ATTENDING TO THE AESTHETIC IN THE MATHEMATICS CLASSROOM

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[Mr. S begins §3.5, "Solving equations with variables on both sides."]

Mr. S: The book says that if you find variables on both sides, eliminate them from one side. [He writes $3/8 - 1/4x = 1/2x - 3/4$ on the board and asks students how they could solve the equation.]

Nicole: Subtract $1/2x$ from both sides of the equation. [Nicole provides more instructions, which Mr. S. follows.]

Mr. S: Is there something we could do to make it more efficient? You'll find that as we go through the book you'll have more techniques – you get weapons to defeat the algebra beast. I would have multiplied the whole thing by 8. That's a secret weapon I have in my pocket.

How do aesthetic considerations play out in the mathematics classroom? This question, and my initial attempt to answer it here, represents a change in focus from previous work, in which I have argued that the aesthetic is fundamental to mathematical thinking and learning (see Sinclair, 2001, 2004, 2006). That work identified three characteristic ways in which mathematicians (including student-mathematicians) engage their aesthetic sensibilities in the process of posing and solving problems. It defined the aesthetic as a guiding 'sense of fit' (see Gombrich, 1979, in the realm of the arts and Wechsler, 1978, in the realm of the sciences) that intermingles cognition and affect – feelings are essential components of aesthetic responses.

Now I wish to shift the focus from the aesthetic responses of an individual doing mathematics to the aesthetic values communicated at the whole classroom level. From a pragmatist point of view, aesthetic values are involved in choices we make about what constitutes satisfactory and desirable outcomes (see Cherryholmes, 1999; Dewey, 1934). Since aesthetic values are subjective and contextualized, they are developed and shared in social interactions. It is in the normative practices of the classroom that students will learn to be aware of aesthetic considerations – or not.

In the short transcript of a classroom episode presented above, even without invoking an explicit language of aesthetics (words such as *beauty* and *elegance*), we see the teacher communicating to students that he values efficiency in solving mathematical problems. His words imply that it is not just the answer that matters, or its correctness, but the way of getting that answer. In fact, more is at stake than efficiency; this is about "defeat[ing] the algebra beast," exterminating those ugly fractions and long, arduous methods of manipulating symbols. By using one simple multiplication to turn the equation above into $3 - 2x = 4x - 6$, the teacher communicates to the students that certain forms are easier to work with than others. When the students see what happens to the equation after he applies his "secret weapon," they respond with "ooohs" and "aaahs" – one students says "I like that way." The computer might not distinguish the two forms, but the thinking and feeling human being would much rather work with integers.

One way in which to view my question about aesthetic concerns in the classroom is to link it to research on the development of socio-mathematical norms. Yackel and Cobb (1996) conceptualise this development in terms of the ways in which students learn what is mathematically different, sophisticated, efficient and elegant. However, they focus primarily on mathematical difference and fail to develop the more aesthetic norms related to mathematical sophistication, efficiency and elegance. The teacher is offering aesthetic guidance, albeit somewhat covertly, and we could ask whether talk about form and efficiency might become normative in the classroom. In other words, using the language of Bishop (1991), we could ask whether the teacher is *enculturating* the students by drawing their attention to relationships to knowledge valued by the mathematics culture. [1]

Framing the school mathematical aesthetic

In taking an aesthetic perspective, I choose to focus more closely on a restricted set of the thoughts, actions and practices of students and teachers in the classroom. I do so in part because aesthetic perspectives have thus far taken a back seat to the cognitive, affective and social perspectives adopted in most research. This despite the claims of writers such as Dewey (1934), Dissanayake (1992), Johnson (2007), Papert (1980) and Schiralli (1989), who argue that the aesthetic should not be overlooked insofar as it is intimately intermingled with all dimensions of human thought and activity, including the cognitive and the social. How then might we account for the fact that the aesthetic is regularly overlooked by classroom participants or even by outside observers?

Perhaps the mathematics classroom simply drives the aesthetic sensitivities of teachers and students away. Perhaps those sensibilities are engaged but in ways that never become explicit or overt enough to be shared and communicated. Or, perhaps our ideas of the mathematical aesthetic are simply too rigid and elitist, calling forth a sense of 'museum mathematics' having inaccessible criteria such as 'elegance' and 'depth.' Even when our ideas of the mathematical aesthetic are attentive to process – and not just the final published work of art – they draw on the practices of research mathematicians who pose their own problems, try to solve problems that may have no solutions, and have to communicate their solutions according to certain disciplinary styles. While mathematical creation may well constitute the primary activity of a research mathematician, it certainly does not constitute the primary work of teachers and students of school mathematics. Teachers explain ideas, work through examples and offer tasks or exercises; they assign homework, interpret textbooks, and evaluate students' responses to questions and problems. Students try to answer questions or work on exercises. What might the aesthetic dimension of this sort of classroom activity have to do with the aesthetics of research mathematics?

Indeed, one could hardly count the simplifying of algebraic equations among the canonically beautiful ideas of mathematics. Yet, both the teacher and the students respond pleasurably to the transformation of $\frac{3}{8} - \frac{1}{4}x = \frac{1}{2}x - \frac{3}{4}$ into $3 - 2x = 4x - 6$. Surely, they are relieved to see those beastly fractions disappear. But perhaps also the lack of denominators reveals a certain structure that was missing beforehand. The technique is not actually more efficient, if one counts the number of steps required to solve for x, and it does not give a more correct solution, but it offers a certain satisfying, and perhaps surprising, reduction of complexity.

In describing his technique as "efficient," the teacher alludes to a characteristic that several mathematicians count as chief among aesthetic qualities in mathematics (Hardy, 1940; Schattschneider, 2006). Geometers used to revel in being able to undertake a construction in fewer steps than Euclid. In this sense, efficiency motivated a great deal of geometric work. The reward was not in being able to save time or make more money, but in finding a new, often more clever way of navigating a problem. With their responses, the students communicate some appreciation for the new tool introduced by the teacher. Maybe only half the class verbalized their responses, but those "ooohs" and "aaahs" were, for a brief moment, palpably present and persuasive. I'm inclined to see this episode as an example of aesthetic sensibility in the mathematics classroom, albeit one that may have passed unnoticed by many observers.

While many mathematics classrooms succeed in anesthetising teachers and students alike, I believe that certain opportunities for developing aesthetic sensibilities in the classroom are being overlooked and underdeveloped. Researchers do not recognise them. Teachers are not aware of them. Students do not count them as relevant or important. We need a new approach to identifying aesthetic possibilities in the mathematics classroom that has greater continuity with ways of framing the aesthetic in other human endeavours, including art. I thus propose Pimm's (2006) pragmatic formulation of the aesthetic, which satisfies this goal while being consonant with known aesthetic aspects of research mathematics. Pimm writes:

> Aesthetic considerations concern *what* to attend to (the problems, elements, objects), *how* to attend to them (the means, principles, techniques, methods) and *why* they are worth attending to (in pursuit of the beautiful, the good, the right, the useful, the ideal, the perfect or, simply, the true). (p. 160; italics in original)

His formulation provides a way to identify forms of communication in the classroom that are aesthetic in nature. The following three sections will thus be organised around these three strategies of the *what*, the *how* and the *why*. My goal in applying the frame is to see whether it offers better insights into the apparent paradox described above, and better strategies for aesthetic enculturation.

The episode introducing this article occurred while I was observing a Grade 7 lesson. In the following sections, I will offer several more examples taken from the same classroom. I visited it twice per week for approximately two months. The teacher is the head of the middle school mathematics group, and is very experienced. He uses technology extensively, frequently participates in professional development opportunities, and has been involved in several university-based research projects. I take his classroom teaching as a case study allowing me to exercise a whole-classroom aesthetic lens, and to probe for examples of or opportunities for aesthetic enculturation.

What to attend to in the mathematics classroom

Teachers are often preoccupied with *what* to teach, if only by virtue of endless standards, frameworks and prescribed learning outcomes that typically offer long lists of *what* students should know. Textbooks offer their own version of *what* to attend to. So, in looking for examples of aesthetic considerations with respect to the *what*, there needs to be something about the problem, element or object that is distinguished or privileged from the background of the given content of the curriculum or textbook. In the lessons I observed, I saw three episodes during which the students were placed in the presence of value-driven choices about content.

The first episode occurred when the teacher was introducing his students to the Cartesian coordinate system. He began by explaining the way in which the coordinate system was broken up into four different quadrants. He then talked about words associated with using the coordinate system, such as domain and range. He also established that the students knew how to identify or plot particular points on the coordinate system. Finally, he said, "one other thing that you don't have to memorise. It's called 'completeness.' There's only one way to name a point [on the coordinate system]." Using a specific coordinate point, he explained that one and only one pair of coordinates was associated with it.

In introducing the notion of "completeness" [2] last, and telling the students they do not have to memorise it (which probably means they would not be tested on it), the teacher's language sets the notion apart from the other ideas he had introduced and explained. If it's not something they have to memorise, then he must be telling them because he thinks this idea is interesting. But do the students appreciate that the whole point and power of the coordinate system is that by virtue of this "completeness," it allows us to visually distinguish different functions. What is amazing about the coordinate system is that it allows us to attach locations to any ordered pair of a real-valued function. What an absurd – though potentially interesting – idea it would be to imagine a coordinate system with only an upper right quadrant, where the point (1,3) also represented (-1,-3). Or a coordinate system that retained regions for non-real numbers.

This seems like an example in which the teacher highlights for students something that is important *in mathematics* – perhaps more so than vocabulary and method. Indeed, the very idea that each point has one and only one coordinate pair exemplifies the highly valued property of uniqueness in mathematics (Davis & Hersh, 1981). The students did not respond to the teacher's comment, and possibly chose to ignore it given that it was not test-worthy.

In the second episode, the aesthetic consideration focuses more specifically on mathematical objects. This episode occurred during a later class, when the teacher introduced the idea of relations and showed the students that relations involve one-to-one mappings of objects. He drew two circular shapes on the interactive whiteboard and then, having asked students to pick some numbers, started placing these numbers in each shape. The first student offered 7, the second 15, and then the third, one million, with the latter number generating quite a bit of laughter in the classroom. After they had proposed six different numbers, including -23.79, the teacher said, "I was sure one of you were going to say 'pi,'" which led to more laughter. When the teacher had finished writing the various numbers down, he asked the students how they should be matched, and then remarked, as he drew an arc from the one million to the -23.79: "I think one million matches perfectly with negative twenty-three point seven nine." More laughter ensued.

I highlight three moves made by the teacher that communicate to the students the arbitrariness of numbers: first, he willingly accepts one million as a number; second, he suggests that the students could have just as easily chosen π; and, third, he claims a perfect match between a million and -23.79. Accepting a million, which is a number the students almost never work with in their algebra class, places it on an equal footing with more common numbers relative to his question. Proposing π further communicates that any number will do, even the most absurd. And making the preposterous mapping between a million and -23.79, indicates that, in fact, the actual numbers do not matter: the mathematical approach is to look past the numbers, accept their arbitrariness and focus on the mapping itself. What one attends to is the relationships, not the numbers; the students could have just as easily chosen to say a number in French or offer a hexadecimal digit – or, as the teacher later illustrates, names of cars.

By suggesting π, the teacher is also being playful, encouraging the students to mess around with the mathematics, and propose numbers as fancifully as they wish. The playfulness may even have begun earlier, when the students propose one million. Conjuring these numbers may well have the effect of endorsing what Le Lionnais (1948/1986) would call a "romantic" aesthetic tendency (as opposed to a "classical" one) that gravitates toward the transgressive, chaotic and bizarre (rather than the ordered and harmonious). It may have been this trangressivity that caused the students to laugh when the number one million was proposed. Suggesting one million and, later, π had violated some kind of unspoken protocol about the numbers that appear in algebra class. Similarly, after his preposterous mapping, the students are laughing about the way in which mathematics can canonise arbitrariness (or even violation). Arthur Schopenhauer's definition of laughing, as "a struggle between epistemological levels, namely between a concept and a particularity," seems to describe the students' response quite well. They are noticing the incongruity between the concept that a relation involves a mapping between two objects and the particularity of one million being related to -23.79. Their laughter is an emotional response to apprehending that incongruity. Thus, insofar as laughter can be seen as an aesthetic form, this episode shows both an instance of the teacher drawing attention to mathematical value and of the students responding aesthetically – in a combination of emotion and understanding (or affect and cognition) – to his prompting.

The next episode relates to the use of mathematical language and notation. The teacher had introduced the idea of inequalities and had worked with the students through solving several inequalities that involved adding and subtracting terms from both sides of the equation. He then asked the students to solve the inequality $29 + g \leq 68$: "Let's pick some numbers that will work." After the students had suggested a few, he continued: "Here's the issue: do we really want to sit here and show all your answers?" The students responded with a chorus of "no." He then showed them that they could simply write $g \leq 39$ as the meta-solution that included all of their specific solutions, and asked: "Is that more useful to us?" Again, the students concurred that it was.

By juxtaposing their initial attempts at a solution with his own proposal, the teacher was asking the students to attend to the efficiency of a particular mathematical notation, as well as indirectly advancing the idea that brevity, succinctness and completeness are valued in mathematics. In focusing directly on these aspects of a solution, the teacher draws attention to the notation itself, and away from the actual numbers that solve the problem. His attention to notation in this case is interesting, since there are many other kinds of notation that he introduces without making the kind of comparison he did here (including the use of letters to name variables or the use of the (x,y) notation to name positions on the coordinate grid). In this example, one wonders whether students might not have come to a greater awareness of the appeal of the new notation had they been forced to live a while longer with their existing way of solving inequalities. Might a student have asked at some point, whether it was really necessary to write all those numbers down? As it was, the aesthetic awareness seemed to go unexploited, with little visible *feeling* of having simplified a complex phenomenon, and with an added sense – yet again – of the arcane ways in which mathematics teachers expect students to communicate.

In all three examples, the teacher made quite mindful choices about presenting aspects of mathematics that are valued, including uniqueness, arbitrariness and efficiency. The second example elicited more visible (and emotionally-charged) response from the students and it was also perhaps the one in which the values in question were least explicitly communicated – in the first example the teacher refers directly to "completeness," and in the third example to what's "useful." In contrast, the second example involves the teacher successfully setting the students up to *experience* something anomalous and perhaps disconcerting, so that the more cognitive concept of arbitrariness was accompanied (or preceded) by an affective response.

Why things are worth attending to in the mathematics classroom

In the third episode, the teacher's statements draw attention to *why* it is worth attending to the new notation. The teacher drew on the criterion of usefulness in order to compare the two ways of solving an inequality. He also later showed the students a "set-theory notation" way of writing the solution ($\{g \mid g \le 39\}$) and said: "That's the formal way. On a test you'll probably get it right [if you write $g \le 39$] except one little [deducted percentage] point to show there's a better way that's the mathematical way."

The language here reveals a new perspective on why the solution might be worth considering. The "formal way" is described as being "more mathematical," but it is also longer and more cumbersome than the previous one ($g \le 39$). The teacher implies that "more mathematical" is more worthy – literally, worth one more point on the test! In telling the students that he would accept both solutions, he suggests that both are correct, but that one is more *right* than the other. The imperative of aesthetic fit trumps the simple certitude of mathematical correctness. The more "formal way" is actually more precise, or communicative, in that it makes explicit the domain of application. Without explicit guidance though, the students are likely to deduce that the new notation is yet another confusing bit of formalism. How often might it be the case that issues of aesthetic nature in mathematics are thus misrepresented to students? How might the teacher have helped students understand why the "mathematical way" might be preferred?

Later, when working on inequalities, the teacher again provided the students with different ways of doing things. This time, instead of appealing to the format of the solution, the teacher summoned a mathematical value that accounts for the frequency with which mathematicians endeavour to find different solutions or proofs to problems that have already been solved. He was showing the students that when an inequality involves multiplying or dividing by a negative number on both sides, one must "flip the sign" in order for the original inequality to continue to hold. He then said, "I want to show you another way of doing this that may make more sense." Starting with the inequality $-3x > 27$, he showed the students that they could simply add $3x$ to both sides of the inequality and solve without having to multiply or divide by a negative number. After obtaining the solution, he asked, "Do I get the same answer?" The students agreed, and he said, "You see further proof that you have to flip it."

In showing the students this new way of solving the problem, the teacher emphasises that both ways lead to the same solution. Of course, teachers often try to explain things to students in different ways, knowing that some students will find certain approaches easier to understand than others. Their motivations are pedagogical. Mathematicians value different explanations to improve their own understanding and to satisfy their desire for solutions that are more simple, elegant, clever and even enlightening. The teacher offers the second approach because it "may make more sense," but also points out that it offers "further proof" of the flip method. The latter statement suggests the mathematical value of rationalism described by Bishop, in which logical connection and cohesion between ideas is deemed important (and inconsistencies, disagreements or incongruities that arise from personal interpretations of ideas are shunned). The truth has already been laid out, and the teacher is now showing how it fits with other ideas with which the students are already familiar.

The third example may seem to contrast with the previous one in that it reveals the importance of a personal relationship to mathematics and the aesthetic experience of understanding. When introducing the idea of relations, the teacher wrote down the textbook definition on the interactive whiteboard: "A set of ordered pairs." He then proceeded to say, "I've copied the definition from the book. But I have got my own way of saying it." He then wrote underneath the first definition: "One group of numbers is matched up to another group of numbers in some related way."

The teacher here didn't offer *his* definition as one that might be easier for the students to understand. He invited the students to consider his definition precisely because it is his, thus suggesting that people may have individual understandings of mathematical concepts and that it is acceptable to work with definitions that are *personally* useful or relevant. In this example, the teacher expresses a clear discomfort with the textbook definition – a negative aesthetic response in the sense that the definition does not fit his way of understanding the concept of relation. [3] Of course, for the students, the teacher's definition may not be any less authoritarian and depersonalised than that of the textbook, in which case the teacher's possessive declaration would hardly communicate the possibility or value of personal meaning.

None of these examples seem to elicit much response from the students: they listened and nodded. However, the teacher is clearly drawing attention to different values related to *why* things are written or explained the way they are (for precision, for connection and for personal meaning).

How to attend in the mathematics classroom

One of the most direct ways in which the teacher talked about the methods used in mathematics came just after he had finished explaining the labeling of the coordinate grid quadrants. He announced: "Mathematicians love to name things because then they can talk about them," and then introduced the terms *domain* and *range*.

The teacher seems to be conveying to the students the idea that naming something can be a means of attending to it in mathematics. Halliday (1978) speaks to the extreme salience of what we choose to name in orienting our attention:

> languages have different patterns of meaning – different "semantic structures," in the terminology of

> linguistics. These are significant for the ways their speakers interact with one another; not in the sense that they determine the ways in which the members of the community *perceive* the world around them, but in the sense that they determine what the members of the community *attend to*. (p. 198)

Humans are born into a world of the already-named and so mathematicians liking to name signals a strong fondness for orienting attention in specific ways for particular ends.

By telling students that mathematicians "love" naming things, the teacher also communicated to them that knowing the names of things – and perhaps coming up with your own names – is valuable in mathematics. Indeed, the teacher alluded to the pleasure that comes from human symbolic agency. The students may not be aware that not everything once had names, and that there may still be things left to name, even on the coordinate grid. Why not name all those grid lines that are parallel to the x-axis? Why not give the diagonals a special name, especially since they signify an interesting transition between slopes less than or greater than one for linear functions?

The teacher drew attention to a rather different technique that can be useful and interesting in mathematics. After he had defined the idea of relations and provided examples (as was described above), he drew two little coordinate systems on the interactive whiteboard. One had a random set of points on it and the other had a set of collinear points. He said, "A relation can be random or could follow a pattern. Algebra is good here [pointing to the linear pattern]. This is more interesting because there is some pattern I can use."

By telling the students when algebra is good, he is casting it as a technique that can be used to describe certain relations. When using algebra, one attends to linear sorts of patterns, not to random arrangements. The teacher also speaks to the *what* to attend to when he describes the linear pattern as being interesting. He even addresses the *why* in saying that linear patterns are worth attending to because "I can use" them. Indeed, the above example illustrates quite clearly the connections between the three aesthetic considerations I have distinguished. In particular, one might expect the *what* and the *why* to intermingle quite easily, with the latter providing the justification for the former. This justification may be the most difficult component of the teacher's work: the curriculum requires that students attend to linear patterns, but why might such patterns be worth attending to? It would be hard to argue that they are the best at describing real, everyday phenomena.

Finally, I consider an example in which the teacher describes the methods used in mathematics to deal with new situations. The teacher was introducing the idea of inequalities. He asked the students how they would write "eight is greater than five" and then wrote "$8 > 5$" on the interactive whiteboard. He then said, "We won't study this a whole lot because there's not something interesting going on. We're going to play around with that a little." After showing the students that they can solve inequalities such as $x + 4 < 2$, which involve adding to or subtracting from both sides of the inequality, he remarked: "It looks like we can add and subtract and things will stay the same. That's nice because we can work with inequalities just like we can work with equations. Why in the world would you do problems like this? I don't know, just for the fun of it I guess. The point is that you have tools in your toolbox to solve these now."

How does one attend in mathematics? The teacher says that it is by generalizing to new contexts techniques that worked before, and by playing around to make things that were too obvious into things that are more complex. The teacher also suggests the rather Hardyesque principle that mathematical machinations need not lead to useful or applicable ideas – they can be just "for the fun of it."

In the previous three sections, I offered examples of ways in which the teacher, in his actions and especially his spoken language, draws on aesthetic values in mathematics. Once again, the students did not seem particularly responsive to the teacher's attempts in that they did not comment, ask questions or show any emotional response. The two exceptions were with the introductory algebra beast episode and the matching numbers example. In the next section, I consider a slightly different way of conceptualizing the aesthetic, in which the teacher tries explicitly to evoke aesthetic responses in the students, like he succeeded in doing with the two exceptions.

Designing for aesthetic response in the mathematics classroom

In describing the work of the artist, Dewey (1934) writes that creating an artifact of aesthetic import "involves the ability to manipulate form (media) in order to express and create within the observer the desired emotional response" (p. 51). Similarly, we can think of the teacher as manipulating form in order to evoke within her students a desired intellectual and emotional response. Since the teacher is trying to manufacture a pedagogically appropriate response, the manipulation of form has to connect the mathematics explicitly with the students.

The episode I will describe lasted only five minutes, but was built on a much longer sequence of shared experiences. The students had begun the year working with equations of the form $x + a = b$, learning how to isolate the variable x by adding or subtracting from both sides of the equation. Then they moved on to equations that involved multiplying or dividing both sides of the equation, before progressing to equations where all four arithmetic operations had to be used to isolate the variable. Finally, they worked on equations that involved variables on both sides of the equals sign. The teacher began the day's lesson by writing the equation $5n + 4 = 7(n + 1) - 2n$ on the board, and asking the students to solve it. The students started giving him instructions about how to proceed, first multiplying out the 7 and then rearrange, and subtract $2n$ from $7n$ on the right, and then subtract $5n$ to both sides. He performed each step carefully and slowly, writing down all the manipulations clearly on the board. As he went along, a growing number of comments were made, such as "uh-oh," and then some facial cringes would be seen, and when he wrote down the final, inescapable line $4 = 7$, the students had become quite boisterous, shouting out "it's impossible" and "but 4 will never equal 7." The teacher smiled and said, "In real-life sometimes there are no solutions."

The students' collective response of surprise, pleasure and understanding provides a first indicator that the teacher succeeded in offering an aesthetic artifact. He purposefully set the students up for their reaction by letting them go along as they had done thus far. He let the final, impossible statement of 4 = 7 emerge out of the course of common, acceptable steps of simplifying, instead of, say, declaring ahead of time the impossibility of solving some equations or trying to get the students to determine *a priori* the problem with the equation. The students' response indicates that they "bought into" the absurdity of the mathematics. And, instead of feeling oppressed, anxious or confused by it, they were genuinely regaled.

I see this as an instance of the teacher manipulating his media – including his performance, his text – in order to elicit an aesthetic response for the students. Like an artist who intends to shock or soothe, he had anticipated the way in which his students would respond, and chose his actions to suit their expectations. It's important to note that the moment of surprise and juxtaposition he offered was only possible because of the previous, extended experience of equations working-out (leading to solutions). It would be impossible to find aesthetic pleasure in constant change and surprise. This realization suggests that aesthetic experiences may not occur too frequently in the classroom. However, as with the long-term effect of a positive affect (Goldin, 2000), these relatively rare experiences can provide learners with enough motivation to overcome periods of drudgery or to seek out similar experiences on their own.

This example focused largely on the nature of the students' emotional responses to a given situation, and on the way in which the teacher manufactured those responses. But of course, as mathematics educators, we are not simply interested in students' emotions; the artifact becomes a mathematically aesthetic one only when the media fits together in some integral way with the mathematics. That is to say, it is the interaction of the emotion with understanding and mathematical meaning that gives rise to the aesthetic artifact. In the example, the students do not just respond with surprise and intrigue because of a silly joke the teacher has told. They respond because something mathematically absurd is going to happen or has just happened, and because their verbal interjections provide a way for them to communicate the fact they understand this. What I mean to stress here is the continuity of the students' responses with their mathematical engagement; their emotions are not mere epiphenomena of the mathematics.

It is possible to read this example in terms of Pimm's framework as well, with the teacher drawing attention to the romance of the mathematically impossible by juxtaposing well-behaved sequences of algebraic manipulation with less well-behaved ones. (Interestingly, it was a similar sense of juxtaposition that provoked the only example of emotional response in the examples of the previous sections.) However, this way of reading the example reveals little of the necessary interaction between the teacher and the students in evoking and nurturing aesthetic responses. What Pimm's framework does offer, however, is a new way of interpreting the elements of school mathematics that may invite students to engage with mathematics at a more aesthetic level.

Concluding remarks

To be sure, the class never discussed the beauty of a proof or the elegance of a theorem. These would be the traditional ways in which the mathematical aesthetic makes itself known in the public educational arena. And, as Dreyfus and Eisenberg (1986) note, these would be difficult endeavours for students, who are often still struggling to understand the proof or who have never even heard the word *theorem* before. This article attempts to develop greater awareness of the different ways in which the aesthetic nudges its way into everyday classroom events. In broadening aesthetic considerations in mathematics from a very narrow focus on certain prized artifacts (proofs and theorems) to a much wider attention to the multitude of choices, preferences and values that pervade mathematics, I argue for an increased access to *who* can particulate in the aesthetic considerations of mathematics (including teachers and students) and *when* they can do so (not only when proving theorems or reviewing journal articles, but in thinking about how to write a definition or why one kind of notation might be better than another).

Dewey's conception of the aesthetic as a theme in human experience, as a way that humans organize and derive meaning from everyday situations in which they find themselves, differs from the usual conception of aesthetics, which deals with the nature of perceptually interesting aspects of phenomena. Pimm's framework provides an effective way of locating aesthetic values in the mathematics classroom – the interesting aspects of school mathematical ideas – and of pointing to specific modes of enculturation that are usually not foregrounded in classroom observations. Having located these values, the question then becomes whether, in the kinds of examples described above, the teacher succeeds in reaching and registering with students? Does the teacher need to be more explicit about these values? In evaluating the examples offered, it seems that the more effective attempts at enculturation occurred when the teacher was able to engage the students' emotions, as Dewey would have predicted. The teacher may point explicitly to instances of simplicity, efficiency and utility, but the importance of such values need to be *felt* – describing and prescribing them may only further contribute to the loss of choice and satisfaction often experienced in school mathematics.

Notes

[1] Values can be either ethical in nature or aesthetic. Bishop does not distinguish between the two, but does point to the ways in which several of the values held in the culture of (western) mathematics have ethical implications. On the whole, mathematicians have recognised aesthetic values much more than ethical ones.

[2] In using the term "completeness," the teacher was referring to what mathematicians call "well-definedness" of the coordinate representation; technically, completeness is a property of the real numbers.

[3] In looking for examples of teachers' aesthetic moves, it might be fruitful to pay attention to ways in which they choose to position themselves relative to their textbooks; in this case, we see the teacher establishing a free-trade zone of local relevance at the border of various imposed definitions. This provides him with some much-needed axiological space. The work of Herbel-Eisenmann (2007) touches on this; she describes different ways in which middle school teachers are subordinate, or not, to their textbooks.

References

Bishop, A. (1991) *Mathematica enculturation: a cultural perspective on mathematics education*, Dordrecht, Netherlands, Kluwer Academic Publishers.

Cherryholmes, C. (1999) *Reading pragmatism*, New York, NY, Teachers College Press.
Davis, P. and Hersh, R. (1981) *The mathematical experience*, Boston, MA, Birkhäuser.
Dewey, J. (1934) *Art as experience*, New York, NY, Capricorn Books.
Dissanakye, E. (1992) *Homo aestheticus*, New York, NY, Free Press.
Dreyfus, T. and Eisenberg, T. (1986) 'On the aesthetics of mathematical thought', *For the Learning of Mathematics* **6**(1), 2-10.
Goldin, G. (2000) 'Affective pathways and representation in mathematical problem solving', *Mathematical Thinking and Learning* **2**(3), 209-219.
Gombrich, E. (1979) *The sense of order: a study in the psychology of decorative art*, Oxford, UK, Phaidon Press.
Hardy, G. (1940) *A mathematician's apology*, Cambridge, UK, Cambridge University Press.
Halliday, M. (1978) 'Sociolinguistic aspects of mathematics education', in Halliday, M. (ed.), *Language as social semiotic*, London, UK, Edward Arnold, pp. 194-204.
Herbel-Eisenmann, B. (2007) 'From intended to written curriculum: examining the "voice" of a mathematics textbook', *Journal for Research in Mathematics Education* **38**(4), 344-369.
Johnson, M. (2007) *The meaning of the body: aesthetics of human understanding*, Chicago, IL, University of Chicago Press.
Le Lionnais, F. (1948/1986) 'La beauté en mathématiques', in Le Lionnais, F. (ed.), *Les grands courants de la pensée mathématique*, Paris, France, Editions Rivages, pp. 437-465.
Papert, S. (1980) *Mindstorms: children, computers, and powerful ideas*, New York, NY, Basic Books.
Pimm, D. (2006) 'Drawing on the image in mathematics and art', in Sinclair, N., Pimm, D. and Higginson, W. (eds), *Mathematics and the aesthetic: new approaches to an ancient affinity*, New York, NY, Springer, pp. 160-189.
Schattschneider, D. (2006) 'Beauty and truth in mathematics', in Sinclair, N., Pimm, D. and Higginson, W. (eds), *Mathematics and the aesthetic: new approaches to an ancient affinity*, New York, NY, Springer, pp. 41-57.
Schiralli, M. (1989) 'Art and the Joycean artist', *Journal of Aesthetic Education* **23**(4), 37-50.
Sinclair, N. (2001) 'The aesthetic *is* relevant', *For the Learning of Mathematics* **21**(1), 25-32.
Sinclair, N. (2004) 'The roles of the aesthetic in mathematical inquiry', *Mathematical Thinking and Learning* **6**(3), 261-284.
Sinclair, N. (2006) 'The aesthetic sensibilities of mathematicians', in Sinclair, N., Pimm, D. and Higginson, W. (eds), *Mathematics and the aesthetic: new approaches to an ancient affinity*, New York, NY, Springer, pp. 87-104.
Wechsler, J. (ed.) (1978) *On aesthetics in science*, Cambridge, MA, MIT Press.
Yackel, E. and Cobb, P. (1996) 'Sociomathematical norms, argumentations and autonomy in mathematics', *Journal for Research in Mathematics Education* **27**, 458–477.

I would like to encourage mathematicians, indeed anyone who has responsibility for the learning of mathematics, to open mathematical activity to include the subjectivity of intuitions, to model their own intuitive processes, to create the conditions in which learners are encouraged to value and explore their own and their colleagues' intuitions and the means that they use to gather them. This seems to me to be a necessary step which provides a justification for, but is prior to, the search for convincing and, ultimately, proof.

(Leone Burton (1980) 'Why is intuition so important to mathematicians but missing from mathematics education?', *For the Learning of Mathematics*, **19**(3), 27-32)