Exploring the Practical Rationality of Mathematics Teaching through Conversations about Videotaped Episodes: the Case of Engaging Students in Proving

PATRICIO HERBST, DANIEL CHAZAN

What might it mean to conceive of mathematics teaching as a *practice*? Can mathematics teaching be seen as something analogous to criminal law practice or family practice in medicine within the spectrum of human endeavors? As Donald Schön (1983) claims, those practices involve more than the application of technical knowledge expressed in declarative form in some professional canon - there are forms of practical rationality (which Schön calls 'knowledge-in-action' and 'reflection-in-action') that enable practitioners to do what they do. Such rationality cannot be reduced to individual wisdom, gift, sensibility or skill, since these are common to people who perform the same job; yet they are not all part of the explicit regulations that describe this job.

As Pierre Bourdieu (1998) puts it, all practices, not just those of professionals, require a "feel for the game" (p. 25), or:

> knowledge without cognitive intent, the mastery that agents acquire of their social world by way of durable immersion within it (Bourdieu and Wacquant, 1992, p 19)

Such practical rationality not only enables practices to reproduce themselves over time as the people who are the practitioners change, but also regulates how instances of the practice are produced and what makes them count as instances. And, often, practical rationality also erases its own tracks, making practitioners believe and make believe that these practices themselves are 'natural', as if similarities and differences among ways of acting were solely the product of similarities and differences among individual actors (Bourdieu, 1985).

This suggests that, if mathematics teaching can be conceived of as a practice, it must rest on the base of a common practical rationality, upon which individual practitioners can build their own mathematics teaching against the backdrop of their personal commitments and the demands of the institutional contexts where they work. In this article, we explore the need to examine this practical rationality of mathematics teaching, how conversations about videotaped episodes of teaching afford a medium to elicit it and how it might be described.

1. The practical rationality of mathematics teaching

Different 'kinds' of mathematics teaching co-exist side by side - frequently presenting themselves as incommensurable with one another in nature, at the same time as being called on to play similar social functions. For us, this observation serves as a warrant for inquiring into a *practical rationality of mathematics teaching*. To postulate that mathematics teaching is enabled and regulated by a common practical rationality which allows the emergence of diversity within similarity - and to find out how such a rationality operates in producing diversity within similarity - seem like useful steps toward understanding and promoting communication and improvement in mathematics teaching.

A case in point that helps to illustrate this observation comes from recent U.S. reform efforts in mathematics education, such as those promoted by the NCTM (1989, 2000) which has put forth a vision for improving the quality of students' opportunities to learn in classrooms. This attempt has involved more than simply changing the content students have opportunities to learn or the techniques used by teachers in their instruction: it additionally seems influenced by a substantial reconsideration of the nature of students' mathematical activity in classrooms. Such reconsideration has involved, in part, reflection about the nature of mathematical activity in the discipline and of what it takes to come to know mathematical ideas.

This reflection on disciplinary knowledge has been inspired by accounts from mathematicians as well as from philosophers and historians of mathematics (e.g. Davis and Hersh, 1981; Gillies, 1992; Lakatos, 1976; Polya, 1945; Thurston, 1994; Tymoczko, 1986). Such accounts of the nature of mathematical activity have resulted in recommendations for the mathematical work that students *should* be doing in classrooms, work quite different from what students traditionally have been asked to do.

Many teachers have been inspired by the vision that students might have authentic intellectual experiences with mathematics in classrooms (Lampert, 1992) and have committed to enabling ways of working whereby responsibility for reasoning mathematically and developing mathematical ideas in classrooms is devolved to students (Chazan, Bethell and Lehman, in preparation; Lampert, 1990). But the panorama of U.S. mathematics education shows that reform ideas do not spread out quickly or widely. In fact, ways of knowing and doing mathematics in classrooms change little

For the Learning of Mathematics 23, 1 (March, 2003)
FLM Publishing Association, Kingston, Ontario, Canada

and slowly, sometimes only cosmetically. (In the context of Californian reforms, see Cohen, 1990; Heaton, 1992; Remillard, 1992.) Small numbers of enthusiasts co-exist with large numbers of others who seem to ignore or even oppose the possibilities opened by the reform recommendations.

As an observer witnesses the various kinds of mathematical activity that obtain in most classrooms, questions arise that relate to the co-existence of and communication across 'kinds' of teaching (e.g. 'reform' and 'traditional'). Reformers tend to use a language of vision as they seek to convince practitioners of the need to change and provide them with the requisite knowledge to improve classroom teaching. Despite our endorsement of that vision, we recognize that real communication across 'kinds' of teaching gets muddled when the presumption is that differences are due to the lack of knowledge or paucity of vision of one camp.

We contend that school mathematical activity is shaped by the various (institutional, situational, epistemological, temporal, material) contexts where it unfolds (see Chevallard, 1991; Popkewitz, 1988) as much as it is shaped by the individual characteristics of the agents who facilitate it. There is consequently a need to devise ways of inquiring into how that play of agency and structure allows (and perhaps produces) very different 'kinds' of mathematics teaching which are evidently alternative, viable ways of doing the work. The object of such inquiry is what we are calling the practical rationality of mathematics teaching.

A case: the place of proof in the mathematical activity of geometry classrooms

One particular domain where there is a visible need to understand the diversity of viable practices concerns the place of proof in the mathematical activity of geometry classrooms. In regard to the role that reasoning and proof both do and might play in mathematics classrooms, the work of Imre Lakatos (1976) has been a source of enlightenment and wonderment for mathematics educators. He gives a relational definition of proof as that which mathematicians of a given time might accept as a fitting way of reducing a problem to a collection of more elementary lemmas, whose truth is understood by the audience. Lakatos' exploration of his claim that there is a *logic* of mathematical discovery suggests that proof plays a fundamental role in bringing mathematical concepts into existence. Proving a naïve conjecture develops the need for concepts, provides the means to calibrate and decide among putative definitions, affords choices for postulates and suggests how to improve both the conjecture and the proof – and even what a theorem should state in order to become true.

Partly in response to that awareness, the NCTM's (2000) *Principles and Standards* have recommended a privileged place for reasoning and proving. They argue that:

> Reasoning and proof should be a consistent part of students' mathematical experience in prekindergarten through grade 12 [... and envision that such work help students] see and expect that mathematics makes sense. (p. 56)

Yet, as regards the place that proof occupies in mathematical activity, comparatively few classrooms offer qualities similar to those that Lakatos attributes to the work of mathematicians (e.g. see the practices described in Chazan, 1995). In the experience of most American students, proof occupies a far more restricted place, limited to their high-school geometry course where it plays a role that has little to do with coming to know. 'Doing proofs' in geometry often implies carrying out a deduction from given premises to a known conclusion, justifying every intermediate 'statement' by a legitimate 'reason' (a previously studied theorem, postulate or definition – see Herbst, 2002b). The informational value of or theoretical interest in the propositions that students prove is often low and unimportant in comparison with the purpose of providing practice in sequential, deductive reasoning (see also Weber, 2002).

Hence, as we think about what it takes for proof to play the role of tool for developing knowledge in the mathematics class, we are also mindful that such a question needs to be formulated against the backdrop of an existing, viable custom of 'doing proofs'. In that context, we ask, what is the practical rationality that underlies how teachers engage and might engage students in proving? And also, how would such practical rationality allow or prevent proof playing a broader role, not just as a formal device to verify truth but also as a substantial device to find out what makes sense? As we discuss what the practical rationality of mathematics teaching means and how one might elicit this rationality, we come back to the case of proof in school mathematics to find examples for those more general ideas.

What could be meant by a practical rationality of mathematics teaching?

The need for reasoned communication and understanding about issues like the place of proof in school mathematics, across 'kinds' of teaching, illustrates the value of conceiving of a rationality that underlies all practices of teaching mathematics in school classrooms. Such rationality is often implicit and characterizes the regulatory mechanisms of practice as a system that socializes its members into ways of thinking and doing adapted to the work they are expected to undertake. Lastly, it enables individual practitioners to resemble as well as differ from each other in how they invest the personal assets, knowledge and commitments they bring to the work they are required to do.

This practical rationality implicates the objective characteristics of the position of 'mathematics teacher', as well as the commitments and knowledge that individual practitioners bring into play in such positions. We thus suspect that such rationality is constituted by elements that are specific to the practice of teaching mathematics in school institutions. It is important to understand theoretically how this rationality works to create and maintain stability in mathematics teaching. Indeed, it may help us understand how the shared sense is created among practitioners that it is possible to speak of one such practice ('teaching mathematics') as different from other, related but different practices (such as 'teaching history' or 'doing mathematics').

To examine the nature of such a rationality is not only helpful as a way to envision what a practitioner's language would need to be like if it were to support reasoned communication across, as well as honest representation within, 'kinds' of teaching An understanding of such a rationality can also help explain why some recommendations for change are perceived as feasible by practitioners, whereas others are seen as unlikely and perhaps yet others still viewed as outrageous. It can also help explain why some recommendations for change enable teachers to develop authentic student opportunities to learn mathematics, whereas others are implemented in ways that conspire against the rationale behind their suggestion. Such an understanding is crucial to foreseeing how particular aspects of pushes for change may get disseminated into the practice, as well as to predict how practitioners will likely position themselves in relation to their colleagues as a result of the 'kinds' of teaching in which they engage.

In this article, we present and exemplify the conceptual underpinnings of a methodology for eliciting elements of a practical rationality of mathematics teaching. Practitioners' conversations about videotaped episodes of teaching practice, when they are set up in particular ways, can become natural windows into this rationality. Those conversations may conjure specific elements which render visible how they function to create a shared sense of participating in a unique practice on the part of mathematics teachers. We use instances from a study that implements this conceptual approach for data collection to illustrate how we describe the practical rationality of mathematics teaching with respect to one particular aspect of the practice of teaching geometry: engaging students in proving

2. Conversations about videotaped classroom episodes

The increased availability of video technology has enabled researchers to focus on videotaped episodes of classrooms to understand better various aspects of mathematics teaching and learning. In particular, the examination of videotaped records of practice from diverse perspectives has become a widespread tool in research on teaching. For instance, Lampert and Ball (1998) use extensive video records of teaching practice in elementary teacher education in order to elicit implicit knowledge about teaching that pre-service teachers have acquired through their direct experiences as students in school Furthermore, these records of practice have been used by scholars of teaching seeking to understand what it takes to teach mathematics in a way that responds to particularly interesting sets of commitments, such as using problems as vehicles to teach about mathematical ideas (Lampert, 2001; see also Ball, 1993; Ball and Bass, 2000).

Videotape episodes have also become common as instruments and probes for communication among teachers, educators and researchers. For example, Nemirovsky *et al* (2001) have developed the notion of a 'videopaper' as an instrument that permits better communication between researchers and teachers on issues of student learning. In communication around episodes featured in a videopaper, participants often treated the episode as data, grounding descriptions and interpretations of specific actions in the episode (e.g. what student X was thinking when he did Y). Such interest in the use of video technology to understand the work of teaching has been fuelled by the usefulness of videotape records to display the diversity of teaching practices found by international comparisons (e.g. Stigler and Hiebert, 1999)

Arguments about the need to build a knowledge base for teaching on the practices of teaching point clearly to the potential of using video episodes in similar ways for accumulating and disseminating such knowledge (Hiebert, Gallimore and Stigler, 2002). All of those uses place an emphasis on the videotape as record and on the situation of viewing videotapes (individually or in groups) as a means for developing an understanding of the meaning of the actions recorded. Videotaped records have thus served various objects of study (teaching, learning or the knowledge of the professional community) and purposes (research, communication or teacher education)

In our own work of understanding mathematical activity in classrooms and educating secondary teachers, both of us have often used video records in ways similar to those described above. Through reflecting on and comparing those uses, we have become aware of how useful the situation in which practitioners are confronted with video episodes to elicit what we have been calling their practical rationality is. In what follows, we show how we think about those situations that confront practitioners with episodes as we design sessions for the collection of empirical data to study the practical rationality of mathematics teaching. To show how these general considerations play out in practice, we work out an example that comes from Herbst's research agenda

A methodological problem in studying how teachers engage students in proving

Herbst's research is focused on understanding the connections between the place allocated to reasoning and proof in geometry classrooms and the conditions and constraints that teachers juggle with as they facilitate students' development of geometric knowledge (Chazan, 1990, reflects an attempt to intervene in this system.) Herbst (2002b) has shown how the expectation that teachers teach 'proof' to students developed historically, imposing demands on the work that teachers do to create and manage opportunities for students to 'do proofs' Herbst (2002a) has used video records of practice to investigate the actual work of the teacher engaging students in proving In doing that work, he has taken the stance of an observer who looks for how objective structures - in this case, the need to negotiate a viable didactical contract for proof (Brousseau, 1997) - condition the actions of teaching

To complement this work, questions remain to be explored that require some attention to the subjective experience of the actors, the particular awareness that makes sense for teachers to invest as they engage students in proving. What dispositions do teachers normally hold as they engage students in proving? How do those dispositions make these practices (that an observer might see as conditioned by the didactical contract for proof) appear as normal and natural to practitioners? These questions, and better understandings of how issues of agency and sense-making mingle with structural conditions and constraints, might help explain why existing practices of proving remain in place in high-school geometry and what it might take to change them.

In trying to implement such a research agenda, an important methodological problem has been that of figuring out how to elicit those often unspoken and highly situated aspects of the subjective experience of practitioners that inform the practice they (and others like them) engage in Chazan's work promoting teachers' use of video records to communicate with others about their practice (see Chazan, Bethell and Lehman, in preparation) and his experience as a broker of conversations about that practice have been valuable precursors. What follows is an attempt to articulate such a methodology and illustrate it by discussing the features of the development and use of an instrument for collecting data on the practical rationality that geometry teachers invest in as they engage students in proving To do that, we consider it necessary to start from a discussion of the generic situation of having practitioners gathered for a conversation around a video episode.

Conversations about episodes of practice: summoning individuals into positions

Picture a group of practicing teachers gathered together by someone in pursuit of a given agenda to watch and discuss videotape episodes of mathematics teaching. For example, a teacher educator might gather a group of eighth-grade teachers to watch a Japanese lesson on 'area of triangles' from the TIMSS study (see Lopez-Real and Leung, 2001, for a related account), with the agenda of discussing different approaches to using problem solving in instruction. The mathematics co-ordinator for a school district might gather the elementary teachers she or he supports to show them video episodes that demonstrate how elementary teachers elsewhere teach a specific unit of a U S Standards-based elementary mathematics curriculum.

These conversations' convenors call upon participants who occupy positions in the educational system (they teach certain kinds of mathematics at a given level, in a particular district, under specific explicit institutional expectations, etc). As the convenor sets an agenda for the conversation, he or she specifies the positions into which participants are summoned. The agenda brings participants into the conversation to represent not only themselves but also the putative position in which they work The situation where such a gathering involves individuals who are taken to represent the same position (e.g. as elementary teachers of city X or algebra teachers of region Y) is especially interesting in developing our methodology. We contend that, in this case, such tacit assignment and assumption of positions establishes a provisional common ground on which all gathered participants may consider themselves as comparable to one another. We argue below that the existence of such constituted 'common ground' sanctions certain ways of speaking as relevant.

Conversations about episodes of teaching are possible because in taking a position to represent themselves, the participants tacitly accept a level at which the records of practice to be inspected are *supposed to be* meaningful to them. Indeed, participants may be expected to interact about those episodes on the presumption that they can regard themselves as people who do the same sort of job as the teachers who appear in the video episode. This taking of positions is important at the point at which practitioners negotiate the meaning of the video episodes in their interactions and as observers examine the meaning of those interactions. We are interested in those conversations in so far as mathematics teaching practice is the subject of the conversation and, we will argue, practical rationality may appear as the intelligence (or the perspective) needed by practitioners to construct accounts of the practice whose records they see in the episode (cf. Simon and Tzur, 1999).

In the example discussed below, where we intended to elicit the practical rationality invested by teachers as they engaged students in proving, we brokered a conversation targeted at those who teach or have taught geometry, who could identify their own practice with the practice displayed in the video records. In order to create a prompt for a conversation to be led by Chazan, we used records of a lesson taught by Herbst where a geometry class had worked collectively to produce a proof.

The agenda for the conversation proposed to discuss:

> the place that proof occupies in the mathematics curriculum and the work that teachers do to engage students in proving.

Chazan started it by pointing out that in spite of:

> all sorts of criticisms of what goes on in the name of proving, it seems that there are things to understand about how [those] involved in that work think about the kinds of proving that go on in high-school classrooms.

The video episode to be watched was framed as showcasing possible ways of engaging students in proving. Chazan commented that, after inspecting an interesting geometric phenomenon that they all might consider adapting into a proof exercise, they would:

> look at a videotaped lesson where a proof related to that phenomenon was developed by the group. [The intention was to] talk through the moves that both the students and the teacher made [... under the assumption that discussing those moves might help everybody in the room learn more about] what it takes for a teacher to engage students in proving theorems

In addition to describing these conversations in terms of summoning practitioners to represent more or less specific positions, as well as interacting in a setting characterized by certain assumptions about those positions, we need to specify further what we have so far been calling video episodes

Video episodes: records, artifacts, cases and probes

By the term 'video episode', we mean a constructed documentary narration about classroom interaction referring to a span of time shorter than a lesson. This narration is done through an assemblage of footage from the actual interactions that constituted the lesson and other sources (such as captions, slides with questions or comments from observers, titles, photographs, etc.). We want to comment on their nature and function in establishing interaction with and among a group of viewers Compared with other forms of narration, the specific nature of video episodes is multifaceted, as episodes are at the same time both records and artifacts – resembling as well as departing from the reality they narrate (Hall, 2000) There is also a dual function for these video episodes They act as cases of a common practice that binds gathered practitioners as colleagues. They also serve as probes that test individuals' sense of belonging to that common practice and help reveal deeper regulatory commitments and dispositions that condition which practices belong and to what extent

Records and artifacts

Lampert and Ball (1998) use the term 'record' to speak about how video episodes create a memory of a practice, thus making that practice available for inspection. For those purposes, it is obvious that video episodes are better than other records of classroom teaching. Videotape shares with other kinds of records the capacity to create opportunities for repeated inspection of a practice at a more leisured pace, than the richer, but also more fleeting, experience of actual practice in real time. All sorts of static records from a lesson pale when compared with video episodes with regard to their capacity to portray the reality they point to as much and as dynamically as possible.

Furthermore, as shown so effectively for the case of photography in Michelangelo Antonioni's *Blow Up* (1966; see also Cortázar, 1959/1983), the records themselves may afford visibility at several scales, supporting at one time the (appearance of) visibility of objects or actions that were not real (or realized) before (see also Ball and Lampert, 1999) Unlike other media, the realism of video (or film) cannot be underestimated as one considers it a record. Film critic Stanley Kauffmann (2002) writes:

> Even trick photography photographs the tricks, sweeps them into the real. [...] The camera certifies the existence of what it looks at. (p 24)

Yet, in spite of all the richness of information that video episodes as records of practice can provide, that information is neither comprehensive nor impartial. Technical and conceptual choices and constraints place bounds on what those episodes may offer As Hall (2000) points out, video recording understood as the production of a narrative implies a multitude of assumptions concerning how the real-world phenomenon being displayed is relevant to the questions being asked. A video episode cannot be thought of as a portrait of a reality that might be checked independently – that reality is done and gone.

Yet, it invites the viewer to reconstruct a possible reality from where that episode could have come and may even provide ground rules for doing so in a coherent way. In this sense, a video episode is comparable to the petrified bones of a prehistoric animal that a paleontologist might find in a rock formation, that could help her notice what the skeleton of the animal constructed from these bones might have been like. The petrification process is so altering that the bones actually found may never actually become part of the reconstructed skeleton. Nevertheless, it is because of that altering process that those bones have been preserved in a relatively usable form

Leaving the question of intention aside, the process of editing videotape is analogous to petrification in that, in order to preserve a record that can be used for certain purposes, it creates an artifact. The episode may be an effective prop for a conversation about teaching practice of the kind recorded because of the creative (thus, altering) work done in editing (adding commentary, emphasizing, filtering noise, labeling, organizing, parsing, shortening). For that very reason, the comments that participants make about what they see in a video episode cannot just be taken at face value, but need to be interpreted (Pimm, 1993).

The interesting quality of video episodes of teaching is that they are at the same time both records of and free-standing artifacts – that they come across as portraits and yet necessarily require reconstruction. Since the viewer must relate to a video episode as both record and artifact, it makes sense to ask what kind of thinking and talking a video episode may bring about related to the actions that it *re-creates* The example we discuss below allows us to illustrate how the dual nature of episodes as records and artifacts may potentially elicit the practical rationality of mathematics teaching

Proving that the angle bisectors of opposite angles of a parallelogram are parallel

The video episode used in the conversation we report on was constructed from footage from a lesson taught by Herbst in a regular geometry class, taken mostly by high-school sophomores. In that lesson, the class had collectively formulated and proved the claim that the angle bisectors of a parallelogram intersect at the vertices of a rectangle (see Figure 1).

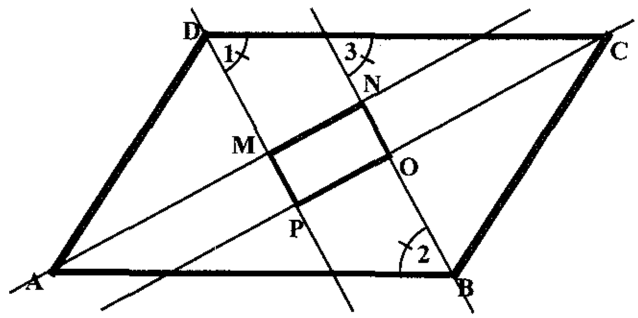


Figure 1: The angle bisectors of a parallelogram intersect at the vertices of a rectangle

The lesson was part of a review section on quadrilaterals that took place shortly before the mid-year exam and it was announced to students as an opportunity to review quadrilaterals for that exam. As the class formulated and sought to justify a conjecture about the quadrilateral determined by the intersection of the angle bisectors, the teacher insisted that they prove it and encouraged the use of the two-column proof format to conceive of and write their argument The video episodes used in the conversation that we report on included selected footage from an eighteen-minute segment This segment started with the teacher writing the following statement on the board:

> MNOP is a rectangle as long as ABCD is a parallelogram

He then issued an invitation to the students: "Let's write a proof for that one"

The teacher recognized one student, Adrian [1], as having an idea for a proof and invited him to come up to the board to get the proof started. (A sketch similar to Figure 1, but without any marked angles, was on the board at the time.) Not recognizing the need to prove that pairs of angle bisectors were parallel, Adrian tried to start from the assumption that quadrilateral MNOP was a parallelogram and offered to prove that it had a right angle. But many in the class protested, pointing out that he could only assume the given quadrilateral, ABCD, to be a parallelogram, not MNOP. As Adrian could not then make an argument for why MNOP would have to be a parallelogram, another student, Eamon, who claimed to know what to do, was asked to go to the board.

Eamon found himself in difficulty. He was not sure whether in order:

> to prove that they're [the bisectors] parallel [he should ….] just say that since they're [the opposite angles] both equal, the bisectors would be parallel.

The teacher limited himself to inviting the class to help Eamon. After a long silence, Anton commented from his seat that his 'problem' was not

> know[ing] how to [use the fact that those were] bisectors [in] proving them parallel.

Eamon spent several minutes and a number of false attempts tinkering with how to use alternate interior angles to prove the bisectors were parallel but to no avail, until he said:

> All I'm saying is I'm stuck on this step. Cause after this step it'll be easy.

The teacher took Eamon's comment as a request:

> Oh, so what you're saying is that if you could presume that the [angle bisectors] are parallel you could continue.

When Eamon agreed, the teacher suggested to the class:

> Why don't we let him go ahead and then we'll complete the details.

He let Eamon take as given that bisectors were parallel and told the class that:

> We will need somebody to come up with an argument for that [later]

As it turned out, Eamon went back to his seat shortly thereafter and Adrian came back to the board to proceed with his original idea for arguing why MNOP would have a right angle. As he was doing that silently on the board, the class discussion went back to why the bisectors would have to be parallel. Eamon and Mark insisted on them being parallel because of their being equidistant, Anton interjected that it just happens that they are parallel as one draws them and Todd suggested thinking of linear pairs but could not complete an idea. Adrian left his proof that the figure had a right angle for a moment to provide an oral argument for the bisectors determining congruent corresponding angles:

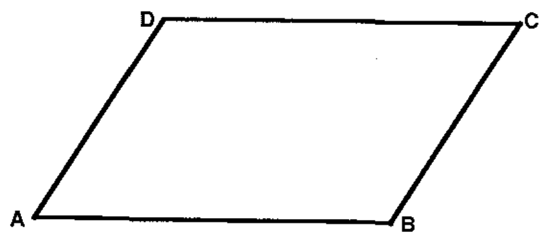
> Because, um, this angle [points to angle 2 - see Figure 1] is equal to this angle [points to angle 3], because of parallel lines [pointing to DC and AB]. And, um, this angle [pointing to angle 2] is equal to this angle [pointing to angle 1], because they're half of an opposite, opposite angles of a parallelogram. [2]

Eamon could finally express orally how Adrian's idea would imply that angle bisectors DM and BO would be parallel. But the argument was never written on the board, as Adrian went back to proving that MNOP had a right angle and the class moved to considering that part of the argument.

Records and artifacts elicit the practical rationality of mathematics teaching

The observation that video episodes are both records and artifacts that require the viewer simultaneously to observe and reconstruct helps us conceive of a methodology for surfacing elements of the practical rationality of mathematics teaching: specifically, for crafting a video episode as a probe into that practical rationality. The above example helps point out how that might work. One of the important choices that we made in selecting footage for the episode included not showing how the lesson had started and, in particular, not showing the diagram that had initially been given to students.

The episode displayed several instances of the difficulty that some students had in developing an argument for the assertion that the bisectors *had* to be parallel (and even recognizing the need to prove it). As participants in the conversation discussed the episode, they attempted to reconstruct those elements of context. They made assumptions, asked questions and passed judgements on how the task had been set up. They recognized that to receive a worksheet such as that shown in Figure 2 or a diagram such as that shown in Figure 3 could make a difference to the students' capacity to consider what was needed for a proof.



ABCD is a parallelogram. If you draw its angle bisectors, what can you tell about the intersection points of those angle bisectors?

Figure 2: Worksheet as originally given to students

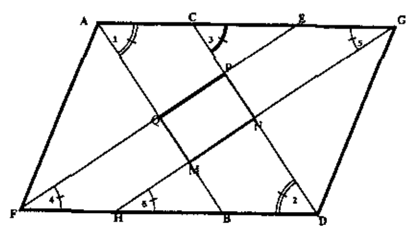


Figure 3: A possible figure given to prove that MNOP is a rectangle

Such a recognition implied differential expectations placed on students – the participants did not think that most students could be expected to look for relationships among elements of the figure unless the relevance of those elements was indicated to them. With this assumption, they provided us with confirming evidence for the conjecture that teachers who are committed to engaging students in proving are disposed to setting up a proof exercise in ways that lay out some of the substance of the proof (see Herbst, 2002a).

We consider the disposition to provide students with some of the substance of the proof as an element of the practical rationality invested in engaging students in proving. This disposition was made visible because the situation of watching this video episode with others who teach students to prove statements in geometry required not only watching and understanding a record, but also investing their practical rationality in recreating crucial missing pieces of the episode. To show how such video episodes may enhance one's chances of eliciting the practical rationality of mathematics teaching, we discuss further how these records/artifacts function in the context of a conversation among practitioners.

Cases and probes

A video episode used to create a context for a conversation among practitioners may create a sense of resemblance with the experience of the viewers, as well as provoke a rupture with it. The efficacy of video episodes in prompting a conversation about the practice of the practitioners involved rests on the capacity of these video episodes to function both as cases and probes.

In observing that the agenda for a conversation summons participants to represent positions, and in assuming that a video episode portrays a reality relevant to those so positioned, we underscore that a conversation refers to more than the particular actions displayed. The agenda – even a default agenda of judging the teaching that is seen – imposes on the gathered practitioners the need to consider the episode as a case of something larger to which they all should be able to relate by virtue of the position they occupy in the educational system.

In order for such talk around a classroom episode to function as a conversation about a practice, participants must identify the characters in the episode with positions that they and their students might occupy and identify the displayed actions with tasks that characterize their own teaching of mathematics. In particular, even when looking at episodes from other cultures, practitioners brought into such conversations are obliged to consider that they might be engaged with an experience that unfolds under comparable institutional conditions as their own teaching. Seen in this way, as an occasion when institutional positions are gathered around a case of their everyday action, conversations open a window into the work of teaching mathematics.

As practitioners' positions are summoned to a conversation, they bring with them taken-as-shared ways of parsing and organizing such experience. Thus, accounts of practice produced through those conversations may yield access to the normal conditions of work that make it possible for teachers to consider words like 'lesson', 'problem' or 'concept' as viable, common categories for talking about practices as different as those of Japanese and American mathematics lessons (Stigler and Hiebert, 1999).

The opportunity to interact about a video episode that records experiences similar to those that participants share and yet requires the reconstruction of pieces of those experiences may lead them to express the categories that ordinarily organize what is believed common in all participants' experience. These categories that practitioners may use so transparently to relate video episodes to their own experience not only include general terms like 'lesson', 'problem' or 'concept'. They may also include standard ways of labeling action (both in terms of what counts as a meaningful action to be labeled and how much detail the label should include; see Mason, 1999).

Geometry or algebra teachers may use expressions like "doing triangle proofs" or "deciding which graphs are functions" to point to actions that they deem to be rather precise. Those expressions which may seem over- or under-specified for people who occupy other positions are, however, sample elements of the common language that practitioners may recognize as appropriate for the position they have been summoned into. A video episode can be seen as a case of their practice in so far as a conversation among practitioners who occupy similar positions to those recorded in the episode might take place in a language similar to the one they might use to express their own experience to colleagues. [3]

But a gathering of practitioners representing a position can only become a real conversation about practice rather than solely about the episode observed to the extent that the practitioners gathered feel they have something particular to say. And this can only happen if the episode can at one and the same time narrate a case akin to what participants do as well as different from what they do. As practitioners are gathered to observe an episode that is represented in ways that come close to their everyday experience, implicit questions are asked of them – "Might this be you?" or "Might this happen in your class?"

The viewing of an episode may create identifications between practitioners and actors in the episode (teacher or students) and precipitate feelings of sympathy as well as feelings of outrage. The episode may as well provoke reactions to the presumption that the episode in any way narrates a story about them (in the sense of being a story about the mathematics teaching that they do). Practitioners may be provoked by the expectation to see action in the video as akin or alien to their own experience. [4]

Thus, as we consider a conversation as a gathering of position representatives in response to an agenda and as appealing to individuals' identification with the episode's characters in spite of the agenda, we think that video episodes are not only cases but also probes comparable to Rorschach ink-blots. That is, they function in a conversation as stimuli for eliciting not so much readings of the objective stories narrated in the episodes but rather as subjective structures that participants use to (re)construct those stories in ways that make sense for those who hold the positions they do.

An episode may indeed probe the sense of resemblance with a viewer's own practice. By negotiating the conditions under which that sense of resemblance can be maintained (or else rejected), viewers may find in their reactions:

> things that [they] might never have owned up to in a cooler discussion. (Pimm, 1993, p 29)

These conversations around videotaped episodes are also interesting because, as practitioners observe and identify with the characters in action, the regulatory presence of the group may help them realize that they are experiencing those feelings in a situation that they participate in by virtue of the position they hold. Thus, the episode is not just a catalyst for unconstrained sympathy, criticism or other personal feelings, but serves as a detonator of constrained reactions, regulated by the need to maintain a sense of resemblance with the group of viewers

These responses expose not just what individuals personally know or feel about the experience recorded in the episode and its relationship to the experiential world of the individual practitioners. Their reactions to the experiences recorded in the video episode also account for why those feelings and thoughts are legitimate for somebody in the position they are representing. By virtue of the viewers' ability to reconstruct the experience of the episode as something akin to his or her own, their reactions to the episode also point at and offer a possible rationale for the issues that are hard to assimilate.

The problem of understanding the meaning of practitioners' accounts of specific events has been addressed from a variety of perspectives. Some have put an emphasis on the subjectivity of the individual practitioner. The presumption is that practitioners' accounts of specific episodes point at more general aspects of their individual identity (say, personal knowledge or beliefs) which explain the opportunities to learn that they might offer their students (e g Cooney, 1985; Thompson, 1992). Other perspectives emphasize structural characteristics of the practice of teaching mathematics and how those characteristics impose conditions on what teachers may do in their classrooms. The presumption is that practitioners' accounts of specific episodes point at more objective structures of instructional systems (say, the didactical contract; Brousseau, 1984, 1997) which explain the opportunities to learn that get developed (Sensevy *et al.*, 2000).

Conversations around records of practice combine both of these perspectives, letting positions speak through the voices of individuals and in so doing flesh these positions out, endowing structure with texture. We suggest that, where the agenda summons participants to engage a language of resemblance, and the episode also probes individuals' sense of resemblance, the kinds of reactions that practitioners manifest as they engage in conversations about video episodes provide glimpses of the practical rationality of mathematics teaching. To illustrate further how the role of video episodes as cases and probes can help elicit the practical rationality of mathematics teaching, here is an extract from the conversation where one geometry teacher responds to an episode by making some comments about his own teaching.

'Moving away from formality' or breaching expectations of order?

The following dialogue shows how, near the end of the conversation about the video episode described earlier, three teachers, Adam, Mia and Paul [5], talk with Herbst and Chazan about students' making problematic assumptions. (Other participants and observers were silent during this interaction – ellipses marked by '…' simply indicate a pause, not excised material.)

Adam: I try to get my class to … move away from some of the formality of the two-column proof. And talk about … you know what we gotta know is this … so let's not get into too much of trying to prove this is true and in some of the idea … And I try to err on the side of … where the crux of the argument really lies and ask them what you have to spell out and not kind of the truer details of it. … So we don't get bogged down with details. And try to get in part of the argument versus get in just some of the details that were sure that step one is right and step two is the right thing and how do you say that and … you know.

Mia: Okay, I'm going to say it. Okay, the one example you gave where the kid just said, okay I know these two angles are complementary so this angle has to be ninety. But he didn't write that formally out into the proof, which would actually take several steps.

Adam: We'd have to dot all the i's and cross the t's

Mia: And you're saying it's better and you try to be more relaxed like that in your own classroom? That's what you're saying?

Paul: But don't you run into 'where you draw the line' problems? Because later on those kids were wrestling with the concept that the bisectors emanating from opposite vertices were parallel and it seemed obvious to them, but do you let them leap over that when …

Adam: No. I think that's where you have to realize where you draw the line. Because you can't leap over that point, because they think that's an important issue to see kind of the chain of the argument and how the chain kind of fits together and to me that was the crux of that particular argument.

Paul: Yeah. I would think that in every classroom you'd have to reach sort of an equilibrium as to … you know. On the one end of the spectrum is spending a whole period just proving that this is a right triangle or this is a triangle that the angles add up to one hundred eighty degrees At the other extreme is just kind of brushing over everything and saying 'Well, it certainly looks true, so let's accept it'.

Patricio: In fact, it was very difficult ... When Eamon said, 'I'm stuck on this step because after this step it will be easy'. What I chose to do was to say, 'well, let's assume it's true and go on', but I had mixed feelings about that all the way through.

Adam: Why don't you think about doing a proof in mathematics? I mean if you take a proof in mathematics you say, 'You know, I'm stuck here, but I'll just assume that if I come back to that I can figure that out, I'll just jump ahead and see if the rest of the thing works out'. And if it works out then you go back and you pick that step up So, I mean, I don't think that's out of the realm of what real mathematicians do

Resemblance to and contrast with what is customary in engaging students in proving

The episode had been presented to the group as a case of engaging students in proving The actions were shown as records from an actual lesson - in the form of five video excerpts Our editing of the video had reduced spans of time in which students were silent, but indicated the amount of time that had elapsed through the changes in video excerpts as well as in the transcript given to participants.

The episode could potentially appeal to participants' identification with the practice displayed in various ways. For example, the task students in the video were undertaking was directly related to concepts that students usually learn in geometry (such as properties of angles determined on parallels cut by a transversal or relations among the angles of a triangle). As with many geometry teachers, the teacher in the videotape expected students to use the two-column format to give a proof The action took place in a regular classroom and students had difficulties knowing how to justify statements in a proof similar to students observed in other situations (see Herbst, 2002a).

As can be seen from the dialogue quoted above, participants took the episode as a case for discussing issues about engaging students in proving which were more general than the particular difficulties encountered with proving the inside quadrilateral to be a rectangle. Expressions such as "the crux of the argument", "the formality of the two-column proof" or "the proof would take several steps" point to a level of discourse more general than the particulars of the proposition being proved At that level, the video episode could create resonance with the proving practices that the participants might consider customary in geometry classes (beyond whether the particular problem displayed in the video episode would normally be solved in a geometry class).

However, the episode showed a lesson where students produced a proof in conditions that in many ways contradicted what Herbst (2002a) has characterized as the customary way the didactical contract on proof is negotiated. The way in which students had come to develop a proof could probe participants' sense of what is normal in engaging students in proving. Through an analysis similar to the one provided by Herbst, we could anticipate why, from the perspective of an observer, some aspects of the presented episode would seem out of place for the participants. Such anticipation built on the notion that the teacher's work engaging students in proving (in U.S high-school geometry, at least) is regulated by a didactical contract that privileges form and sequence in proving.

Using that notion to analyze the task and its enactment, we had identified several aspects of the work of the teacher in the episode that showed him breaching the conditions imposed by the didactical contract. In particular, the teacher had not told the students that they had to prove that the bisectors were parallel. We conjectured that viewers might see how, having drawn the bisectors themselves, students might take for granted that the bisectors just happened to be parallel, and thus not realize that there was a connection with the fact that they had started from a parallelogram, somethng which they should prove

We further thought that viewers would not deem to be normal the teacher's failure to ensure that students acknowledge the need to prove the connection at the outset (e.g. by indicating explicitly what conditions had to be proved) Furthermore, the suggestion that Eamon assume the bisectors to be parallel and that he go on with the proof was something that we anticipated would fly in the face of normal practice

Similarly, we hoped the lack of the presentation of reasons in Adrian's final argument for the parallelism of two bisectors might be considered on the fringes of standard practice Finally, something that we thought was particularly probing about the teacher's management of the production of this proof was the long time he spent having students work on one problem The teacher might have reduced that time had he taken a more active role in pointing out which angles to consider or which properties to use.

We thought, therefore, that those features might probe the participants' sense of resemblance with the teaching recorded on the video, in particular their views of how the proof for the parallelism of the bisectors was developed. Based on that analysis, we introduced each video excerpt with questions or comments meant to focus attention on events that might precipitate participants' reactions. For example, we titled one of the excerpts *I'm stuck on this step, 'cause after this step it will be easy* and came to a cut right after that utterance. We titled the subsequent excerpt *Letting Eamon assume bisectors are parallel: why would you do that?* and included Eamon's utterance once again, followed by the teacher's suggestion that he make that assumption.

In making those choices, we foresaw eliciting participants' sympathy with the teacher's situation of having a co-operative student in difficulty doing a proof at the board We also hoped to probe participants' sense of resemblance with the ensuing teacher action, prompting them to discuss whether and how this episode was similar to but different from what they customarily do Finally, we expected that as participants engaged in such discussion they would explore with us elements of the shared but often unspoken rationality that they invest in constructing what is normal or customary in engaging students in proving

They might, thus, bring to the surface and enable us as observers to start to ascertain what constitutes the boundaries of the mathematics teaching practices ordinarily enacted, as well as the regulatory mechanisms of these practices customarily enacted by those in the positions summoned into the conversation. We might also be able to find out how those mechanisms allow individual practitioners to bring the different commitments, beliefs and knowledge they might hold into their practice in addition to coming to know how things are seen as legitimate within the shared sense of resemblance constructed by a group of peers in the educational system.

In a way, all conversations around video episodes could be examined as providing some degree of access to the rationality that is specific to the practice of those positions summoned to speak The example that we show intends to illustrate that these conversations can also be used as a blueprint to create experiments to explore hypotheses about the practical rationality of mathematics teaching In the case discussed, the conversation provided pilot data yielding some confirmation of and enrichment to the conjectures made about the work that a teacher is required to do when engaging students in proving

3. Practical rationality and proving in geometry

A careful analysis of the conversation we have been referring to is beyond the scope of this article, as our present purpose is simply to illustrate how video episodes can elicit conversations that bring to the surface elements of the practical rationality of mathematics teaching. We want to note, however, how what Adam, Mia and Paul said provides an observer with a chance to see elements of that practical rationality activated and used in a particular situation both to value and regulate variations in the way practitioners engage students in proving.

These participants were able to contribute to the conversation because they had to confront an episode that in specific moments probed their shared sense of resemblance with the teaching displayed, inviting them to react. As they did so in the context of a group conversation, they were pushed toward not just reacting on the basis of what they personally believed, but also on the basis of what might be acceptable for them to say to their colleagues in the group.

What are some of the elements of practical rationality that are visible through those quoted exchanges? Many issues could be commented on, but we concentrate on just one Adam seemed disposed toward appreciating classroom mathematical activity in so far as it relates to "what real mathematicians do" which appeared in particular to justify why it would be acceptable to encourage students to make provisional assumptions.

In the process of arguing that those were indeed grounds for appreciating the episode, he shared the rationale under which the episode would have been regarded as odd. His observation that the class was doing what real mathematicians do was juxtaposed with the presumption that a teacher would customarily make students spell out "the truer details of it", making sure "that step one is right and step two is the right thing and how do you say that"

Without necessarily opposing the relevance of "what real mathematicians do" in valuing classroom activity, Paul pointed to a possible risk in allowing students that choice, noting that, as a result, students might not even recognize that something had to be proved because it seemed obvious to them. According to Paul, it might become a problem, or at least an extra expenditure, for a teacher who allows students to make provisional assumptions to be able later to enforce that those assumptions need to be questioned. Paul pointed to the complications added to the work of the teacher by the need to engage in such negotiations, complications that may feel avoidable when the universal expectation is to proceed through a proof in sequence, justifying everything.

At a previous point in their conversation, Mia indicated that she often tries to persuade her students that geometric proofs are useful to them beyond geometry, that proofs enable students to develop habits of clear and orderly reasoning that they might use elsewhere - a variation of the century-old, mental discipline argument for justifying the need to study geometry (Herbst, 2002b) This helps to understand why she would suggest, in a later exchange with Paul and Adam, that students do see disorganized proofs as outside the realm of what mathematicians do

The disorganization that Adam considered productive and authentic seemed to raise problems for Mia who, thinking on behalf of her students, might adduce that activities that promote disorganization are not those that provide the discipline sought Thus, dispositions toward the authenticity of the activity of proving, the legitimacy of the end product and the possibility of claiming that the activity and the product are useful for students are brought to the discussion as Adam, Mia and Paul discuss the making of a provisional assumption as a step toward proving a conjecture.

The inspection of a conversation like this one can help explain phenomena that are observed in ordinary practice. Why might it be unlikely that a geometry teacher would do what the teacher in the video episode did, namely assign a task that contains several unspecified things to be proved and encourage students to assume they had taken care of one in order to go on? The conversation points to a possible tension that such an action might confront the teacher with. Seeing students engage in activity that exhibits valuable, authentic mathematical characteristics might delight a teacher. But, at the same time, a teacher might also realize that such an action could jeopardize students' ability to know when a result has been proven

A teacher's commitment to mind the mathematical qualities of a task and the requirements of his or her position to mind the legitimacy of the product shaped through the task might come into tension if a teacher were to encourage students to make these provisional assumptions. Thus, the precaution, willingness or reluctance of a teacher to promote such way of working could be justified in terms of how a teacher prioritizes those conflicting elements in the particular circumstances where he or she works

Such prioritization might, according to the situation in which the class is involved, also depend on what the teacher implicitly feels it makes sense to do. Because the episode enabled participants to observe students' mathematical engagement with a proposition which they had to formulate and justify and, at the same time, was predicated as a case of the institutionalized, general skill of 'doing proofs', opposing dispositions were activated

One can observe in classrooms a customary separation between the activities of coming to know (or discovery) and of proving. It might very well be explained as an outcome of the need to exorcise the tensions that those opposing elements might create. If, in contrast to this separation, the symbiotic relationship between proving and coming to know that obtains in the discipline of mathematics were to take hold within school geometry classrooms, teachers may need to learn how to live with, and manage, that tension

The opportunity to foresee how potentially conflicting elements might regulate what teachers (think they might) do in their classrooms is an outcome of the methodological choices we made, namely confronting a group of colleagues involved in similar practices to a video episode where some ordinary characteristics of that practice are altered. [6] Such a way of working has the potential to uncover in the long term a host of the dispositions that regulate a practice such as teaching high-school geometry. In the next section, we discuss how this example and the methodology we use can help conceive of and study more generally the notion of a practical rationality of mathematics teaching.

4. Describing practical rationality: various dispositions

Many educators point out the limitations of standard ways of speaking about teaching (e g. Chazan and Ball, 1999) and suggest that we need better language for describing teaching As Lampert (2001) says:

> But there is as yet no coherent tradition of scholarship whose purpose is to look across stories and identify the complexities of practice in a way that is as multifocal as the work itself; nor is there a professional language that goes very far beyond the anecdote or 'case' for talking about practice in a way that captures the multiple ways in which any teaching action may be working to link students and content. (pp. 27-28)

Yet, it is Lampert herself who has made key contributions in this direction. In her seminal piece, 'How do teachers manage to teach?', Lampert (1985) describes teaching as a practice that requires teachers continually to manage dilemmas in which different commitments enter in conflict Lampert's notions of commitments and dilemmas have found use primarily by scholars studying their own teaching practice and who have access to a large set of records of practice (e.g. Ball, 1993; Chazan and Ball, 1999). In these circumstances, the practitioner has direct access to his or her own commitments and experiences of tensions in teaching. Furthermore, he or she may be able to claim that there exist relationships between the commitments in tension in an episode of practice and dilemmas that are operative beyond the particular actions in the episode.

But what is a dilemma for one teacher is not necessarily a dilemma for another And teachers who might seem to share commitments often mean quite different things despite using the same words Furthermore, sometimes people act in ways that an observer can describe as coping with (or avoiding) a tension or dilemma, though they are not necessarily conscious of such a situation (Cohen, 1990; Herbst, in press). To what extent, and on what basis, can relationships between commitments and dilemmas be hypothesised to exist beyond the limits of one person's practice or consciousness? To what extent are those relationships between commitments and dilemmas arguably likely or unlikely in particular *kinds* of teaching whose general characteristics are being sought at the same time that its dilemmas are being identified in specific episodes?

As suggested by descriptions like those of Ball (1993), Chazan and Ball (1999) and Herbst (in press), the notion that teachers have to manage dilemmas or tensions that are characteristic of the practice of teaching is a useful tool to generate *post-hoc* explanations of teachers' actions Yet, to be able to account for change and stability in classroom practice requires developing conceptual tools to *anticipate* those actions that are possible or can be expected and to estimate the likelihood of those actions. Indeed, all conceivable actions are not created equal: some may feel natural, others acceptable, yet others regrettable and others still not even viable.

With these goals in mind, we return to the context of conversations with practitioners around video episodes of classroom practice. We have outlined how conversations around video episodes might open windows into the practical rationality of mathematics teaching Practitioners' words and deeds in the context of such conversations may amount to saying 'This is how I feel about what I have seen and this is why it is proper or improper for somebody in my position to feel like that'. The institutionally bound position and the experientially situated person are able to speak at the same time, contributing both the stable, formal relationships of the position and the transient feelings of kinship of the person to the conversation. They can do that because a practical rationality, one characteristic of the roles that actual teachers play in their positions through time, is shaping those actual utterances so that they are acceptable for the many voices that they have to articulate.

Building on Lampert's notion of commitments, we draw on the notion of *dispositions* from the reflexive sociology of Pierre Bourdieu as a term to describe a scholar's hypotheses about elements of practical rationality. Dispositions are "categories of perception and appreciation" (Bourdieu and Wacquant, 1992, p. 11) that help a scholar reconstruct the regulatory mechanisms of action in context. Dispositions are neither individual commitments nor institutional requirements; they are *like* requirements in so far as they create a sense of intersubjective normality but they are implicit; they are *like* commitments in that they accommodate personal preferences, though they are also transposable among people who do similar work. [7]

Dispositions are shaped through time as agents work in institutions, but are activated for particular situations lived by individuals. We postulate that conversations about a specific event activate certain dispositions in practitioners that help them say what they think or feel within the situation where they are saying it. By making hypotheses regarding what those dispositions could be, through inspecting and comparing specific utterances in these conversations, we can describe practical rationality as a network of dispositions activated in specific situations and anticipate possible tensions that might surface in the context of particular lessons that embody those situations

Practitioners' conversations about video episodes in which competing dispositions have been hypothesised as being activated can help confirm, refute and refine those hypotheses. Dispositions uncovered in the way individual practitioners respond to episodes might also help predict what kind of dilemmas teachers might have in the future as they engage in actions that activate those dispositions Such conversations are places to look for more general statements of the elements in tension that produce dilemmas (in a practice of teaching broader than the actual episode, broader than the practice of the individual featured in the episode). This may help express dilemmas not just as proper to an episode at hand but rather as endemic to the work involved in (a 'kind' of) mathematics teaching (see Ball, 1993).

Postulating the existence of a practical rationality of mathematics teaching helps conceive of it as a unique practice that accommodates diversity within resemblance at the price of positioning and valuing different 'kinds' of teaching in different ways To pursue a theoretical characterization of this practical rationality as a network of dispositions can provide the means not only to describe action in context and explain its phenomena after the fact, but also explain in advance how some of those phenomena result from conflict or differential prioritization of activated dispositions

A 'kind' of teaching might thus be exemplified in particular ways of acting in particular contexts - and it could be described as a network of differentially prioritized dispositions that produces it. The consistency of such a characterization could be probed and improved on through prediction and experimentation that makes structured use of conversations about video episodes Such a way of building a theory of the practice of mathematics teaching can also help understand problems of change and stability in teaching practice, as well as appreciate better what it takes to enhance communication across 'kinds' of teaching.

Acknowledgements

The authors acknowledge comments from Elizabeth Moje on a previous version of this article Patricio Herbst acknowledges the support (REC 0133619) and Dan Chazan the partial support (REC 9805289) of NSF grants in the writing of this article.

Notes

[1] All student names are pseudonyms

[2] Angles were not marked or numbered on the board; the authors have introduced those marks to facilitate the readers' understanding of Adrian's indexical expressions

[3] Similar things may occur in more diverse groups of observers as well, but the dynamics might be a bit different Imagine, for example, being a former teacher in a diverse group of educators and feeling that one must speak from that position, regardless of whether or not one is still, truly, a teacher.

[4] By saying this, however, we are not suggesting that practitioners must feel a complete kinship with the teaching that they see in a video episode We are suggesting that some aspects of episodes might elicit these feelings and those might be interesting beyond their relationship with the agenda of the convenor of the conversation

[5] Mia and Adam were at the time teaching geometry and Paul had taught it in the past (all teacher names are pseudonyms).

[6] The ethnomethodological notion of 'breaching experiment' (Mehan and Wood, 1975, pp 23-27) has been a source of inspiration in thinking about this strategy.

[7] Terms like 'commitments' and 'requirements' are particularly useful as interface terms to inquire about dispositions - they support the interaction between observer and practitioners in structuring a conversation (giving language as well as creating ways to refine hypotheses): for example, "Your initial joy seeing that Ms. B pursued S's idea seemed to indicate your commitment to take students' thinking seriously, but then I sensed some of you were expecting her to do something other than what she did"

References

Antonioni, M. (1966) *Blow Up* [Film], produced by Carlo Ponti.

Ball, D (1993) 'With an eye on the mathematical horizon: dilemmas of teaching elementary school mathematics', *The Elementary School Journal* **93**(4), 373-397.

Ball, D. and Bass, H (2000) 'Making believe: the collective construction of public mathematical knowledge in the elementary classroom', in Phillips, D (ed.), *Constructivism in Education: Opinions and Second Opinions on Controversial Issues* (Yearbook of the National Society for the Study of Education), Chicago, IL, University of Chicago Press, pp. 193-224.

Ball, D. and Lampert, M (1999) 'Multiples of evidence, time, and perspective: revising the study of teaching and learning', in Lagemann, E. and Shulman, L (eds), *Issues in Education Research: Problems and Possibilities,* San Francisco, CA, Jossey-Bass, pp 371-398.

Bourdieu, P. (1985) *Distinction: a Social Critique of the Judgement of Taste* (trans Nice, R.), Cambridge, MA, Harvard University Press

Bourdieu, P. (1998) *Practical Reason,* Palo Alto, CA, Stanford University Press

Bourdieu, P. and Wacquant, L. (1992) *An Invitation to Reflexive Sociology* Chicago, IL, The University of Chicago Press.

Brousseau, G (1984) 'The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics', in Steiner, H.-G (ed.), *Theory of Mathematics Education* Occasional paper 54, Bielefeld, Germany, IDM, pp 110-119

Brousseau, G (1997) *Theory of Didactical Situations in Mathematics. Didactique des Mathématiques 1970-1990* (Balacheff, N., Cooper, M., Sutherland, R. and Warfield, V, eds and trans), Dordrecht, Kluwer

Chazan, D (1990) 'Implementing the Standards: students' microcomputer-aided exploration in geometry', *Mathematics Teacher* **83**(8), 628-635

Chazan, D. (1995) *Where Do Student Conjectures Come from? Empirical Exploration in Mathematics Classes,* East Lansing, MI, National Center for Research on Teacher Learning, Michigan State University.

Chazan, D and Ball, D (1999) 'Beyond being told not to tell' *For the Learning of Mathematics* **19**(2), 2-10

Chazan, D, Bethell, S and Lehman, M. (eds) (in preparation) *Embracing Reason. Egalitarian Ideals and High School Mathematics Teaching.*

Chevallard, Y (1991) *La Transposition Didactique: du Savoir Savant au Savoir Enseigné* (2nd edn), Grenoble, La Pensée Sauvage [The Didactical Transposition: from the Knowledge of the Expert to the Knowledge Taught]

Cohen, D (1990) 'A revolution in one classroom: the case of Mrs. Oublier', *Educational Evaluation and Policy Analysis* **12**(3), 327-345

Cooney, T. (1985) 'A beginning teacher's view of problem solving', *Journal for Research in Mathematics Education* **16**(5), 324-336.

Cortázar, J. (1959/1983) 'Las babas del diablo', in Cortázar, J., *Ceremonias,* Buenos Aires: Seix Barral, pp 201-215. [The devil's slobber]

Davis, P. and Hersh, R. (1981) *The Mathematical Experience,* Boston, MA, Birkhäuser.

Gillies, D. (ed) (1992) *Revolutions in Mathematics* Oxford, Clarendon Press

Hall, R (2000) 'Videorecording as theory', in Kelly, A and Lesh, R (eds), *Handbook of Research Design in Mathematics and Science Education* Mahwah, NJ, Lawrence Erlbaum, pp 647-664

Heaton, R. (1992) 'Who is minding the mathematics content? A case study of a fifth-grade teacher', *The Elementary School Journal* **93**(2), 153-162
Herbst, P. (2002a) 'Engaging students in proving: a double bind on the teacher', *Journal for Research in Mathematics Education* **33**(2), 176-203
Herbst, P. (2002b) 'Establishing a custom of proving in American school geometry: evolution of the two-column proof in the early twentieth century', *Educational Studies in Mathematics* **49**(3), 283-312.
Herbst, P. (in press) 'Using novel tasks in teaching mathematics: three tensions affecting the work of the teacher', *American Educational Research Journal*
Hiebert, J, Gallimore, R and Stigler, J. (2002) 'A knowledge base for the teaching profession: what would it look like and how can we get one?', *Educational Researcher* **31**(5), 3-15.
Kauffmann, S. (2002) 'Photography and facts', *The New Republic* **227**(2-3), 24
Lakatos, I. (1976) *Proofs and Refutations: the Logic of Mathematical Discovery,* Cambridge, Cambridge University Press.
Lampert, M. (1985) 'How do teachers manage to teach? Perspectives on problems in practice', *Harvard Educational Review* **55**(2), 178-194.
Lampert, M (1990) 'When the problem is not the question and the solution is not the answer: mathematical knowing and teaching', *American Educational Research Journal* **27**(1), 29-63
Lampert, M. (1992) 'Practices and problems in teaching authentic mathematics', in Oser, F, Dick, A and Patry, J.-L. (eds), *Effective and Responsible Teaching. the New Synthesis,* San Francisco, CA, Jossey-Bass, pp 295-314.
Lampert, M. (2001) *Teaching Problems and the Problems of Teaching.* New Haven, CT, Yale University Press
Lampert, M. and Ball, D (1998) *Teaching, Multimedia and Mathematics: Investigations of Real Practice,* New York, NY, Teachers College Press
López-Real, F and Leung, A. (2001) 'Reflections on a TIMSS geometry lesson', *For the Learning of Mathematics* **21**(3), 25-31
Mason, J (1999) 'The role of labels in promoting learning from experience among teachers and students', in Burton, L (ed), *Learning Mathematics: from Hierarchies to Networks.* London, Falmer Press, pp. 187-208
Mehan, H. and Wood, H (1975) *The Reality of Ethnomethodology.* New York, NY, John Wiley and Sons
NCTM (1989) *Curriculum and Evaluation Standards for School Mathematics,* Reston, VA, National Council of Teachers of Mathematics
NCTM (2000) *Principles and Standards for School Mathematics,* Reston, VA, National Council of Teachers of Mathematics
Nemirovsky, R, Lara-Meloy, I., Earnest, D and Ribeiro, B. (2001) 'Video-papers: investigating new multimedia genres to foster the interweaving of research and teaching', in van den Heuvel-Panhuizen, M. (ed.), *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education.* Utrecht, The Netherlands, Freudenthal Institute, **3**, pp. 423-430
Pimm, D. (1993) 'From should to could: reflections on possibilities of mathematics teacher education', *For the Learning of Mathematics* **13**(2), 27-32.
Polya, G (1945) *How to Solve It, a New Aspect of Mathematical Method,* Princeton, NJ, Princeton University Press.
Popkewitz, T. (1988) 'Institutional issues in the study of school mathematics: curriculum research', *Educational Studies in Mathematics* **19**(2), 221-249.
Remillard, J (1992) 'Teaching mathematics for understanding: a fifth-grade teacher's interpretation of policy', *The Elementary School Journal* **93**(2), 179-193.
Schön, D. (1983) *The Reflective Practitioner: How Professionals Think in Action,* New York, NY, Basic Books.
Sensevy, G., Mercier, A. and Schubauer-Leoni, M.-L (2000) 'Vers un modèle de l'action didactique du profeseur: à propos de la course à 20', *Recherches en Didactique des Mathématiques* **20**(3), 263-304
Simon, M and Tzur, R (1999) 'Explicating the teacher's perspective from the researchers' perspectives: generating accounts of mathematics teachers' practice', *Journal for Research in Mathematics Education* **30**(3), 252-264.
Stigler, J. and Hiebert, J (1999) *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom.* New York, NY, Free Press.
Thompson, A. (1992) 'Teachers' beliefs and conceptions: a synthesis of the research', in Grouws, D (ed.), *Handbook of Research on Mathematics Teaching and Learning,* New York, NY, Macmillan, pp. 127-146
Thurston, W (1994) 'On proof and progress in mathematics', *Bulletin (New Series) of the American Mathematical Society* **30**(2), 161-177 (Reprinted in *For the Learning of Mathematics* **15**(1), 29-37)
Tymoczko, T (ed) (1986) *New Directions in the Philosophy of Mathematics: an Anthology,* Boston, MA, Birkhäuser.
Weber, K (2002) 'Beyond proving and explaining: proofs that justify the use of definitions and axiomatic structures and proofs that illustrate technique', *For the Learning of Mathematics* **22**(3), 14-17.