SPEAKING MATHEMATICALLY: ATTUNEMENT

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> Systems in social interaction are always mutual *interpreters* of each other's actions. Precisely because they can neither have access to each other's experience nor determine the content of each other's experience, they have to express themselves and make sense of each other's expressions *consensually*, yet each within their own cognitive or experiential domain. Indeed, expression in social interaction is *con*-sensual, but without experience, strictly speaking, being shared. The necessity to maintain cognitive integrity requires mutual attunement rather than mutual instruction or mutually imposed regulation. Hence, social interaction involves "agreement in action"—to borrow a Wittgensteinian notion—but not propositional agreement or equal mental content. (Baerveldt & Verheggen, 2012, pp. 188–189)

I am interested in how mathematics teachers develop a culture in their classrooms where social interaction, speaking mathematically, develops through mutual attunement, agreement in action, doing mathematics. In Maturana's (1988) words, I am concerned with children's move into the domain of explanations within a mathematics classroom. In other words, the children are speaking mathematically through becoming attuned to hear through responses to each other's actions. What one child says and does triggers others to say and do things related to their awarenesses and, over time, there are patterns in behaviour that an observer could describe as acting mathematically. The role of the teacher in a mathematics classroom in creating the classroom culture could be seen as being an active observer of the children's behaviours.

After a brief introduction to my own developing worldview, I explore mutual attunement in speaking mathematically through a focus on acting consensually and metacommenting.

Becoming an enactivist

Alf Coles and I met around 1995 when I started to visit his school once a week. We have continued to work together ever since. This early research was written up as papers for local and international mathematics education conferences. At that time, we described ourselves (Brown & Coles, 1996, p. 151) as working within what Bruner (1990) called a "culturally sensitive psychology":

> [which] is and must be based not only upon what people actually *do* but what they *say* they do and what they *say* caused them to do what they did. It is also concerned with what people *say* others did and why. [...] How curious that there are so few studies that [ask]: how does what one *does* reveal what one thinks or feels or believes? (pp. 16–17)

This position became even more focused on action when, at the suggestion of David Reid, current editor of *For the Learning of Mathematics,* I started reading Maturana and Varela's work, especially Varela, Thompson and Rosch (1995). Varela introduces the enactivist idea that, centrally, all doing is knowing and all knowing is doing. By 1999, I had successfully obtained a national research council grant for an enactivist research project and was writing enactivist contributions to PME research reports, research forums and working groups. I have described myself as an enactivist ever since.

The opening quotation is from two authors who are thinking deeply about what they call enactivist cultural psychology. The idea of attunement stood out for me as having something to do with the developing culture in a classroom. In what follows, I first explore an example from mathematics teaching and learning related to acting consensually. This will be followed by an exploration of an observed behaviour, called metacommenting, of a teacher of mathematics supporting the developing culture of children speaking mathematically in their classroom. Acting consensually and metacommenting are related to the emergence of attunement in a mathematics classroom.

Acting consensually: see what they say

Look at Figure 1. What do you see?

> On your own, in silence, pay attention to what you see. After a while, share what you see, the task being to see what others see. Pay attention to when you are aware that you are confused and ask for more detail. Don't point, work on descriptions. As time goes on, pay attention to questions that are arising and see what progress you can make on them by structuring your vision. As you look, you will see more. (Adapted from ATM, 2016, pp. 7–8)

The image in Figure 1 is a 2D representation of a hypercube: a 4D cube. It fits into a regular octagon. I used this image at the beginning of a Masterclass with a hall full of 12-year-old children from different schools around Bristol, UK.

The invitation is for one child to say what they see and for the others to see what they say. So that a language can develop for this morning in this room, it is important to

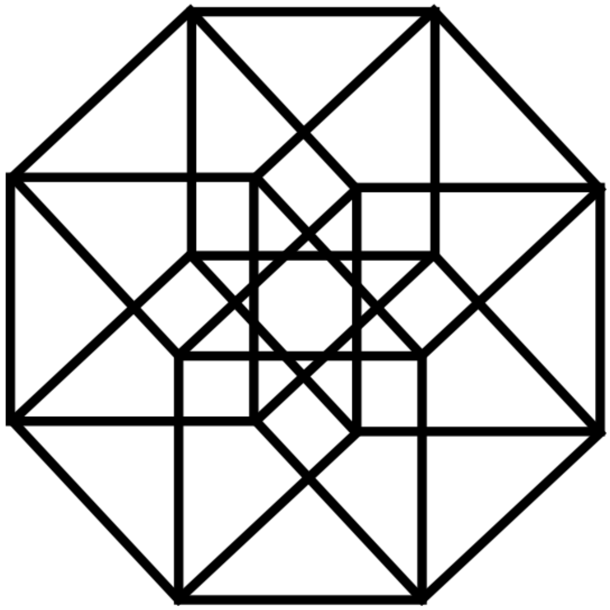


Figure 1. What do you see?

spend time, without pointing, to let descriptions be heard and revised. On this morning, the first child to speak says that they see a hexagonal prism. No-one has ever offered this on the previous occasions when I have used this image and, as if for the first time, I see a hexagonal prism in the diagram. It turns out to be a different one from the hexagonal prism that the student sees, leading us to work as a group on how many hexagonal prisms are in the diagram. What is the relationship between a hexagonal prism and a hypercube? Try seeing more in the diagram rather than using algebra or previous knowledge. What other questions arise?

So many of our concepts are formed and are still under formation because they come up against disruptions. In this consensual domain, am I really seeing what they say? The responses to sharing what each other sees, given what each says, leads to the awareness of different children seeing different hexagonal prisms. How many different ways are there of seeing the hexagonal prism that you said?

One culture of a mathematics classroom that is familiar to many is of the teacher as instructor, doing an example or explaining, with the children copying or listening followed by practice. What is effective mathematical behaviour in this kind of classroom culture? As a 17-year-old, I was a successful mathematician until I came up against using the quadratic formula and got all the examples wrong. I had to relook at doing operations with negative numbers. There were holes in my previously effective use of negative numbers thrown up by this more complex use. Behaviour is effective when it happens smoothly. I do not need to pay attention to it. The examples used in school before I met the quadratic formula were not so complex; whatever I had done to operate on them created no disturbance. Effective behaviour within one environment does not have be so within a changed environment. So, when effective behaviour is compromised there is the opportunity to learn.

How could a teacher create a culture in their mathematics classroom where children and teacher are speaking mathematically, making "sense of each other's expressions *consensually*" leading to "agreement in action" as the authors of the quotation above describe? The activity of "see what they say" could be seen as mutual attunement in such a culture, with what the students and teacher see, hear and do being shared with each other, triggering questions and ideas for further exploration. In the next section, how one teacher develops what he calls an "exploration space" in his classroom through "metacommenting" gives an example of what is possible.

Metacommenting

The focus of this section will be two excerpts taken from co-teaching/co-researching in Alf Coles's mathematics classroom (Brown & Coles, 2008), looking in a little more detail at strategies to support speaking mathematically through mutual attunement. In the department in the school in which Alf worked, the teachers began each year by sharing "actions related to becoming a mathematician" in their first department meeting. The teachers became attuned to seeing these actions in their teaching, hearing what the children say, and reporting back a description of their actions to them. The process of sharing such observations we called metacommenting. These metacomments were not judgements about the mathematics but descriptions of mathematical actions, such as "getting organised", "collecting examples", "spotting patterns", "convincing" (leading to proving), as ways of "thinking mathematically" or "being a mathematician". For instance, at the start of the school year with a new class, Alf would make many comments in each lesson about behaviours of the children both inside and outside of doing mathematics, commenting to the whole class about, say, a group getting organised, an individual looking for a counterexample, or that sharing ideas together seemed to be useful when stuck. As the year went on, what struck us was that these metacomments, at a different level to that of the mathematical discussions, were less frequently made by Alf, but that some would start to be used by the children, such as, "we're stuck so we need to have a time sharing ideas". The culture of each of Alf's classrooms over the years developed in different ways. That is, the "agreement in action" for each class had a different set of emergent effective behaviours that developed consensually from this process. Each class was noticeably Alf's, however, because of the way the children were attuned to hear such phrases, supporting their doing of the mathematics.

Excerpt 1. From teacher metacomments to attunement in a domain of explanations

The image shown in Figure 2 is taken from the book *Hearing Silence,* the story of the research focused on Alf's development as a teacher (Brown & Coles, 2008, p. 64), and shows a "collage of students' comments and written work to illustrate the mathematical interaction" during some work on algebra.

The comments in Figure 2 are not illustrating "mutual instruction" or "mutually imposed regulation", they capture

Nathan's conjecture: If T + S equals L + R then there is an unlimited amount of solutions.
Liam's conjecture: If T + S does <u>not</u> equal L + R then there are <u>NO</u> solutions.

Algebra made my conjecture turn into a theory.

I have learnt that being mathematical you have to ask questions from finding out one thing leading to another. You have to ask why all the time or how because you want to find out more. Chris found a conjecture and I asked how? and why? I noticed a difference, and then noticed the difference of difference.

N → NN
N → 2N + 2
N → N + N + 2
N → 2xN + 2
N → N xN + 2
N → 2 x (N + 1)
N → 2(N + 1)

Students: ~ How could you draw it?
Teacher A: ~ Well, it would be a sixth of a unit. Very small.
Students: ~ If you drew it really big so one square was 6 ...
Sir, what would just a straight line be?

Figure 2. Excerpt from Hearing Silence *(Brown & Coles, 2008, p. 64).*

how the children share their thoughts with each other. In Alf's classroom, the plenary discussions were recorded on what we came to call a "common board". Alf might write students' conjectures publicly, without judgement (see Nathan's and Liam's conjectures) or the students might write up their (different) expressions for a rule themselves. This process had happened since the start of the academic year and, at this point, there was already evidence of the children using words like conjecture and theory themselves, as ways of being mathematical. Alf did not need to make metacomments anymore. Asking questions seems to be an important part of the process, not by being focused on getting a right answer, but as a journey of finding out more through difference. The creative conversation shown at the bottom right corner of Figure 2 is an example of such a journey, in which asking questions, acting and being mathematicians themselves, leads children into an exploration of scale.

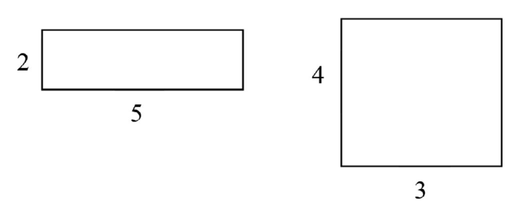
The children in Alf's classroom learnt to act mathematically through solving problems with a range of actions. One of the children would suggest an action that the group can engage in, a process through which the group can make progress, with a group dynamic that supports speaking mathematically. This was not a conversation mediated by the teacher. The children were being mathematical. They seemed to be attuned to hearing meta-messages that they interpreted consensually whilst each having their own individual interpretations.

How is each classroom culture established? The culture is established for all the students and is a "*we* culture", in which the children are attuned to hear each other and, in particular, to the sorts of comments that govern their behaviour. The children are attuned to hear metacomments linked to what they do to do mathematics. The experience of each child within this culture can be expressed as: "I become attuned to hear through knowing what to do, and we are attuned to hear when we know what to do". The particular metacomments that a particular class of Alf's students would have as their agreement in action are unique, developed over their time together.

Excerpt 2. A teaching strategy for setting up the classroom culture—same/different

Alf convened a cross-curricular group of teachers in his school. The sessions involved one of the group actively teaching using a strategy or strategies they would use in their classrooms, followed by discussion. In the following excerpt from *Hearing Silence* (Brown & Coles, 2008, p. 83), Alf is talking about "using student responses":

> The single idea that, in the cross-curricular group, we have found has the most widespread application is that of using "same/different" *i.e.,* getting students asking questions, or starting a discussion by presenting them with contrasting images or examples. In mathematics, a lesson start I have used is to draw the following images on the board and ask the class; "describe what you see?" or "what is the same and what is different about these rectangles?":



Two rectangles - what's the same? what's different?

> A discussion has always ensued about the property that these rectangles have the same perimeter but different areas. Often a question such as, "what's the biggest area we could get with this perimeter?" has come naturally

from a student and has provided the motivation for a number of lessons' work.

Strategies such as "see what they say" or "what's the same? what's different?" can provide the focus on what students say to form a culture in the classroom where students are used to hearing each other. In the early days of the year, strategies to develop depth of discussion are also central. In observing Alf, two striking repeated patterns of behaviour, different to teaching strategies seen in other mathematics classrooms, were apparent. When a student offered a comment or shared what they saw, Alf did not repeat the comment or comment on the comment but simply moved his right arm to include the class sweeping slowly to his right, inviting other contributions. On its own this could lead to a set of disconnected comments, not leading to the consensual attunement characterising Alf's classroom. The expectation is that before going on to offer a new contribution, the previous one needs to be explored first. In the early days of the year, Alf would ask someone who wanted to contribute whether their comment was related to the one just made. If the answer was negative, then another comment was invited that was either a question about or a comment on the previous one. Later in the school year, new observers to Alf's classroom would comment on the quality of the discussions, not knowing that this had been set up from the start of the year.

Coda

The title of this writing is taken, in part, from David Pimm's book *Speaking Mathematically.* The title of this endpiece, in honour of David's retirement, celebrates the love of music that David and I share. The videos created at the Open University at the heart of his book showed secondary mathematics classroom practice. The great dodecahedron poster reproduced on the cover of my copy of the book brings vividly to mind a teacher developing the strategy of a hot-seat, encouraging in-depth discussion of what one person (on the hot-seat) sees in the poster, with the rest of the group asking questions, letting go, for the moment, of what they themselves had seen. In *Four Loves,* C. S. Lewis (2012) talks about friends facing in the same direction with a shared focus. The structural coupling over a professional lifetime, with a focus on mathematics classroom practice, speaking mathematically, and, in both our cases, editing *For the Learning of Mathematics,* shows our attunement. Richard Barwell wrote, after reading a draft of this writing without this coda: "The writing triggers lots of ideas in me […] how it isn't the words, exactly, that matter, but what happens with them, and yet the words do matter, not because of anything about the words themselves, but because over time and groups of people shared attunements have emerged". So, David, here's to more singing in our respective retirements and more mutual attunements!

References

ATM (2016) *Mathematical Imagery.* Association of Teachers of Mathematics.

Baerveldt, C. & Verheggen, T. (2012) Enactivism. In Valsiner, J. (Ed.) *The Oxford Handbook of Culture and Psychology,* 165–190. Oxford University Press.

Brown, L. & Coles, A. (1996) The story of silence: teacher as researcher – researcher as teacher. In Puig, L. & Gutierrez, A. (Eds.) *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 2, 145–152. PME.

Brown, L. & Coles, A. (2008) *Hearing Silence: Learning to Teach Mathematics.* Black Apollo Press.

Bruner, J. (1990) *Acts of Meaning*. Harvard University Press.

Lewis, C.S. (2012). *The Four Loves* (reprint). HarperCollins.

Maturana, H.R. (1988) The search for objectivity or the quest for a compelling argument. *The Irish Journal of Psychology* **9**(1), 25–82.

Varela, F., Thompson, E. & Rosch, E. (1995) *The Embodied Mind.* MIT Press.

I had shown [her] a circle with some lines, red ones intersecting the circle and green ones not doing so.

What could you say about the red lines?

Well they are fighting—sort of cutting up the circle.

And the green lines?

They are protecting—yes, they are guarding.

I had then drawn a tangent in pencil and asked what colour it might be given. There was a pause. Then:

It's green escaping from red. A brief pause.

Or red escaping from green. Another pause.

Oh dear, it's a helpless man.

Who is to say without examining the idea further that perceiving geometric incidence and proximity in terms of threat and attack is not a *mathematically* useful way of viewing such situations? (p. 11)

— David Pimm, from p. 11 in the section on meaning in Chapter 1 of *Speaking Mathematically: Communication in Mathematics Classrooms* (1987) Routledge & Kegan Paul.