Knowledge at the Crossroads

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Rigorous mathematics began with the Greeks. Greek mathematics began with Thales. What did Thales see that no one before him had seen?

There are texts by Diogenes and by Plutarch, though they give different details, that tell of Thales measuring the height of a pyramid by the shadow it casts. Auguste Comte suggested that the science of mathematics arose in the first place because people could not measure directly most of the heights and distances they wanted to know. From this point of view the geometry Thales used was a ruse, a device to reach something beyond experience. The ruse (the geometry) is, as it were, a model, a scale, a type of ladder; it copies the inaccessible into the realm of the accessible. The operation of measuring has been transported from one realm to the other.

We might say the height of the pyramid is inaccessible because we cannot touch it, so in order to measure it, using a ruse fashioned by our reason, we imagine an accessible substitute. We attempt similar transportations to find the altitude of the sun, the distance across a river, the distance to the horizon. Mathematics is our name for the paths taken by these ruses. But even direct measurement of an accessible object involves more than touch: the alignment of a measuring instrument has to be checked by sight. Thales's method of measuring the pyramid's height gives us a visual representation of what we cannot reach. He offers us a theory that involves "sighting" (the Greek terms remind us that to theorize is to see), lines of sight, the casting of shadows. The underlying idea is again that of transportation.

As the sun moves, the shadow of the pyramid moves too. The shadow of the fixed pyramid gives information about the moving sun, and hence about time, like a sundial. But Thales reverses the implication and uses the sun to give information about the pyramid. Time is stopped and geometry begins.

That roughly paraphrases the first four pages of a chapter by Michel Serres, the French historian of science. A handful of his essays have been collected and translated into English in the book under review[1]. Available for ten years, it does not appear to have generated much discussion by mathematicians or mathematics educators. Difficult to summarize because it is already the result of several condensations, Serres's thought is startling in its juxtapositions but luckily not, as in the customary French manner, strenuously opaque.

A feature of his style is the returning again and again to a particular point of interest, always by a different route. There are three - or is it four? - characterizations of Thales's theorem in the short passage represented above. The chapter continues with more.

The pyramid reminds us of the Egyptians, and of the folklore that tells us Greek merchants acquired much of their knowledge from the Egyptian priests. Thales' theorem becomes a myth about the origins of geometry.

The theorem is only true in the space of similarities. The pyramid builders must have used homothety and therefore must have "known" Thales' theorem, which is like a secret hidden in the hands of the builders, in the shadow of the pyramid. The theorem dramatizes the connection between the shadow and the sun, between practice and theory, but in the last resort measurement can only compare like with like, shadow with shadow, and cannot resolve the question of the practice-theory relationship.

The insubstantial shadow carries information about the solid pyramid. How else can we study volumes but by working on their plane projections?...

Weaving a web of autonomous "readings" is an essential part of Serres's method. It yields even more unexpected illumination when the contributory strands draw upon alternative locally-valid knowledge domains. The struggle for knowledge can rely only on these regional epistemologies, so coherence and rigour are regional too. The usual classifications of knowledge break down. Knowledge accumulates in pockets at the intersections of multiple paths, at the crossroads where several independent lines of enquiry meet.

Five of Serres's major writings are named for the mythical figure of Hermes. The multiple roles of the god in Greek mythology are indicators of Serres's methodology. Hermes is the divine messenger, the traveller's guide and the guardian of crossroads; he is the god of orators and of thieves; he is cunning and ingenious; he is the protector of shepherds.

Serres holds that the physical and human sciences are not inherently divorced and at odds with each other, but the connections are tortuous and difficult to trace. His writings about the elucidation of the connections call on powerful geographical images of awkward terrain and dangerous seas. A volume in the *Hermès* series is titled "North West Passage"; he talks of human knowledge as "islands sown in archipelagos on the noisy, poorly understood disorder of the sea"[2]. Traversal of this kind of world demands an epistemology of displacements and journeys rather than of concepts and progressive logical structures, of movement rather than fixedness, of complexity rather than unity. He speaks warningly of the "dogmatism of unified and systematic knowledge". The books of human knowledge are better regarded not as treatises but as encyclopedias.

Another metaphor that Serres frequently calls on is that of a Rosetta Stone. One side of such a "stone" carries the leg-