teach certain aspects of problem solving, and if so, should we? In what ways can an understanding of symmetry in general (not merely in geometry) assist in the learning of mathematics?

Problem:

What is the nature of the problem solving process in mathematics and how is it related to the analogous processes in other subject areas?

Is there such a thing as *the* problem solving process in mathematics? (The analogous question in science, namely, "Is there such a thing as *the* Scientific Method?" has been answered in the negative to the satisfaction of many emminent scientists and science educators) If there is, then is it anything other than a synthesis of various problem-solving processes? Are these problem solving processes in mathematics exclusive to mathematics or are the same processes evident in other disciplines? If not the same, then are there analogues? If not the same and if not analogous, then why not? What is it about mathematics that is "special"?

Assuming that the problem-solving process in mathematics does exist, is it learnable? Is it teachable? If it *is* teachable, *should* we try to ensure that it be learned? What are the pay-offs? What are the sociological *dis*advantages if any, of a situation in which every person, or even most people, are expert problem-solvers in mathematics?......

Department of Mathematics and Science Education
University of British Columbia
Vancouver, B.C., Canada V6T 1Z5

The following appeared as an editorial in *Investigations in Mathematics Education* (Volume 16. Number 2; Spring 1983) and is reprinted with permission

THOMAS CARPENTER

At the turn of the century, Hilbert posed 23 problems whose solution would lead to fundamental advance in mathematics. It has been proposed that this exercise serve as model for identifying the critical problems in mathematics education. Richard Shumway asked the athors of chapters in *Research in Mathematics Education* [Shumway, 1980] to follow Hilbert's example and identify a small number of significant problems based on the research reviewed. Recently, David Wheeler invited a number of colleagues in mathematics education to go through the same exercise so that he might arrive at a synthesis of critical problems that would give direction to research in mathematics education much as Hilbert's 23 problems did for mathematics

A case can be made that the failure of educational research to provide definitive answers to serious educational problems results from the fact that the problems have not been clearly articulated. Platt [1964] has argued that the areas of science in which the most dramatic successes have occurred are those in which the practitioners have invested a substantial effort in identifying and analyzing the cricical problems. It is not clear, however, that educational problems can be subjected to the same level of analysis or be as clearly solved as problems in mathematics, microbiology, or high energy physics. Cronbach [1975] argues that conclusions in social service are generally not absolute. He proposes that "we cannot store up generalizations and constructs for ultimate assembly into a network" (p 123). In other words, even if fundamental problems in mathematics education could be identified, it is not apparent that they could be clearly solved.

In the last 10 to 15 years, a number of research areas promised to provide answers to fundamental questions in mathematics education, but what specific changes in mathematics instruction have been based on the extensive body of Piagetian research or research on discovery learning or aptitude treatment interactions?

I believe that research is unlikely to provide definitive answers to broad fundamental educational questions. I think that the most progress will be made if we are more modest in our goals, our research is more clearly focused, and our conclusions are more carefully qualified. I am suggesting that research be directed at developing what Shulman [1974] calls middle range theories. These theories fall between the task-specific working hypotheses that are generated to explain individual behaviors and the comprehensive theories that attempt to encompass all of instruction in mathematics

For the most part, I believe that attempts to draw all encompassing conclusions from educational research at best have not been terribly productive and at worst have been misleading. For example, I think that the claims for academic learning time and direct instruction must be highly qualified if one acknowledges that the goals of instruction include understanding and problem solving. Broader conclusions based on research in this area could potentially lead to many inappropriate decisions about effective teaching. On the other hand although I am generally sympathetic to the finding that teaching for understanding facilitates retention and transfer, I think that this conclusion is so broad that it has had relatively little impact on instruction in mathematics.

The kind of direction that I am suggesting is illustrated by a discussion at a recent conference on concept learning. In one of the working groups, the thesis was put forth that a certain sequence of positive and negative examples was most effective in teaching mathematics concepts. Alan Schoenfeld proceeded to identify a number of concepts that everyone present agreed would be most effectively taught using only positive examples. In fact, for every sequency of positive and/or negative examples the group could come up with, he was able to find a concept for which that sequence would be most effective. The point he was making is that conclusions about concept learning in general are not appropriate. The most effective way to teach a particular concept depends on the concept.

Research is beginning to provide a picture of how specific mathematics concepts are acquired and is beginning to provide an understanding of the instructional process in particular contexts [Romberg & Carpenter, in press]. But much of this research is currently descriptive and it is not clear that it can readily be captured in 23 critical problems. This does not mean that careful analysis is not necessary. A great deal of sloppy thinking is excused on the grounds that it is necessary to be flexible in clinical or ethnographic research. The clearest insights have come when the research was guided by some theory, and it was possible to put structure on the results. Thus, I do believe that it is necessary to identify the critical problems within a specific domain, but I am not sure that these critical problems will encompass all of mathematics education.

I would like to end this editorial with a disclaimer. There was nothing in either Shumway's or Wheeler's requests for critical problems to preclude the kinds of limits on problems that I have proposed. The straw person that I have attempted to knock down is my own creation, not theirs. Furthermore, I do not intend to disparage the search for larger questions. I do not believe that it is either naive or a waste of time. Like the knights of the round table searching for the holy grail, the search itself can prove instructive. The larger questions are worth asking; I'm just not sure we are going to find definitive answers that will significantly influence instruction. The questions themselves, however, may provide direction to the more clearly focused research, but we must be sure they do not limit it.

References

Cronbach, L. J. Beyond two disciplines of scientific psychology. *American Psychologist*, 1975, *30*, 116-127

Platt, J R. Strong inference. *Science*, 1974, *146*, 347-353

Romberg, T.A. & Carpenter T.P. The teaching of mathematics. In M C. Wittrock (Ed.), *The third handbook of research on teaching*. Chicago: Rand McNally, in press

Shulman, L. J. The psychology of school subjects: A premature obituary? *Journal of Research in Science Teaching*. 1974, *4*, 319-339

Shumway, R. (Ed.), *Research in mathematics education*. Reston. VA: National Council of Teachers of Mathematics. 1980

Department of Curriculum and Instruction
University of Wisconsin-Madison
Madison, WI 53706, U.S.A.

W.M. BROOKES

When I got your letter about research problems, I felt it was an invitation to be artificial. Clearly, I said, what I have chosen to become personally engaged in is the most important thing. On the other hand, someone might say: "Ah! But what would you prefer to do?" My difficulty is that I don't think I believe in problems in the abstract, and certainly object to comparing "mathematics education" to mathematics in the Hilbert sense.

For me the word "education" implies a political stance. The origin of the idea of education as political is evident in Plato's writing. He has been referred to as the inventor of political philosophy and, given the anxiety to promote the welfare of the world, it seems that the notion of "educated man" has been confused with that of "developed man". The right to education is recognised, by those who talk of such things, as a universal human right; but the degree of development in a country can be measured, by those who want to, by the period devoted to compulsory education of children. The longer the better! What is this thing that, being compulsory, is simultaneously a sought-after right?

So for me the phrase "mathematics education", when we invented it in the early sixties, was a convenient slogan. It slipped neatly between the operational words "teaching" and "learning". Sufficiently neutral to avoid bigotry; sufficiently sober to deserve attention; but always at risk when people enquired too closely. "But where are the theorems?" said one Professor of Mathematics, no doubt ironically, but with some truth.

I do not believe that we should fall victim to the transcendality of the words. It is bad enough for the word "mathematics" to be accepted as describing something which easily crosses national and linguistic boundries. This is increasingly dubious—or should I say, the manifestations of what is called "mathematics" which appear easily to transcend context may not, in our best interests, be what we should always wish to recognise as mathematics? There is something metaphysical here which is hiding and is kept in hiding by a conspiracy that urges silence through the slogan "Metaphysics implies woolly thinking"

All this amounts to a personal refusal to believe in a generalised object called "mathematics education". There is only a need to have this noun phrase *because* difficulties have arisen with something that had hitherto been taken for granted. If there had been no difficulties, we would never have needed the phrase. So I do not believe that I can formulate a *"specific problem"* (sic) independent of my context.

At 12 I hated mathematics and was terrified. I used to excuse myself from lessons and sit in the lavatory for lengthy periods. For 2½ years I was miserable with maths. I enjoyed school and everything else I did. My father changed his job and I changed school. After two years, when I took the School Certificate, I *knew* I would get 100% in the Algebra paper. I *knew* that if I had stayed at the other school, I would not have coped with mathematics.

This experience made me (a) believe that the concept of an innate ability for mathematics is not particularly useful; and (b) believe that it is possible for people to be frightened of mathematics who should not be.

I do not wish people to waste time attempting to convince me I was wrong, for although they may enjoy the activity, it won't impress me. This *does* mean that I am biassed. It means that I believe a lot more people could cope with mathematics than do so, and that a lot more needn't be frightened.

Once I move away from what seems to be commonly accepted—that is, that to be able to do mathematics is to be unusual and that there will always be some people who are afraid of mathematics—I have difficulties. I cannot accept what is normally done. When I meet teachers who accept (consciously or not) that these two propositions hold, I do not see how they can share my difficulties.

Even when I have attempted to talk and work with those who appear similarly committed, I have found rejection more often than acceptance. I have been forced to believe that we are much more idiosyncratically driven the further we move away from the normatively tolerant stable state of accepting.

Those people who know my recent work know that a lot of mileage has been achieved by recognising that the question "What is a problem?" can be replaced by "When is a problem?" (ATM Supplement No. 19). The nature of the question we ask will govern the consequences. To reflect on the form of a question rather than believe that a question is no more than a tool to get at what we wish to know seems vital. Sometimes it is necessary *not* to ask a question. Certainly curiosity for its own sake, so often casually invoked as a scientist's raison d'etre ("I suppose I am a scientist because I am curious..."), is inappropriate to practitioners concerned with their practice. A practitioner needs to question as a consequence of some disturbance in his experiencing of his practice. Given our current linguistic states (believing in words rather than in actions), the question he articulates is often the first manifestation of the disturbance. In such circumstances it is hardly the best basis for a deeper investigation. There is a need to transform the question through the sequence of disturbances which has given rise to it so that there is a chance of acting relevantly.

Experience with the question "How do you prove that minus minus is a plus?" is a case in point. A well-known mathematician/mathematical educator on hearing this referred to by an experienced inspector said to me, "I can send him a proof on a postage stamp." On hearing this I said, "But how do you know that that would be anything to do with what he is really asking?" If we reflect on our experiences with the collection of sets of words forming questions about "minus minus is a plus", we discern a map of spaces with differing discourses. The question "How do you teach minus minus is a plus?" is undecideable without a context, without a teacher and a taught.

I do not approve the sophistry which observes that a question is decideable in principle because if a context were given then it would be decideable. What appears to me as an increasingly observable failure to provide for disciplined interpretation can lead only too easily to the ignoring of the necessary caveat: "in principle". There is a difficulty because we all have had many experiences when the context of such a question has been obvious and so the question appears to be decideable. But when the context is not obvious, or the obvious assumption turns out to be wrong, then the time-place-people dependence operates and transcendental questions, which have appeared useful, have to be brought down to a "when-where-who" context.

So I suppose I say that *any* efforts to deal with consequences of the slogan "mathematics education" are grist to the enquiry mill—except those demanding the global metaphysic implied by accepting a transcendental state for an object called "mathematics education".

On the other hand I can detect behind the questions posed in your letter an earnestness which wants to identify significance in possible enquiry. I cannot help feeling that this represents an uneasiness in the recognition that practitioners and researchers are discrete sets of people and the question is essentially "researcher"-grounded. "Practitioner"-grounded questions have a totally different quality. Confusion develops because both sets find it difficult to avoid the transcendent quality of analytic Western languages and hence tend to pose differently-grounded questions with the same words. The disturbances felt by a practitioner in his practice are not the same as those felt by researchers, despite the protestations of the latter.

"Researchers" are tempted by the transcendancy of the words which describe the object of their research into an implicit acceptance of that global metaphysic which appears to give meaning to their work. "Our field" is referred to by a number of your correspondents as well as by you yourself.

When I say that questions have to be brought down to a "when-where-who" context, I really mean that *unless* they are then any productive consequences can only be a spin-off. I believe that given our present linguistic state with respect to the way we organise ourselves, productive consequences *only* occur as spin-off. It seems so easy to institutionalise abstract objects like "research", "problem" and "mathematics education" and be in danger of misdirection and being wasteful in the way resources are deployed.

So I devote my energies to the promotion of a "time-place-people" policy in the development of practitioner-based enquiry. And it turns out that in the positive achievements of this that general theory about learning and teaching mathematics has no particular place, and in any case often turns out to be surprisingly empty. Practitioner's theory is, however, vital, though idiosyncratic, and as a consequence I believe I find some help in Feyerabend's Dadaism.

The theoretically general questions appear only at the meta-level and are to do with how language, people and systems interact. A theory of mathematics learning which has local virtue—that is, it is highly appropriate to a particular context—is fine when kept within bounds. By extending it to a generality beyond its range of application there is a risk of compromising its virtue. This seems to happen from a kind of endemic urge to be as generalised as possible. In this way Piaget's work was pushed into other cultures on the assumption that it could cross cultural boundaries.

I will find it useful to observe which of your correspondent's suggestions correspond either to local issues dependent on context, or to genuinely general issues at a meta-level which are useful over a range of contexts.

I believe that the issues broached by your questions are of a multi-dimensional kind and have a complexity for which topological dynamics may offer appropriate metaphors. Our older classical mathematical metaphors are worn out beyond repair.

School of Education
University of Southampton
Southampton, S09 5NW, U.K.