Snapshots of a Mathematics Teacher: some Preliminary Data from the Mathematics Teaching Project

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The Mathematics Teaching Project, a three year research investigation based at the Polytechnic of North London, aims to characterise good mathematics teachers. A detailed description of the rationale and aims of the Project are published elsewhere [Hoyles et al, 1984]. The following is based on material collected during the pilot study The subject of this article, Ms X, is an experienced teacher in a girls' comprehensive school in North London. The school is streamed after the third year, and the class taken as a focus for study was a high ability fourth year group Ms X knew the class well having taught them for several years.

Her views of the class were ascertained as were the pupils thoughts about her. Details of how she related to the class and interacted with individual pupils were also recorded. It is acknowledged that the kind of style identified here for this teacher might be influenced by the particular type of pupil in this class. However, it is hoped that the "snapshots" presented here will promote an understanding of the methods used and provide useful insights and suggestions for practising teachers.

The teacher perspective

Research on the teacher perspective is based on the belief that pupils build up a concept of themselves as learners from inferred attributions and expectations embedded in the classroom interactions with their teacher The research therefore attempts to probe the teacher's general perception of her pupils and how this perception is applied to and varies between individual pupils. The three sources of data are: an attribution survey in which for each pupil in the class the teacher is asked to hypothesise about possible causes for success or failure in a planned mathematical test; the Rheinberg test which seeks to establish whether the teacher assesses a pupil's achievement in relation to the average level of the class or to the pupil's prior performance; and a repertory grid of the personal constructs that the teacher holds of the pupil population in the class under study.

During the study of Ms X's perspective it became apparent that she knew her pupils well and cared about them deeply as individuals. She perceived the general ability of the class to be high and was hopeful and optimistic about her pupils' chances of success. With this background in mind, she tended to cite effort as the predominant influence on test performance. Pupils were assumed to work hard throughout the year but it was expected that special efforts would be made in revision prior to any test. Ms X assumed that most pupils would make maximum effort during the test itself. Lack of effort as a reason for failure was seen to arise largely from general disaffection with school or overall laziness.

Although Ms X perceived all her pupils as able, she did in fact differentiate according to ability and gave this as the second most important factor affecting test performances. The specific ways in which her very able pupils used their ability to attain success included the application of existing knowledge to unfamiliar situations, persistence in task, speed, thoughtfulness, a questioning approach and a self-sufficiency with respect to mathematics. Lack of ability was not however seen as significant as a cause for failure. Instead, Ms X referred to the nature of a particular mathematical task as an attributional reason for failure (and just occasionally for success). If those pupils, generally perceived as able, failed, Ms X would suggest that the failure was related to characteristics of the specific topic being tested, the poor quality of her teaching or the material supplied. One "very able" all rounder was particularly seen to be turned "off" or "on" by the nature of the task while others were seen to "miss the point" occasionally.

The element of chance was Ms X's third main attribution area, although she was reluctant to attribute an unexpectedly good result to luck She would however see that failure for some could be the result of "jumping to conclusions" and careless work; misjudgement of the minimum amount of work necessary for success in the test and the inability of pupils, who require slow but thorough hard work for success, to secure the necessary time and facilities.

Finally Ms X referred to anxiety, lack of confidence and panic as influencing test failure for certain pupils This attribution was, to a certain extent, linked with a content variable, i.e. specific topics were seen as being prone to cause confusion Ms X's attributions were certainly influenced by her perceptions of her pupil's personalities, e g. impulsive, anxious. She recognised the influence of emotional response of all kinds in mathematics learning; for example, she suggested the antagonism of one pupil towards herself as a reason for failure.

A concern with affective as well as cognitive influences

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on learning was also very evident in the more general perceptual framework Ms X had built up of her pupils as identified in the personal constructs elicited and clustered in a repertory grid.

Table 1 below lists the constructs elicited together with brief examples of the meaning of the constructs. Table 2 illustrates how these constructs link together using simple cluster analysis. The cluster analysis displayed in Table 2 was shown to Ms X and several underlying or superordinate clusters emerged in discussion.

Cluster 1 and 6 (68.75%)
The superordinate construct which appears to give a meaningful interpretation to the clustering of these two constructs could be designated "attitude to mathematical work". There thus seem to be two dimensions of attitude distinguished here — effort/persistence/determination, and interest/enjoyment/liking.

Cluster 1, 6 together with 7 (56.25%)
This cluster could be interpreted as "attitude to school teacher/class", i.e. a superordinate construct which includes the attitude to the subject but also involves a more general aspect of pupil response to the teacher and the conditions and constraints of the classroom.

Cluster 3 and 2 (81.25%)
This cluster appears to be related to the teacher-pupil interactions in the mathematics classroom. It distinguishes pupils who are able and willing to engage the teacher in dialogue, who have some autonomy in their work and who will initiate interaction with the teacher in contrast to pupils who will only "receive" from the teacher and want always to be led rather than take any lead.

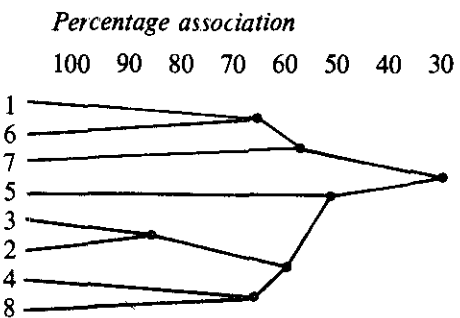
Cluster 4 and 8 (62.5%)
The underlying construct of these two constructs appears to be the teacher's assessment of the pupil's self-image or self-concept of mathematical ability. Construct 4 is more

Table 1
MS X'S PERSONAL CONSTRUCTS OF HER PUPILS

Order in which constructs elicited	*Bi-polar constructs*	*Comments*
1	Consistent effort ——— Avoids effort	Concerned with reaction to "challenge" in mathematics, i.e. determined effort or giving up.
6	Interested ——— Lacks interest	Concerned with interest in mathematics and mathematical activity.
7	Co-operative ——— Unco-operative	Concerned with the extent of co-operation with the teacher as a "manager" of the class.
5	Quite ——— Exuberant	Concerned with a perceived personality trait of the children.
3	Accepting ——— Questioning	Concerned with how the pupils "received" the mathematics introduced in the class.
2	Teacher dependent ——— Flair	Concerned with autonomy and risk-taking in mathematics.
4	Anxious ——— Confident	Concerned with pupil self-image of mathematical ability.
8	Neat ———Untidy	Concerned with the presentation of mathematics in written form.

Table 2
CONSTRUCT CLUSTERS

Consistent effort	Avoids effort	1
Interested	Lacks interest	6
Co-operative	Unco-operative	7
Quiet	Exuberant	5
Accepting	Questioning	3
Teacher dependent	Flair	2
Anxious	Confident	4
Neat	Untidy	8



concerned with how this self-concept is manifested in the classroom and construct 8 with its manifestation in written work.

Cluster 3, 2 with 4, 6 (56.25%)
The underlying construct for this cluster appears to be "perceived mathematical ability", with a tendency for mathematically gifted pupils to be seen as questioning, confident, and with flair and perhaps being untidy. Less gifted pupils are more likely to be seen to have the characteristics of lacking initiative, accepting the teacher's exposition, wishing to follow instructions, tending to be anxious and with an inclination for an undue focus on neatness and presentation.

Cluster 3, 2, 4, 8 and 5 (50%)
The association is not strong here but could suggest some linkage of mathematical giftedness with exuberance. It is also of interest that this construct is not related to construct 7, which implies that it does not concern co-operation with the teacher in the general classroom situation.

Ms X's overall construct system separates into two general clusters that represent important distinctions made in her view of pupils. One cluster is based on perceived attitude and one on "ability", particularly as this is manifested in interactions with her in the classroom. Both distinctions are very related to mathematics and the mathematics classroom as opposed to discriminations influenced by more general pupil characteristics. Thus for Ms X, the pupil's "relationship" with the subject, mathematics, and their relationship with her as a mathematics teacher are of great significance.

In summary, all the data collected in the teacher perspective points to Ms X's concern for commitment to good work in mathematics but also to her sensitivity to affective issues in the classroom and the potential influence of anxiety and panic. Ms X places great importance on her own role in the learning process and is willing (rather unusually) to assume responsibility for failure rather than attribute this to the shortcomings of her pupils.

Table 3
DESIRABLE CHARACTERISTICS IN A MATHEMATICS TEACHER

A good mathematics teacher:

(1) is kind, sympathetic and easy to talk to
(2) has the ability to explain clearly and in more than one way
(3) is completely fair and has no favourites
(4) is an enthusiastic and able mathematician
(5) has a good sense of humour
(6) treats pupils as responsible people
(7) uses methods which make mathematics more "alive" and interesting
(8) has patience with those who need more help
(9) understands pupils' problems with their work
(10) arranges work to suit the pace of the pupils.

Table 4
THE PAIRED-COMPARISON TEST

Below are listed a number of characteristics desirable in mathematics teachers; you will notice that they are arranged in pairs (and that each characteristic appears several times in the test). Your task is to consider each pair separately, and to choose which of the two items you consider *more* important in deciding how good a mathematics teacher is. Put a tick beside the item that you have chosen, then move on to the next pair.

A good mathematics teacher:

1 (a) has a good sense of humour ________
(b) is kind, sympathetic and easy to talk to ________

2 (a) treats pupils as responsible people ________
(b) is an enthusiastic and able mathematician ________

(continued with all possible pairs from the list of characteristics in Table 3).

Table 5
RANKING OF DESIRABLE CHARACTERISTICS IN A MATHEMATICS TEACHER

2. has the ability to explain clearly and in more than one way
8 has patience with those who need more help
9 understands pupils' problems with their work
*
10. arranges work to suit the pace of the pupils
7 uses methods which make mathematics more "alive" and interesting
1 is kind, sympathetic and easy to talk to
*
6 treats pupils as responsible people
5 has a good sense of humour
4 is an enthusiastic and able mathematician
*
3. is completely fair and has no favourites

* The gap between the clusters of factors signifies quite a substantial "jump" in the rating: the cluster of three factors 2, 8, 9, for example, was seen as being of much greater importance than cluster 10, 7, 1.

Table 6
FREQUENCY TABLE ($N = 28$) SUMMARISING PUPILS' RATINGS OF MS X

Factor	Very Good	Good	Average	Bad	Very Bad
(1)	20	7	1	0	0
(2)	24	4	0	0	0
(3)	19	3	6	0	0
(4)	23	5	0	0	0
(5)	14	12	2	0	0
(6)	18	7	3	0	0
(7)	5	18	3	2	0
(8)	23	5	0	0	0
(9)	20	5	3	0	0
(10)	7	14	7	0	0

The pupil perspective

The study of the pupil perspective can be divided into two sections, the first part concerned with the elicitation of the characteristics considered to be of general importance in mathematics teachers and the second with pupil perceptions of those characteristics in their own mathematics teacher. Thus before ascertaining the pupils' views of Ms X, their ideal preferences were studied. Below is Ann's response to the direction:

> *Imagine that you have "ordered" an "ideal" mathematics teacher who has now arrived to teach your class. I would like you to describe this teacher to me.*

Ann The teacher would be a competent mathematician, able to explain and project things clearly, interestingly and with a sense of humour. The teacher would spend the right amount of time with everyone owing to the person's need. The teacher would be willing to help with specific problems outside the lesson — approachable! The teacher would be understanding to people's difficulties with maths. The teacher would keep the subject "alive", e.g. not choosing too many monotonous questions, using different material sometimes — from another book, etc. However at the same time keeping within the syllabus and the class's ability. A teacher like this should have no trouble keeping discipline within a class!

The pupils in this and in a second class participating in the study (a lower C.S.E. class with a different teacher) all provided such descriptions, and these were analysed to establish factors which seemed to be of general importance. It should be emphasised that the chosen characteristics are not those which emerged as most "popular" in the main study, where a much larger and more varied sample was surveyed. The experience of the pilot study brought to light the need for more precision in the definition of variables. For example in characteristic (4) below, two factors "enthusiastic" and "able" were put together. It emerged however that these were not necessarily regarded by pupils as being closely related or indeed of equal importance.

Bearing this weakness in mind, it is nevertheless interesting to see what this class required of their perfect mathematics teacher. The pupils were then asked to complete a paired-comparison test, the beginning of which is given in Table 4 for clarification; the results were analysed using Kendall's methods. [1975] In this case, Kendall's rigorous technique allowed the researcher to draw two conclusions:

(a) that the individual pupils had completed their tests with a high level of consistency ($\delta \geq 0.75$ for all but 4 pupils)

(b) that there is a highly significant level of agreement between the members of the class (coefficient of agreement, $u \sim 8.30$; significance $\sim 10\ 07$).

Results from the paired comparison test given in Table 5 showed clear priorities in the class for different characteristics in their teacher.

Having discovered the priorities of the pupils, their views about Ms X were first investigated by asking them to make a judgment of her with respect to each of the factors under consideration. The pupils were asked to consider each factor in turn and rank Ms X on that factor using a five-point scale. The confidentiality of their judgments were stressed, as was the importance of considering each pair separately

The results summarized in Table 6 show clearly the high regard in which Ms X was held by her pupils.

The final phase of the research in the pupil perspective was concerned with investigating the range of specifc events which lead to a particular positive attribution being made. A method of network analysis [Bliss and Ogborn, 1983] is used in the interpretation of the pupil interviews in the main study, but for the purposes of this paper a few extracts from the interviews are given as "snapshots" of how Ms X was viewed. Each pupil was asked to describe a particular time when Ms X exhibited one of the characteristics on which they had judged her highly. It is evident from the extracts that this initial request only provided a framework for the interview and pupils tended to talk spontaneously of a range of positive instances.

EXTRACT 1

Jane: She will try to let you come to a discovery yourself. She won't just tell you the answer, which is very good. With me, I went to her for help with probabilities, er, some kind of chart where you had to work out the pattern and fill in the next two lines, and she said, "Just look at it," and she said, "Now, look at those numbers," and she. .... First of all she told me to look at the whole chart, then she narrowed it down because I wasn't getting anywhere, then I was able to see the pattern, when she'd narrowed it down, which was very good; and it doesn't leave you feeling that you're a real thicko because you made that discovery yourself it, it makes you feel good, so she's very sensitive in that way and she doesn't crush anybody

EXTRACT 2

Susan: She spent about half and hour, explaining to me and my friend just about one little thing about πr^2 and why it worked, and we didn't even need it, but we wanted to know, to experiment.

Interviewer: Can you tell me exactly what it was she was trying to explain?

Susan: About the area of the cone, why it was πrl, and we drew little things, and we cut things out just to explain this point and why it worked.

Interviewer: So, what was your problem? Can you tell me exactly what it was that you didn't understand?

Susan: We didn't know why it was πrl that this whole thing was, that made you get the, um, area of the curved surface.

Interviewer: So what did she do?

Susan: She cut out a circle, and she cut out a section of it, and she explained that the circumference... that there was a certain way of getting it, and then when she took a bit away and put that together there was a completely different circumference, and we ratioed the two and things like that (laughs) and eventually got the answer

Interviewer: How did you think that what she did was particularly helpful?
Susan: Because, I suppose, she actually did the actions, and actually went through it with the paper and things rather than just explaining it, and she was very patient about it.
Interviewer: And what were you doing while she was explaining?
Susan: Um, I see. Well we didn't actually take part, but we, like there were, people sort of joined in part way through as we were sort of having this big debate really on it. ....and as they had questions she would answer them. That's when I thought she was patient 'cause she sort of answered them very methodically, right through it all....
Interviewer: And how did you feel at the end of it. How did it make you feel.
Susan: Very satisfied, actually. It was really good, 'cause we did feel a sense of satisfaction.... also you can remember the formula a lot better 'cause you know how it got there.

There is not enough space here to give a comprehensive set of examples, covering all the characteristics but the extracts given above and at the end of the article serve to illustrate the wealth of data and insight available.

The mathematics classroom
As well as undertaking detailed investigation of the teacher and pupil framework as described above, Ms X was regularly observed with the class under study and the interactions and classroom strategies recorded and analysed. The research for this perspective is in the ethnomethodological tradition. A preconceived schedule of observation was not used and the focus of study emerged after many hours of observation. It was felt that an audio transcript of teacher and pupil interaction did not convey adequate information of the complexities of the classroom situation. "Reconstructions" have therefore been made of a selection of lessons in which key portions of the transcript are combined with the observer's notes on actions, non-verbal gestures, etc.

In the main study, seven categories have emerged as a means of analysing the data and comparing the teachers' classroom practice. Here, three of these categories will be described with reference to parts of a "reconstructed" lesson.

The class were using an O-level text book, and in this lesson the teacher was preparing them for an exercise which needed the concept that the scale factor for the area of similar figures was the square of the linear scale factor. Before the lesson the teacher had said to the observer that she would teach the scale factor for volume as well, but since that was not necessary for the exercise and she found herself running out of time, she did, in practice, limit the work to area. The influence of "the bell" is quite apparent in the complete reconstruction and the style of the lesson noticeably changes at it approaches the point at which a suitable conclusion must be reached and homework set.

Two rather distinct "modes" of teaching can be identified — an "instructional" mode and an "interactive" mode. The former is identified when the teacher is giving the class information (forming part of the teacher's original "plan" of the lesson) whilst the latter occurs when the teacher is genuinely endeavouring to respond to a pupil initiative.

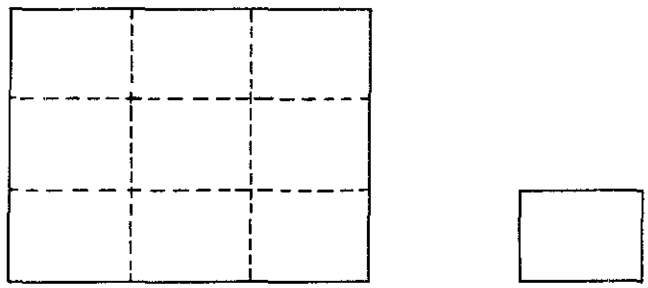
An example from the reconstruction shows the teacher using the instructional mode.

T So . . What would happen if you hadn't reduced the length by a half but you'd reduced the length by a factor of 3? (...) What would happen to the area then? Mary?

It later emerged that the teacher sometimes asks Mary questions as a sort of yardstick. Mary often has difficulties in understanding and can alert the teacher to any more general conceptual problems in the class.

Mary (It would be six times)
T All right, let's see it. .
P's (No, No)
T All right. .
P's ( )
T Right. Lots of offers. So let's see ...

At this point the teacher drew on the board and explained:



T Here's the one. Its only, its length's only a third as long as that one, its width only a third as long as that one, so . if you can fill in these. . Right. How many of the smaller ones can you fit in? 9? Right.

It could be argued that the teacher was responding to Mary's mistake, but from the style of the explanation — no probing as to why Mary said six — it seems that the explanation is aimed at the class, not specifically at Mary, and fits in with the teacher's intended explanation. This contrasts with the following example which has been categorised as being in the "interactive mode".

Jane (interrupting) Ms X. Is it that you square it every time?

The teacher seemed to be a little disconcerted by this interjection as it came *before* her explanation.

T Yes. Yes, it is. Why?
J (You think of it... Cos you can....)
T It's difficult to, it's not easy to explain....

The teacher then used her diagrams to explain.

T Here. This small one fits in.
P (.....square root....)
T Four times. Here, this one fits in nine times. Why?

The teacher then asked Ann, who is considered to be able and who had her hand up, to answer

A Because when you do an area, a side times a side. If you reduce each side to a third of itself....
T Right
A It's a third times a third reduced.
T Good. Right. How? Did you follow that?

This time the teacher was specifically responding to Jane rather than following a predetermined way of explaining the point, and for this reason this exchange has been categorised as an example of the interactive mode. In this instance the teacher is using Jane's interjections to stimulate the others in the class into explaining what is happening.

The relative proportion of the two modes (this teacher had a ratio of 65:35 instructional mode to interactive mode) and the ways in which they are deployed vary considerably both between teachers and for the same teacher taking different classes. This is seen therefore as a fruitful approach to the difficult problem of defining and categorising individual teaching styles; it is also one way in which teachers may be encouraged to reflect constructively on the classroom practice.

Another aspect of classroom practice that is studied are the assumptions made by the teacher about the pupils' thought processes during the lesson. This is not always easy to assess, even with the help of the teacher later reconstructing and talking about what she had been thinking in the classroom, but it is, with a reasonable degree of confidence, possible to infer some assumptions. For instance in the example quoted above, the observer noted at the time (and this was later confirmed in discussion) that the teacher seemed surprised by Jane's response, which showed a more rapid understanding of the problem than she had expected. Her assumption was therefore that most of the class had not yet grasped the concept and she acted accordingly.

Some of the most interesting assumptions are about the pupils' problems and errors, e.g.

Eve So can't you just...? Those numbers always do that anyway.

The teacher looked puzzled and appeared not to understand. There was laughter from the teacher and some pupils.

T Those numbers always do that anyway?
P (.. ....)
T Well, it's...
Eve But aren't they only in a square because...?

The teacher then made an assumption, based on apparently very little evidence, about where the problem lay, i.e. that the pupil had not grasped that for similar figures the dimensions have to be increased by the *same* scale factor. She therefore launched into the instructional mode.

T Yes, but..... But what's, but it's because I've said that each dimension has got to be increased by the same amount. I'm going from one shape to a similar shape. It would be perfectly possible to say, Right, well, I'm going to start with the label and I want to make it three times bigger that way and twice as big that way; that would be perfectly possible... But I'm not actually asking for that. I'm saying, if I want to increase all the lengths by the same number, what happens to the area?

The nature and validity of the assumptions made by the teacher emerges in the analysis and how responsive she is to the individual pupils will prove to be a touchstone of good classroom practice.

Another area in which enlightening comparisons can be made between different teachers and/or different teaching situations is concerned with the teacher's (and pupils') use of language and in particular in the use of technical mathematical terms. In the lesson examined here, for example, it was observed that considerable use was made by the teacher of terms such as "factor," "similar," and "ratio," introduced without any explanation. In addition there were several instances when terms, after initially being used correctly, were then used in a somewhat idiosyncratic way.

T Supposing you started with that shape and you increased its length by two, what would have happened to the area?

Clearly in this case the class understood what was meant (i.e. multiplication by 2, not addition). It seems that many teachers use individual "shorthand" expressions of a broadly similar kind and the implications of this in terms of pupil understanding needs investigation, as well as why a teacher operates in this manner.

Conclusion

The three perspectives build up a picture of Ms X as a competent and sensitive teacher who likes mathematics and is confident that her pupils will achieve well. She enjoys teaching and her pupils respond to her enthusiasm. She tends to be quite directive in her teaching as exemplified in her classroom practice. This is consistent with her own perspective of her teaching where she largely takes on the responsibility for the progress of her pupils. It is also consistent with the tendency she herself acknowledges, of focusing on her pupil's strengths and possibly therefore sometimes making unjustified assumptions as to their understanding and glossing over their difficulties. Although judged very highly by her pupils, the finding that some pupils feel that she could arrange the pace more to suit individuals could be interpreted as a perceived need for more interactive exchanges.

There are interesting threads in the findings, linking reactions to an individual pupil in the classroom with how Ms X sees her and how that particular pupil judges Ms X. A case in point is Jane. Jane is judged by Ms X as having flair and individuality, and as being bright but slightly erratic — in fact, as having many of the attributes of what is perceived as mathematical giftedness. In the classroom there is evidence that the teacher *does* respond to interventions from Jane (in contrast with other pupils). She does not, however, treat these interactions as a yardstick for the class and will often, after an interactive episode with Jane,