I have had an interest in children’s mathematical learning in collaborative group work since my career as an elementary school teacher and later as a researcher into the development of exploratory talk. Following ideas from exploratory talk studies (e.g., Mercer, Wegerif & Dawes, 1999), I have worked with teachers in encouraging young children to engage critically and constructively in solving mathematical problems. Research has suggested that exploratory talk impacts positively on achievement in mathematics (Mercer & Sams, 2006), but I was still keen to explore the relationship between learning and talk, particularly with young children.

Mercer (2000) has related talk to learning by proposing that exploratory talk creates a dialogic space within the zone of proximal development. A key aspect of exploratory talk is reaching a consensus and, in order to reach this consensus, ideas need to be made explicit for others to access. This explicitness can then enable an exchange of ideas and provide not just interaction, but the sharing of psychological processes that support conceptual change (Littleton & Mercer, 2013). In my view, this sharing of psychological processes is further explained by shared intentionality (Tomasello & Carpenter, 2007). In sharing an intention, an individual conveys their meaning or significance of an object of knowledge in a way that can be shared with others. In other words, one person’s meaning of an object is communicated in order to influence another person’s meaning of that object. My interest in this article is in how young children express their ideas when collaborating on a task, and whether they are able to make ideas explicit in a way that enables shared intentionality.

The examples presented in this article have come from a project that I worked on, with teachers of young children (aged six and seven years old) in the south-west of England, to encourage collaboration and exploratory talk in mathematics. The children worked together on mathematics tasks set by their teachers within the numeracy work that was part of their classroom teaching over a period of two months. Although there was evidence of children collaborating on tasks, the children did not seem to elaborate verbally their mathematics ideas. Rather than justifying ideas using explicit explanations, we noticed that the children pointed to the images, signs, or representations they were working with and communicated with phrases such as “It’s that one” or “Like that”.

Initially, we were concerned that the mathematics tasks had not encouraged exploratory talk. The children’s communication seemed to be typical of incipient exploratory talk (Rojas-Drummond, Gomez & Velez, 2008), as it was characterised by opinions and pointing, rather than the more elaborate exploratory talk and verbal description of ideas. However, there were several instances where the children decided on the meaning of a mathematical object together and the use of pointing seemed key to their interaction in making these decisions. So did this use of incipient talk enable the children to share intentions, and if so, how? Whilst it seemed plausible for elaborate exploratory talk to have the potential for sharing intentions, was it possible that simply pointing and stating “Like that” could also enable sharing of intentions that would influence another child’s meaning?

In more elaborate talk and verbal descriptions, we would expect the use of content words, such as nouns and verbs, related to a specific mathematical context; for example, “add” or “times”. However, the children’s talk did not contain such content words when they were making decisions. Instead, the talk often relied on the use of function words such as the pronoun “it” and the demonstrative “that”.

According to Halliday and Matthiessen’s (2004) theory of systemic functional linguistics (SFL), it is these function words that are seen as the powerhouse of meaning-making. In making meaning, the speaker makes systemic (although not always conscious) choices in which function words to use. So, in analysing the children’s interactions, I examined their use of language in a functional way. That is, I examined how they used function words, and in particular “it” and “that”, to construct phrases and link content words grammatically, and how this use related to shared intentions.

I was also aware that talk was only one of the multiple semiotic modes that could be used to make meaning in small group collaborative group work. Other modes, including symbols, diagrams, and gestures, may also be employed in communication and in meaning-making. Such multiple modes ground or embody abstract mathematics ideas (Roth,
Deixis and intentionality
The children used the pronoun “it” and the demonstratives “this” and “that” to point to the objects they were working with. The function of these terms is deictic, from the Greek word “deixis” meaning to point. Along with the gesture of pointing, these terms provide an index, or a sign, that has a connection with an object (Peirce, 1955). As an aspect of indexicality, deixis links speech to a context by establishing relationships between speaker and listener, but where meaning is created within the context. Hence the children used deictic terms to point out elements to each other and to link their speech to the context, and the interpretation of meaning depended on the context.

Demonstratives can be used anaphorically to refer to something that has been said or happened already in the discourse, or they can be used spatially to draw attention to an object that is visible to the speaker and listeners. In either form, their use draws attention to aspects of a context, and brings them into a collective consciousness (Radford, 2002). The pronoun “it” also acts deictically as a “linguistic pointer to a shared idea” (Rowland, 1992, p. 47), but the pronoun “it” is an indicator of something that is held in the mind and collectively understood. The pronoun does not refer to an object in view, hence its use is anaphoric.

Examples of mathematical group work
In this section, I present four excerpts of transcripts, as examples of the children’s group work, in order to illustrate their use of deixis. These transcripts were analysed according to the deictic terms used in an utterance. I also made reference to video recordings of the children’s group work, in order to analyse their gestures or actions, along with the mathematical representations (diagrams or symbols) used in the task.

Excerpt 1: Double seven
In Excerpt 1, three children, Emma, Olwen and Diane, were given a large card with an image of two baskets and a set of images of eggs that could be manipulated and stuck onto the baskets. The children were looking at the function “double seven”. They decided to stick seven eggs in each of the two baskets (Figure 1). They then realised they were required to determine the total number of eggs.

1.1 Diane: No, we need to know what it equals.

Diane points with one index finger to each basket moving the finger back and forth across the two baskets several times

1.2 Olwen: Oh yea, Emma’s good at that. Seven ... fourteen ... two, fourteen ... equals.

Olwen waves both hands in front of her and then holds them under her chin.

1.3 Diane: Fourteen.

Diane holds one hand over one basket. Her fingers are moving but not touching the basket or eggs. Diane then holds her hand still and gives the total.

1.4 Olwen: Yes, exactly, you don’t have to count it, just work it out.

All three children remove the eggs from the two baskets.

In Excerpt 1, the deictic use of “it” and “that” was anaphoric in referring to the total number of eggs, or the process of finding the total number of eggs, as a shared idea. For example, in utterance 1.1, Diane referred to the idea of double seven that was modelled by the eggs and the two baskets and, in utterance 1.2, Olwen’s use of “that” was anaphoric, with reference to Diane’s “it”, in finding double seven. In utterance 1.4, Olwen referred to the process of finding the total or “it”.

The three children understood that the purpose of the task was to find the solution to double seven, so the use of “it” and “that” was understood by the children. Some deference was given to Emma as an authority (utterance 1.2), but Olwen and Diane arrived at a solution without Emma. Diane appeared to use a counting strategy to enumerate the number of eggs, and Olwen used recall of a fact. The children’s consideration of the different strategies for finding the solution to double seven suggested some elaboration of mathematics ideas, but whether they all accepted the solution is not so clear. Olwen’s “Yes exactly” (utterance 1.4) seemed authoritative enough for the children to remove the eggs ready to start another problem.
Excerpt 2: Matching cards

In another task, the three children, Emma, Olwen and Diane, were matching word problems with number line representations. In Excerpt 2, the group were deciding on two word problems: “15 flies were on a cake, 5 more came along, how many were there altogether?” and “A ladybird has 6 legs, how many legs would four ladybirds have?” along with their matching number line representations (see Figure 2).

2.1 Emma: 15 flies were on a cake, 5 more came along, how many were there altogether?

Emma holds the card with the word problem

2.2 Emma: Do you think it’s that one or that one Diane?

Emma points her index finger to each of the two number line cards.

2.3 Emma: …. That one?

Diane points her index finger to one number line card. Emma points her index finger to the same number line card.

2.4 Olwen: It’s definitely that one, Emma.

Olwen points with her index finger to the same number line card.

2.5 Diane: I don’t know if it’s this one, A ladybird has 6 legs, how many legs would four ladybirds have?

Diane picks up the other word problem card. Diane reads the word problem card and then puts it down.

2.6 Diane: … I’m checking, 6, 12...

Diane picks up the other number line card and holds the card in both hands.

2.7 Olwen: 18 … 24

Olwen has her hands folded on the table whilst Diane looks at the number line card.

In Excerpt 2, the demonstrative “that” was used spatially. For example, in utterances 2.2, 2.3 and 2.4 Emma and Olwen’s use of “that” was associated with pointing gestures as they drew attention towards the two number line representations in question. Their use of “it” was then anaphoric in relating to a correct match. In utterance 2.5, Diane referred to the other word problem, possibly challenging the choice made by Emma and Olwen. Again, Diane’s use of “it” was anaphoric in referring to the idea of the correct card. Diane’s use of “this” then related to the number line card that she was holding and directly attending to herself. The card was not visible to the other two children. In utterances, 2.6 and 2.7, Diane skip-counted the multiples of six on the number line card. Diane was looking at the card but holding it with both hands. The card was not visible to Olwen who had her arms folded as she

Figure 2. Number line representations.

completed the count. There seemed to be no gesture to enumerate the skip-counting on the number line representations.

In Excerpt 2, the children chose between two representations in relation to the two word problems. Demonstratives “that” and “this” were used spatially and referred directly to the word problem cards and the number line cards under scrutiny. The demonstratives were used to indicate to which card each child was referring. The children did not elaborate on the word problems or gesture towards features of the number line, but Diane and Olwen did follow the skip-counting for the multiples of six on the number line. This skip-counting seemed to help the children associate the number line card with the word problem.

Excerpt 3: Ten tens

The spatial and anaphoric uses of deixis were evident in the interactions of the other groups of children that I present in the next two excerpts. In Excerpt 3, Ben and Paul were working together. Ben picked a card with the multiplication $10 \times 10$. Paul was expected to answer this problem and then explain his answer to Ben.

3.1 Ben: Ten tens what’s that?

Ben holds the card with the multiplication problem $10 \times 10$.

3.2 Paul: You can count up in your fingers, so imagine 10 20 30 40 50 60 70 80 90 100 like that …

Paul holds his right hand up as a fist then opens up each finger as he skip counts. Paul then moves the count to his left hand, opening each finger. All ten fingers are spread in the air as he says “like that”.

3.3 Paul: … and if it’s on there, that means times.

So ten times …

Paul points using his index finger to the multiplication symbol on the card that Ben is holding.

3.4 Paul: So if it says first then it times out whatever it is.

Paul moves his hand away from the card and points with his thumb and index finger together in the air.
3.5 Paul: ... So it will be a hundred.

Paul brings his fingers together in a loose fist.

3.6 Ben: So can you do it to me now? You pick up one.

Paul picks out a card for Ben.

In Excerpt 3, deixis was mostly anaphoric. In utterance 3.1, Ben’s use of the pronoun “that” was not associated with pointing. “That” was used anaphorically in referring to the product. In utterance 3.2, Paul’s use of “that” was anaphoric in referring to the skip counting and to the product. However in utterance 3.3, Paul’s uses of “it” and “that” were spatially deictic as he directed attention to the multiplication symbol on the card and what the sign meant. Utterances 3.4 and 3.5 indicate a more elaborate generalisation in association with anaphora. In utterance 3.4, Paul was referring to the multiplicand and the multiplier but pointing (with thumb and index finger together) into the air. The use of the word “it” suggested a generality. Rather than the numbers given on the card, the pointing was to any number, not necessarily 10, suggesting use of variables. In utterance 3.5, Paul brought the fingers of his hand together in a loose fist as if making the product and used “it” anaphorically to refer to a product. Ben then used “it” anaphorically in referring to the process of the task, or his turn (utterance 3.6).

Apart from the use of “it” and “that” when Paul pointed to the multiplication symbol on the card (utterance 3.3), Paul’s use of deixis was anaphoric and referred to an abstract idea, something he held in his head. Pointing into the air (utterance 3.4) suggested he was pointing to a mathematical object, a generalisation or variable, and this object was not visible. Paul’s explanation of multiplication and his reference to variables suggested some elaboration of mathematics ideas, but after the explanation, Ben asked to move on to his turn in carrying out the task.

Excerpt 4: More than or less than

In another task, Paul was working with Fran and Iris on a task using inequalities. The teacher had talked with the children in deciding what the inequality signs meant and the children recorded the inequalities $5 < 30$ and $10 < 80$ on a white board. This white board was left with the group so that they could reference the meaning of the different signs (Figure 3). In Excerpt 4, the children were given a sheet of paper with the inequalities $> 50$ on one side and $< 50$ on the other (see Figure 3). The children were told to write numbers on each side that would be true for each inequality.

4.1 Iris: Is that more than or less than, which one?

Iris points to $> 50$ on the worksheet with index finger

4.2 Paul: That is...

Paul points to $> 50$ on the worksheet sheet with a pencil.

4.3 Fran: If we look at, if we have a look...

In contrast to Excerpt 3, there was no anaphoric use of “it” in Excerpt 4. In utterances 4.1 and 4.2, “that” was used spatially. Iris pointed directly to the inequality sign on the worksheet and all three children’s attention was focused on the symbol. Paul also pointed directly to the sign with his pencil. In utterance 4.4, Iris used “that” spatially as she directed attention to the signs. She moved her finger from side to side over the two inequalities, possibly focusing on the direction of the inequalities. In utterance 4.5, both Iris and Fran used “that” spatially in relation to the sign on the worksheet. Fran pointed between the whiteboard and the worksheet as if to direct attention back to the sign that was in question. “That” was then used spatially as Paul directed attention to the sign on the worksheet (utterance 4.6).

In this excerpt, there was no verbal elaboration of mathematics ideas. Nonetheless, scrutiny of the meaning of the sign was carried out by directing attention to the inequalities on the whiteboard and on the worksheet. The children used the demonstrative “that” along with pointing to draw attention to the inequality sign under scrutiny.

The conjunction of talk and gesture in sharing intentions

In these four examples, the children were collaborating, in that they were focusing on and completing a task together. However, there were distinctions in the use of deictic terms.
One distinction was between the use of anaphoric deixis, particularly the use of “it” in Excerpts 1 and 3, and the use of spatial demonstratives “this” and “that” in Excerpts 2 and 4, and I wondered if this distinction suggested the children were sharing intentions in different ways.

One aspect that I noticed was the way the tasks had influenced the children’s use of deixis. In Excerpts 2 and 4, the children were making choices between alternatives, either in matching the cards or in deciding the meaning of the inequality sign. These contexts led to decision-making processes. In making these decisions and arriving at a consensus, the children directly referred to the symbols and diagrams. This direct reference meant that not only did the signs and representations ground the mathematics ideas, they were also the objects under scrutiny. The use of the demonstrative “that” was accompanied by pointing gestures, as the children directed attention to the number line or the inequality sign. That is, gesture and deictic words were spatial and demonstrative.

In contrast, it seemed that the need for consensus was not so prevalent in Excerpts 1 and 3. For example, in Excerpt 1 the image of the eggs in the basket embodied the function double seven, but the images were not the object under scrutiny; the calculation of the function was under scrutiny. Also, in Excerpt 3, the example $10 \times 10$ embodied the idea of a product, but the symbols or the product were not the object under scrutiny. It was the generalisation from the product that was the object under scrutiny. In these excerpts, the children were enumerating a doubling function or generalising a product; they were not choosing or deciding about the meaning of an object. Also, the doubling function or generalisation of a product were not mathematical objects that could be pointed to directly, and so gesture and deictic words tended to be anaphoric.

From this analysis, I suggest that the use of gestures along with the reference of the deictic terms was different according to the nature of the task. Gestures play a role in helping the speaker to formulate speech and to embody or ground abstract ideas in mathematics (Roth, 2009). In these examples, the children’s gestures in relation to both anaphoric and spatial deixis grounded the mathematics ideas in the context of the task, be it a representation, an image, or a symbol. However, I propose that with the distinction between spatial and anaphoric deixis, there was also a distinction in how the intentions were shared, and that the use of gestures reflected the way the intentions were shared.

The gestures the children used in spatial deixis were to point and direct attention towards a symbol or diagram that everyone could see, and this symbol or diagram was the mathematical object under scrutiny. For example, in Excerpt 2, the children pointed directly to the cards with the number problems and the number line representations as the objects to be matched. Also in Excerpt 4, the children pointed directly to the inequality signs, both on the whiteboard and on the worksheet, as they decided on the meaning. Not only did the number line representation and the inequality sign embody a mathematics idea, they were also the objects under scrutiny in determining the mathematics idea.

Instead, in anaphora, reference is made to an idea that is held in the speaker’s head, not something tangible that all can see. So, in Excerpts 1 and 3 the symbols or images that the children could see may have grounded a mathematics idea, but they were not the objects under scrutiny, and the use of gestures reflected this. For example, when Diane moved her finger across the two baskets (utterance 1.1) she was not referring to the baskets themselves but to the function of double seven that the baskets embodied and that she held in her head. When Paul pointed in the air in generalising a product (utterance 3.4), he was pointing to imaginary numbers held in his head and not the $10 \times 10$ that was on the card that Ben was holding.

I suggest that these distinctions related to the way intentions were shared. Demonstrative pointing situated the context by indicating directly a tangible object that was present. The gestures and deictic words were used deliberately to share intentions and change another’s thinking. However, in anaphoric gestures the sharing of intentions was not so direct and deliberate. For example, Diane moved her finger in counting the eggs (utterance 1.3), Paul opened his fingers in skip-counting tens (utterance 3.2), Olwen raised both hands to her chin in recalling a double (utterance 1.2), and Iris moved her finger from side to side over the inequalities on the whiteboard (utterance 4.4). It seemed to me that these gestures were embodying ideas such as numeration, recall of a fact or the idea of inequality, but they were not used deliberately to direct another’s attention. Whilst anaphoric gestures had the potential to indicate intentions, they were used to help the speaker to formulate their ideas and their speech rather than to indicate a tangible object, so they were not used deliberately to change another’s thinking.

Economy of language and cultural mediation

I have indicated that the children used demonstrative pointing and spatial deixis deliberately to change another’s thinking, that is, to share their intentions. But how could such a seemingly limited use of language be effective in influencing the thinking of another in understanding a mathematical idea? After all, any verbal elaborations were limited; the children simply pointed out objects. Bangerter (2004) proposed that in deixis the use of gesture suppresses the need for verbal descriptions. There is a “trade-off” between verbal usage and gestures. In the examples I have presented in this article, it would seem the children were using this “trade-off” between words and pointing. They used talk and gesture in an economical way to direct attention to an object.

In the case of spatial deixis, the children directed each other’s attention in an economical way to the sign or representation under scrutiny, and their interpretation of “that” depended on the immediate physical context of the task. Although the children’s use of words may have been suppressed, they made their intentions tangible by using both talk and gestures (McNeill, 1992). This economical conjoined use of language may have given the children freedom to arrive at a consensus without recourse to specific mathematical language. It seems to me that this conjunction of talk and gesture is especially relevant in mathematics. Mathematical objects are conceptual, they cannot be seen. Nonetheless, in Excerpts 2 and 4, the children could share their intentions of concepts by pointing directly to ostensive examples of abstract objects (the number line representations and the inequality signs). And so the children...
were pointing to the objects as examples of abstract concepts and sharing their individual intentions, or the meanings that they associated with the object.

Whilst I suggest that this deliberate and direct use of ostensive examples enabled the children to share intentions, I also pose this as a potential problem. Mathematical objects have a cultural, conventional meaning. The signs and representations that the children were drawing attention to were endowed semiotically with a cultural meaning. However, intentions have a subjective element. As Radford (2006) has indicated, intentions are two-sided, both subjective and cultural. By sharing intentions, the children were directing attention to what they saw subjectively as significant. I contend that in scrutinising the object and arriving at a consensus, the children were also relating to a meaning, that is, they were attending to what was intrinsically cultural about the number line representation or the inequality sign. The arrival at a shared meaning was mediated semiotically and culturally within the context of the problem.

To illustrate this point, I refer to Excerpts 2 and 4, and the way the meanings of the number line representations and the inequality sign were organised within the context of the tasks. By matching and choosing, the children were working towards a consensus. Elaborations or descriptions of features were limited to skip-counting from the number line representation, and there were no verbal elaborations of the inequality sign. Even so, in spatial deixis, the children were pointing to an ostensive example and scrutinising the cultural meaning of the example using language in an economical way. Within this scrutiny, they were sharing intentions; that is, one child’s meaning was communicated in order to influence another child’s meaning, but they were also relating to a cultural meaning.

On the other hand, in the tasks for Excerpts 1 and 3, the children were (at least in some ways) elaborating strategies for doubling or for generalising the idea of a product. To do this, they referred to abstract objects anaphorically elaborating the cultural meaning of the example using language in an economical way. In order for deixis to be used appropriately, the referent must already exist cognitively “(that is the referent is presupposed) for the interlocutors” (Wertsch, 1985, p. 169). It seems to me that in the tasks that did not involve reaching consensus in matching meanings, the children’s individual intentions might have been understood sufficiently to be effective in the collective consciousness, but they were not scrutinised. As there was no scrutiny of a tangible object, an individual’s meaning was not mediated semiotically or culturally, and it was not clear how (or even if) the children were influencing the thinking of another.

Concluding remarks

In the examples I have presented in this article, the children were able to use language economically in order to share intentions and influence the ideas of others in certain tasks. The children were able to embody intentions through deixis, either spatially by directly pointing to an ostensive object, or anaphorically to a generalised idea. In relating to the conjoined use of talk and gesture in sharing intentions, I maintain that deixis has a role in children’s learning in mathematics. I further propose that this economical use of language is not instead of the mathematical thinking but it is the mathematical thinking, and that talk and gesture are not just ways to externalise personal understanding, they are functions in creating meaning.

The nature of the task was crucial in determining the way deixis was used. I indicated how the tasks that required scrutiny of a mathematical object in reaching a consensus, tended to encourage spatial deixis, and that this use of spatial deixis had potential for cultural mediation. With reference to Radford’s (2002) idea of meaning-making, the children’s use of spatial deixis suggested an interface between the spoken and the seen and this meaning-making process was social, semiotic and cultural.

In carrying out a multi-modal analytic approach, I was able to relate meaning-making and shared intentions to an interpretive framework of learning that conjoined gesture, semiotic and linguistic methods to the “joint orientation to the same object” (Roth, 2009, p. 104), where this conjunction was seen to mediate cultural understanding. Further study of children’s use of gestures in conjunction with functional language within different tasks is needed to understand how young children can make meaning of abstract mathematical objects and share intentions regarding these meanings, when collaborating in group work.

References


