

- Sarason, S. B. *The Culture of the School and the Problem of Change*. Boston: Allyn and Bacon, 1971
- Sawyer, W. W. On being your own teacher. In W. W. Sawyer (Ed.), *Mathematics in Theory and Practice*. London: Odhams Press Ltd., 1948
- School Mathematics Study Group. *Mathematics for Junior High School Volume I*, (Teacher's Commentary). New Haven: Yale University Press, 1961
- Servais, W. Continental traditions and reforms. *International Journal of Mathematics Education in Science and Technology*, 1975, 6, 37-58
- Sharron, S. (Ed.). *Applications in School Mathematics 1979 Yearbook*. Reston, Virginia: National Council of Teachers of Mathematics, 1979
- Shulte, A. P. (Ed.). *Teaching Statistics and Probability*. Reston, Virginia: National Council of Teachers of Mathematics, 1981
- Smith, B. O. Teaching strategies: historical and contemporary perspectives. In T. J. Cooney (Ed.), *Teaching Strategies*. Columbus, Ohio: ERIC Center for Science, Mathematics, and Environmental Education, 1976
- Sobel, M. A. and Maletsky, E. M. *Mathematics II*. Toronto: Ginn and Company, 1972
- Stake, R. E., Easley, J. A. *Case Studies in Science Education* (2 vols.). Center for Instructional Research and Curriculum Evaluation, University of Illinois, 1978
- Steen, L. A. Mathematics today. In L. A. Steen (Ed.), *Mathematics Today. Twelve Informal Essays*. New York: Springer-Verlag, 1978a
- Steen, L. A. Math is a four letter word. *The Mathematical Intelligencer* 1978b, 1, 171-172
- Stevens, J. G. and Garfunkel, R. Summary and curricular implications: An outgrowth of articles by Thom and Dieudonné. *The Mathematics Teacher*, 1975, 68, 683-687
- Swetz, F. (Ed.), *Socialist Mathematics Education*. Southampton, Pennsylvania: Burgundy Press, 1978
- Thorndike, E. L. *The New Methods in Arithmetic*. New York: Rand McNally, 1921
- Thwaites, B. *The School Mathematics Project: The First Ten Years*. London: Cambridge University Press, 1972
- Usiskin, Z. *Algebra Through Applications*, 2 vols. Reston, Virginia: National Council of Teachers of Mathematics, 1979
- Usiskin, Z. What should *not* be in the algebra and geometry curricula of average college-bound students? *The Mathematics Teacher*, 1980, 73, 413-424
- Von Neumann, J. The formalist foundations of mathematics. In P. Benacerraf, and H. Putnam (Eds.), *Philosophy of Mathematics*. Englewood Cliffs, N. J.: Prentice-Hall Inc., 1964
- Waller, W. *The Sociology of Teaching*. New York: Wiley, 1965
- Wang, H. Kurt Gödel's intellectual development. *The Mathematical Intelligencer*, 1978, 1, 182-184
- Watson, F. R. Aims in mathematical education and their implications for the training of mathematics teachers. *International Journal of Mathematics Education in Science and Technology*, 1971, 2, 105-118
- Weber, M. *The Theory of Social and Economic Organization*, (T. Parsons Ed.). New York: The Free Press, 1964
- Wheatley, G. H. Calculators in the classroom: a proposal for curricular change. *Arithmetic Teacher*, 1980, 28(4), 37-39
- Whitehead, A. N. The aims of education — A plea for reform. *The Mathematical Gazette*, 1916, 8, 191-203
- Wilder, R. L. *Evolution of Mathematical Concepts: An Elementary Study*. New York: Wiley, 1968
- Williams, E. An investigation of senior high school students' understanding of the nature of mathematical proof. *Journal for Research in Mathematics Education*, 1980, 11, 165-166
- Wilson, B. J. Change in mathematics education since the late 1950's — Ideas and realisation (West Indies). *Educational Studies in Mathematics*, 1978, 9, 355-379
- Wootton, W. *SMSG: The Making of a Curriculum*. New Haven Connecticut: Yale University Press, 1965

Attention

JOHN MASON

"Johnny, pay attention!"

"I must attend to . . ."

"Mind your own business!"

"Nicholas, concentrate!"

Just what is meant by this word attention? It is used frequently with such words as attract, call, draw, arrest, fix and pay, and according to the Oxford English Dictionary the verb *attend* appears in each of its usages both as a transitive and intransitive verb. Common expressions and idioms often give a hint of deeper meaning, and in this case there is similar evidence. The prevalence of both active and passive forms suggests uncertainty as to the nature of attention. Is it some *thing* which I control, or is it a more intimate part of that I? Just what is expected of a soldier when ordered to stand at attention? Paying and attracting attention suggest some thing is given or expended, but when Johnny is ordered to pay attention, is he paying for something? Is knowledge being poured in his "attention spout"? Or does he pay attention in order to receive attention from the teacher? Has the teacher reciprocated by paying attention to Johnny? Minding your own business, or your step, suggest that attention is part of, or resides in the mind, whatever that is! Exhorting Nicholas to concentrate suggests that

he is doing the opposite, but is he diluting, or is he skittering butterfly like, unable to settle down?

I have recently come to the conclusion that attention is one of the most important aspects of learning. This observation is clear if not trite in retrospect, but it raises an interesting question. Just what is attention? More particularly,

Are there levels or varieties of attention?

Is attention a voluntary matter?

What follows are some rough laconic notes as a first attempt to expose these questions, beginning with some reasons for why attention is of importance to people concerned with the learning of mathematics.

The importance of attention

At the end of the article "When is a symbol symbolic" [1] I came to the conclusion that the idea of three modes of representation — enactive, iconic, symbolic — helps me to explain or rather to describe events in which students fail to accomplish tasks that a teacher expects them to be able to do easily. A classic example in physics teaching involves the use of ratios or proportions, and a similar situation occurs when a student is asked to write out the question before he begins, but is having trouble with his writing. In both cases

the student's attention is totally occupied by one aspect of the task at hand, and so no attention is available for the remaining aspects which are essential to complete the task. If I am being asked to manipulate symbols which are for me symbolic, in the sense that they are but weakly associated with meaning, whether in images or in recent experience with confidently manipulable objects, then all of my attention must be devoted to trying to hang on to the meaning and import of those symbols. It is therefore most unlikely that I will have any room for observing pattern, or relating my symbols to the original question, or having enough attention to undertake and direct the manipulation of those symbols towards some goal. C. Gattegno [2] says something similar:

Thus our process of learning in the realm of knowhows leads to automatisms which make awareness available for other realms and for the use of one's past...

A similar observation is made by Brian Greer [7] who points out the general phenomenon of students responding to "surface features" of a question rather than to a logical or deeper structure, as exemplified particularly in researches by Krutetskii and Janvier. Surely this is connected with the range and focus of students' attention. Attention absorbed by basic skills triggered by a question can only cope with surface features. It does not permit any deeper contact.

One of the first functions of any teacher with a class must be to get the students involved in some relevant task. Many discussions and descriptions of how to get students motivated have appeared, but those that I have looked at have always seemed to me to be unsatisfactory. I recall in the early days of the Open University hearing experienced teachers-turned-authors talking about providing an example at the beginning of a section in order to motivate students. Their aim was to get the students involved somehow. It seems to me that this is really a question of focusing or attracting attention.

A common conflict between Mathematics Departments and Education Departments which arises in planning courses for would-be or in-service teachers is, I believe, based on a fallacy connected with attention. The mathematicians claim that the best way of improving the teachers is to provide them with mathematics courses which extend the courses they have previously studied. In this way the student will gain a greater appreciation of mathematical structure, of where the mathematical ideas that they teach can lead, and a better grasp of mathematics as a process. Educationalists on the other hand tend to concentrate their courses on broader educational interests and frequently leave very little room for specific mathematical instruction. Presumably this is a response to noticing that student teachers are unable to relate the content of mathematics courses to the problems of teaching at their particular level. The mathematicians in my view fail to appreciate that the kinds of courses which they generally provide for teachers make great demands on the students' attention. So much so that the students have to work very hard just to keep up with the content, and there is no time or attention available to be aware of the general mathematical process. Indeed it is true

that very few mathematics courses ever refer explicitly to processes at all. Consequently it is not surprising that students of such courses have only the vaguest sense or appreciation of the nature of mathematical inquiry. Rarely is time given to reflecting on the difficulties of learning or on the processes of investigation. To put it in another way, one of the hardest things to do is to "learn from experience" because usually attention is totally embroiled in the experiencing! Unless time for reflection is provided, learning from experience does not take place.

What is attention?

I have struggled with this question on and off for some time, and have been unable to answer it. Attention is a veritable "will o' the wisp". I become aware that my attention has been drawn to a sudden sound, to a movement, or to an old problem which has been sitting at the back of my mind. In each of these cases it seems that attention, whatever it is, is drawn or attracted. It does not seem to be under my control. If someone says to me "Pay attention" then my attention is drawn by the sound to the event at hand. But very soon my attention wanders off again. A classic example of this is the watch-watching experiment attributed to P. D. Ouspensky. To try it, focus your attention on the sweep second hand of a watch. See how long you can fix your attention totally on the second hand.

Have you tried it? Most people find that they can accomplish it for only a few seconds! A minute or two is exceptional. Frequently there is a pink elephant phenomenon — for how long can you *not* think of a pink elephant? In the case of the watch there slips into my head the statement "I am watching this watch", or "Look at me; how well I am paying attention", which of course immediately indicates that not all of my attention is focused on the watch. I will return to the question of split attention shortly.

Being able to make little progress by a direct attack, I have found the question changing, in true Polya style, to a similar question based on some writings of J. G. Bennett [3]. He introduces the idea of PRESENT MOMENT, which is best explained by examples

- (1) *A fly is buzzing at the window. My hand reaches for the fly swatter and my entire world focuses on the buzzing of the fly and trying to hit it. Thoughts and activities of a few seconds ago vanish. The totality of my awareness in the moment constitute my present moment.*
- (2) *In front of me is a page of symbols trying to capture the essence of a problem. Something seems to be wrong with the calculations. As I stare at the page and fiddle, an inner voice begins to give the lecture that I may give on this topic when (if!) it is ever solved.*
- (3) *I gaze at a field of wheat waving in the sunshine. I become aware that this will be harvested, threshed, ground into flour, baked into bread and eaten. I also become aware of the long history of farming stretching into the past, and I have a sense of connection with all of those people.*

In each of these examples the field of my awareness is what Bennett calls present moment. You can see that there is a sense in which the present moment can vary tremendously

in scope — it can be focused intently on one fly, or it can expand to encompass a long period of time. Notice though that *in the moment* there is no sense of scope because the present moment is by definition the totality of my awareness.

It is true that in each of the examples I have given there is an ambiguity, because it is not clear whether I am describing something after it has taken place or whether I am describing a perception in the moment. In fact in each case I suddenly became aware of what I have described. So at that moment my present moment altered its scope, encompassing both the previous awareness, and awareness of that awareness.

How are attention and Present Moment related?

Is Present Moment the object of attention? This seems to be a reasonable question, yet it is based in the subject-predicate-object structure of English which is frequently misleading. In this case it also implies a subject which attends. Now many authors, including Gattegno [4] and Bennett [5] identify attention with will. However, to come to this perception in the face of attention being constantly drawn and attracted without any sense of control requires accepting that I am more will-less than will-full. It seems to me that a helpful staging point along the way to considering whether attention is a voluntary matter would be to *identify* Attention and Present Moment. Treating them as the same has two virtues. Firstly it changes the question of attention, in true Polya style, into a similar but different question, because it is much easier to become aware of and to recall the extent of my Present Moment than to catch my Attention as a thing to be observed. Secondly it avoids the subject-object hypothesis and so delays the thorny question of will.

Levels or kinds of attention

Very careful observation shows that many experiences which we would like to call levels of attention actually take the form of attention on one thing (let us say the page of mathematics) being diverted to another thing (say the fantasy of giving the lecture) followed by a vague awareness of the diversion. A truly split attention in which the fantasy continues in parallel with awareness of the continuation of the fantasy is actually rare. Rather, along with the fantasy there is a vague awareness well below the level of articulation. In my experience there is a broad spectrum of awareness which is best illustrated by the experience of being stuck on a problem. I sit in front of a page of paper, perhaps with some scribbling on it. The page is at some distance from me and I feel separated from it. There is a gulf between me and it. A vague sense that no progress is being made begins to grow inside me. I become aware that I am not making progress, but the awareness is so diffuse that it takes the form of a gnawing than of anything which can be articulated. This awareness gradually increases to the point of articulation that “I am stuck.” Still I remain at my desk. As the strength behind the statement “I am stuck” increases, the gulf becomes more tangible and less all-engrossing. At a certain threshold of intensity it becomes possible to seek for assistance, in the form of a colleague, a text, or a change of activity. In contrast to this spectrum of

unfocused awareness there are moments when my attention is focused on something, as in each of the three examples given earlier, and suddenly there is a simultaneous focused attention which is aware of my attending to a particular object.

Readers of Castaneda [6] will know of the importance placed on the split attention induced in him by Don Genaro and Don Juan. Readers of Gattegno [4] will know of the importance placed by him on awareness of awareness. It seems to me that a good deal of care is required in order to appreciate what is intended.

Motivation and concentration

Another aspect of the nature of attention is most easily observed in what we might call the strength of concentration. Normally we speak of someone who concentrates intensely as being highly motivated. Now to motivate means to move or to provide energy. However, in respect of attention it is not clear to me whether it is more appropriate to say that students are moved, which implies somehow pushed, or whether their attention is being attracted and focused. An image of a donkey and carrot comes to me. Is attention like beating the donkey or otherwise forcing it to move, or is it like the carrot attracting the donkey?

Inner attention

Having been attracted by a variety of things going on inside my head when ostensibly working on a mathematical investigation, I think I can distinguish three kinds of activities. Firstly there is inner chatter, often described as “the chattering monkeys.” My lecture-giving fantasies are generally of this form. There are also inner images: not necessarily intense pictures but perhaps better described as forms in the platonic sense, or as a sense of pattern or relationship. There is a strong analogy here with dreams of both night and day varieties, which comprise images. A third form of inner activity is a sense of my body in space. This includes both the sensation of my weight on the floor and of the objects around me. In order to come to grips with attention, it seems likely to me that something will have to happen regarding these inner activities.

I seem to have come some distance from questions about learning mathematics. However, as I collect more and more protocols of my work, more and more inner snapshots of the states which arise during mathematical investigation, I am struck by the importance and significance of the question of attention. I look forward to seeing more lucid and helpful comments by other readers on this question.

References

- [1] J. H. Mason, When is a Symbol Symbolic? *For the Learning of Mathematics* 1, 2 (1980): 8-12
- [2] C. Gattegno, The Facts of Awareness, *The science of education* chapter 3, p. 15. New York: Educational Solutions Inc., 1977
- [3] J. G. Bennett, *The dramatic universe* vol III. London: Hodder and Stoughton, 1966
- [4] C. Gattegno, Awareness, *The science of education* chapter 2. New York: Educational Solutions Inc., 1977
- [5] J. G. Bennett, *Deeper man*. London: Turnstone 1980
- [6] C. Castaneda, *The tales of power*. New York: Simon and Schuster 1974
- [7] B. Greer, Cognitive Psychology and Mathematical Thinking. *For the Learning of Mathematics* 1, 3 (1981): 19-26