CAN DIDACTICS SAY HOW TO TEACH? THE BEGINNING OF A DIALOGUE BETWEEN THE ANTHROPOLOGICAL THEORY OF THE DIDACTIC AND OTHER APPROACHES

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Every human activity taking place in society (in particular, scientific activity) is conditioned by several systems of values and regulated by more or less explicit norms. Concerning scientific activity, one could distinguish between *epistemic* and *non-epistemic* values. Epistemic values are linked to the standards of scientific truth: objectivity, internal coherence, compatibility with other accepted theories, predictive power, *etc.* Among those non-epistemic values affecting scientific activity might be: social utility, social cost, accessibility, *etc.*

Epistemic and non-epistemic values, and the corresponding rules derived from them, affect scientific activity as a whole. They have an effect at different moments of this activity, including the choice of research problems, the selection of empirical data and their treatment, the formulation of scientific results and their interpretation, dissemination and use.

According to Bourdieu (2001), social sciences are strongly exposed to non-epistemic values because their object of study is too important, from the point of view of social and symbolic order, to allow for autonomous development.

Some of the features described by Bourdieu for the whole family of social sciences are particularly present in the case of didactics. For instance, the influence of heteronomous verdicts is very strong, especially the demand of *social usefulness* and the obedience to the *dictates of the predominant pedagogical ideology.* These verdicts are meant to support the social importance of education and the needs appearing in the practice of teaching. In fact, the conviction that didactics can (or even must) provide rules to improve teaching-learning processes or to optimize the running of the education systems is broadly shared.

At this point a subtle and complex question appears: can research in didactics, as a scientific activity, provide value judgements and norms for the regulation of educational processes? In other words: Does didactics have (or should it have), *as a science,* a prescriptive/proscriptive character concerning the activity of education? A look at the literature shows that most of the works in didactics give an affirmative answer, *de facto,* to this question.

Given the importance of this issue, we decided to gather responses from several colleagues, to the following question:

> *To what extent, how, under which conditions, can (or must) didactics set value judgments and normative prescriptions in order to provide criteria about how to organize and manage study processes?*

This question was posed, via electronic message in September 2015, to scholars working in different approaches in didactics of mathematics. We received answers from Guy Brousseau, Michèle Artigue, Ed Dubinsky, María Trigueros, Ricardo Cantoral, Koeno Gravemeijer, and Juan D. Godino. This list is varied, but it does not aim to be exhaustive, as space limitations prevent us from including more perspectives. In any case, we regard this work as the beginning of a dialogue in which all theories are welcome. The next sections will be devoted to the answers [1]. Needless to say, we have the permission of our colleagues for the inclusion of their responses here.

On terminology

What do we mean by *normative prescriptions*? Brousseau pointed out in his answer, that the term *norm* can be regarded as "a statement and description of the state of ordinary and/or relatively stable characteristics of a system" or as "an obligation". In our question we intended *normative prescription* as a recommendation that is authoritatively put forward, so we were thinking of *norm* as something related to *obligation*. The general form of a didactic norm would be as follows: "The teaching of K must be of type M" (prescription) or "The teaching of K is not allowed to be of type M" (proscription) or "K must (not) be taught", where K is a piece of knowledge, M a type of teaching, and "must" and "allowed" is not used to express a legal, factual or practical injunction, but a normative one, typically addressed to teachers by researchers in didactics. Of course, these norms need not be intended as legal, official, laws. We can interpret them as saying that if teachers want to do things correctly, then they have to act according to these norms. Thus, according to this interpretation, "The teaching of K must be of type M" would mean "If teachers want to do things correctly, their teaching of K should be of type M".

Norms and *value judgments* are two sides of the same coin. Indeed, the normative prescription "The teaching of K must be of type M" can be regarded as equivalent to the value judgment "The teaching of K is right if, and only, if it is of type M". Conversely, the statement "The teaching of K of type M is right" is equivalent to the statement "The teaching of K is allowed to be of type M" or "The teaching of K of type M is not proscribed". Therefore, value judgments and normative considerations despite their different logical forms, can express the same content.

On the other hand, our question admits two interpretations that are equally important. It can be understood as questioning value judgments and norms in didactics either from the point of view of their *moral legitimacy* or their *scientific legitimacy.* In short, the question can be understood either as "Is didactics morally legitimated to set normative prescriptions for teachers?" or as "Is didactics scientifically legitimated to set normative prescriptions for teachers?" We were thinking of this second interpretation but, as a matter of fact, some of the colleagues consulted mentioned also moral aspects which, as we will see, are sometimes linked to scientific aspects.

Let us start by considering normative prescriptions and proscriptions from the point of view of scientific legitimacy.

Scientific legitimacy

Dubinsky, well known for his development of APOS theory, rejects the possibility of considering norms as a kind of scientific knowledge. He does not say so explicitly, but this is implied by the allegedly exhaustive character of the list of things a scientific theory can do:

> A theory can: support prediction; have explanatory power; be applicable to a broad range of phenomena; help organize one's thinking about complex, interrelated phenomena; serve as a tool for analyzing data; and provide a language for communication of ideas about learning that go beyond superficial descriptions.

As one can see, stating norms is *not* one of the things a theory can do.

Brousseau, from the perspective of Theory of Didactic Situations, says that "only the observation of the singular phenomena that govern the acquisition of knowledge in the conditions specific to them can lead to an understanding, an explanation, and perhaps an improvement of the learning and teaching of mathematics". Of course, *improvement* is a term linked to value judgments, and so closely related to prescriptions and proscriptions (see above). But this does not mean that Brousseau considers norms to be results in research in didactics. Norms can "perhaps" be obtained from results in didactics, but they should not be regarded as genuine results of didactic research, as scientific knowledge. In 2013 we wrote to Brousseau asking the following question:

> *Do you think that didactics is (or should be) a normative science?*

In his answer he said: "Whereas to decide or to choose social rules does not belong to the sphere of scientific activity, science can be used to that end". But he insisted that norms "cannot take part in scientific activity as pieces of knowledge". Brousseau was very clear about that:

> I don't know what a 'normative science' could be. […] I think there is a kind of contradiction, opposition, between the choice of rules [in the sense of norms] and the statement of scientific facts. A science has pieces of knowledge coming from its observation and reasoning, and they can be changed if they turn wrong in new circumstances.

In her answer, Artigue points out that:

> Didactics, when it was set up as a field of research, was not constituted with normative ambitions, especially in France. […] Research was built on the conviction that priority should be given to understanding the functioning of didactic systems, to clarify the processes of teaching and learning of mathematics which alone could be the basis of reasoned action.

Artigue is an expert in didactic engineering, a strategy developed in the Theory of Didactic Situations. One could attribute to didactic engineering the ambition of improving a didactic system (which would imply the appearance of value judgments and norms), but Artigue explains that didactic engineering is not really a tool to act on didactic systems but rather a research methodology to study phenomena. She is "very reticent" about a possible normative function of didactics not only because she develops her research activity in a country in which didactics does not have a "normative ambition", but also due to her vision of science.

Dubinsky, Brousseau and Artigue appeal to their vision of science to exclude norms from the category of scientific knowledge. This is also our our point of view, from the perspective of Anthropological Theory of the Didactic (ATD).

Let us explain a bit further (An extended discussion of this issue will appear in Gascón & Nicolás, 2017). First, to take a stand on the issue, we think it is important to clarify the possible logical form of a scientific statement. Indeed, science is important to society to the extent that it formulates *laws* which support non-trivial and general explanations. A law is an objective statement of the form "Every occurrence of type A is also of type B", where the mentioned occurrences are objective. For an occurrence to be *objective* it has to be *intersubjective* (*i.e.,* perceptible by everybody under standard conditions) and *substantive* (its occurrence does not depend on the perception of someone, that is to say, it can occur even if it is not perceived by anybody). Thus, assertions about non-objective occurrences are not scientific. All this is coherent with academic research in philosophy of science concerning scientific discourse (Díez & Moulines, 2008). Thus, a scientific law in didactics, a *didactic law*, would be an objective statement of the form "Every occurrence of type A is also of type B" where A and B are objective occurrences related to educational processes.

Notice that a value judgment of the form "This teaching of K of type M is wrong" does not deal with objective occurrences. Indeed, it neither expresses an intersubjective property (as two individuals can have different opinions about how good or bad is a certain type of teaching of a certain piece of knowledge) nor a substantive one (as evaluation must be preceded by perception). Since, as shown above, normative prescriptions or proscriptions can be reformulated in terms of value judgments, we conclude that norms do not deal with objective occurrences either.

Therefore, in accord with Weber's thesis on social sciences (Weber 1917/2010), didactics, to the extent that it is regarded as a science, can only formulate statements about means that are rationally suitable to reach previously determined ends (whose validity cannot be rationally set) as well as the consequences (desirable or not) of an action.

Gravemeijer, who approaches didactics from the perspec-

tive of Realistic Mathematics Education, observes that norms "cannot be seen separate from ideas about what goals are important". We also believe that, whenever one finds a norm in a research work in didactics, there is an underlying end. In his answer, Gravemeijer shows some examples of possible ends, for instance, that "mathematics should be useful for students" or "that the logical mathematics has to be taken as the basis for structuring mathematics education" or "that one should learn pure mathematics first and learn to apply it later". He adds that any choice of ends entails a series of normative prescriptions. In short, behind every norm there is a choice of ends, and behind every choice of ends there is, inevitably, a corresponding series of norms.

Assertions expressing that something is an end can never be considered a piece of (scientific) knowledge. Nevertheless, to make explicit the choice of ends is helpful in order to transform normative prescriptions and value judgments into scientific statements. Indeed, consider the value judgment "The teaching of K of type M is right", and let E be the underlying end. Actually, the judgment is intended to say that "To reach E, the teaching of K of type M is a good means". Consider now the normative prescription "The teaching of K must be of type M" with underlying end E. After making explicit E, it would become "To reach E, the only means is to carry out a teaching of K of type M". Now, to assert the suitability of a means in order to reach a certain end is susceptible of being expressed as a scientific law (experimentally refutable), at least if E, K and M are expressed in objective terms.

This is very much in the line of Weber's thesis on social sciences, according to which science cannot tell you what you have to do, but only what can be done and what are the consequences of the corresponding actions. This function of science corresponds perfectly with, what is called in ATD, the study of the *ecological dimension* of a didactic problem (Gascón, 2011), namely, the study of the conditions required in order to enable or to constrict certain types of teaching.

Norms regarded as part of didactic knowledge

All the answers seem to agree that results in didactics could be used to support norms. The difference is that some of them consider that norms cannot be regarded as results of didactics (because didactics is a science and norms are not scientific statements), and some others consider that norms *can* be regarded as results in didactics. Let us examine this last point of view.

Trigueros, who works in APOS theory, says: "In general terms, I think didactics should not propose value criteria, and still less normative ones, about the organization and management of educational processes". Later, she insists: "In general, it is not the role of didactics to create rules for [the] organization and management [of educational processes]".

The first point is that she differentiates between *normative* and *value criteria* when she says "less still". She shows an opposition to norms stronger than to value judgments. As we said before, we do not think there is a difference concerning content: the content expressed by a norm can be equally expressed by a value judgment, and vice-versa. Possibly, Trigueros agrees with us, but perhaps she still notices a pragmatic difference: the imperative character of a norm might be regarded as more unfriendly or awkward than the indicative or expressive character of a value judgment.

The second point is that, perhaps as a precaution, she says "in general". This entails that, in her view, normative prescriptions or value judgments could, exceptionally, be part of the corpus of results in didactics. This exceptionality is intriguing: if exceptionally, why not frequently? Therefore, it seems that her reticence to admit norms and value judgments among the results of didactic is not due to her vision of science, but to some considerations of a different nature. Indeed, if one considers that a certain type of statement is *not* a scientific statement, then it is *never* a scientific statement. On the other hand, she states that "APOS theory was created with the ultimate goal of improving learning of mathematics". The term *improve* is strongly connected with value judgments like "This is right/wrong" and norms of the form "You have to do this". However, one can understand that she says that the improvement of learning of mathematics is, for some reason, not intended to be done by producing norms and judgments, but rather through some kind of results able to support norms and judgments. This seems to be the case, especially after her sentence, "Diffusion of research results in journals not necessarily devoted to research, in the format of summaries of relevant results, may have an impact on the establishment of criteria for organizing educational processes", where she differentiates between *research results* and *criteria to organize educational processes.* In any case, as we saw before, the door of didactics seems to be still slightly open to normative statements.

For Cantoral and Godino, this door is wide open. Indeed, Cantoral, who works in Socioepistemology of Mathematics Education, declares that it is "a theoretical approach with a pragmatic slant" and that "this theory looks for strategies in order to democratize learning of Mathematics". Whatever *democratize* means, it seems that this is intended to produce a change in learning, stipulating the type of learning that is or is not permitted. He does not seem to find any problem in including norms in the corpus of the results produced by a science: "our prescriptions […] should be limited to local systems of action". He even uses the expression *normative science* without observing any kind of contradiction in terms. At this point, Cantoral is at the opposite pole from Brousseau who says: "I don't know what a 'normative science' could be".

Godino, from the Onto-semiotic Approach, uses the expression "techno-science" meaning that "the nature of the knowledge to be constructed [in didactics] has both a scientific and technological character" which implies that, among other things, didactics "takes part in those [teaching and learning] processes to increase their effectiveness" and so it has a "technological-prescriptive component". He says that, "didactic principles of preferred [teaching] actions" are a kind of "*didactic knowledge*" (notice that the term "preferential" is linked to norms and value judgments). Later he says that these didactic principles are "regarded [as] suitability criteria about which there is a certain consensus in the Mathematics Education community". Thus, according to Godino, there is no problem in saying that norms constitute a kind of *knowledge* in didactics and that, actually, some of these norms are already shared and advocated by many researchers.

Non-scientific reservations: moral and practical aspects

Cantoral says that "didactics of mathematics […] is not a normative science". But, in his view, this is only due to practical, counterfactual, aspects, not to the fact that didactics is a science, as we saw in the previous section. Actually, he asserts that "didactics of mathematics, even being a fundamental science, is not a normative science". According to Cantoral, his approach in didactics aims to be normative, as it aspires to be a fundamental science (whatever *fundamental* means in this context), but as a matter of fact this is not possible yet because:

> We still have to learn how to observe and analyze learning processes in mathematics […] We still have to get a better understanding of teacher empowerment processes and the social construction of mathematical knowledge. There is a lack of studies, in collaboration with teachers, about the questioning of school mathematics and so our prescriptions and diagnosis should be limited to local systems of action.

Thus, it is due to practical aspects, that Cantoral shows a certain reluctance to global prescriptions.

This is also the case in Godino's answer: "those criteria are to be used locally, and so the teaching system has to make adjustments and interpretations".

Researchers who do not include norms in the category of scientific didactic knowledge, like Brousseau and Artigue, acknowledge that the spread of normative prescriptions around it is somehow inevitable. Brousseau says that

> Scientific progress in many sciences has drawn attention to all kind of 'weaknesses' of the 'outcomes' of this classical [didactic] scheme. Disputes about teaching have increased, and they have been exacerbated and have quickly incorporated *demands for results* [...] But the pressure of these demands has motivated the appearance of many proposals.

Artigue points out that

> However, the vast majority of didacticians are not just researchers; They are also teachers of mathematics, and/or teacher trainers […] Their work […] is nourished by their visions and didactic knowledge […] This application of didactic […] is in no way a normative application, even when it makes value judgments and has a transformative aim.

In any case, they agree with Cantoral and Godino in saying that there are practical reasons for being cautious with normative prescriptions, regardless the question of whether they are scientific results.

Brousseau considers the imposition of norms on teachers by researchers in didactics morally unlawful. Indeed, in his answer Brousseau says:

> To impose norms concerning phenomena (which we don't understand and we don't control) in order to be able to accuse the actors of the system of violation of these norms, would entail a process of enslavement as old as civilization itself.

Therefore, from Brousseau's viewpoint, the fact that we do not understand didactic phenomena invalidates normative prescriptions.

Several times Brousseau points out that our understanding of learning processes is still very poor. For instance, he says that "nobody really knows how we learn or how we make discoveries", that reliable results to improve education are "manifestly impossible to obtain with the present knowledge in this domain" and that "Only the observation of the singular phenomena that govern the acquisition of knowledge in the conditions specific to them can lead to an understanding, an explanation, and perhaps an improvement of the learning and teaching of mathematics". Thus, according to Brousseau, the imposition of norms to enhance mathematics education could be possible some day, but after a necessary development of didactics, and, in any case, not as laws announced by researchers in didactics to be obeyed by teachers:

> Fundamental research should be advanced resolutely, but it should remain circumspect about its results until applications of this research can provide teachers with appropriate procedures that will provide reasonable security for pupils and teachers who respect them, without relinquishing their personal awareness, which will always be indispensable.

Thus, teachers, and even students, would be expected to have an active role in testing and improving didactic norms.

Also Artigue prefers a kind of collective bargaining about proposals to improve teaching:

> But my reservations are also of a different nature, because in the idea of normative didactics, the question of the relations between the actors in the field arises. A didactic source of norms would cover what? Which norms would be imposed? Who would be judged by whom? Who would introduce what obligations, what relations of subordination? In an educational system like ours, so dominated by hierarchies of all kinds, so top-down in its functioning, we are slowly moving towards a more collaborative vision of work, of relationships between actors and communities, of responsibilities. This seems to me to be an essential issue and I am not at all convinced that claiming a normative dimension for didactics could help.

A proposal for a dialogue: principles, means and ends linked to teaching

As we defended above, under *Scientific legitimacy*, to turn a value judgment or a normative assertion into a statement susceptible of expressing an objective truth, you have to make explicit the underlying end. In this way, you can get a statement about the objective validity of a *means* to reach a certain *end*. Let us list some considerations about the connections between means and ends (and also principles) linked to research and teaching:

a) We distinguish two main types of *research ends* in didactics, depending on whether they are devoted to understanding phenomena about educational processes or to fulfilling teaching ends. In turn, we distinguish different kinds of *teaching ends*: just

transmitting knowledge without showing any preference for a particular way of doing it, transmitting knowledge according to a certain pedagogical paradigm (for example, so-called constructivism), indoctrinating (for instance, to promote a certain political identity), *etc.* Of course, research ends may differ from one approach to another. Also, each approach can assume one or several teaching ends as its own, and can even create new ones. In any case, whenever one finds a value judgment or a norm promulgated by some approach, there is an underlying teaching end.

b) Research approaches in didactics (and in any other science) have what we call *principles*. They are methodological assumptions and postulates concerning what are the basic objects regarded by the approach and the relationships between them. There are obvious links between principles and research ends. To begin with, you cannot formulate a research end without using the vocabulary provided by the principles of the approach you are working in. Thus, if, according to your principles, mathematics is described in terms of *mental constructions*, your research ends, and the assumed teaching ends, necessarily deal with mental constructions. For example, one teaching end would be to help students to form the required mental constructions in order to understand a certain part of mathematics, and a corresponding research end would be to study which kind of educational activities help students to form the required mental constructions. To provide a scientific character to research in didactics, it is essential to differentiate between principles, research ends and teaching ends. Unfortunately, making this differentiation is difficult.

c) As has been said in (a) and (b), research ends are linked to principles and sometimes directed to certain teaching ends. Occasionally, the underlying principles and the assumed teaching ends are not made explicit in publications in didactics. As a consequence, the corresponding research ends seem to be poorly motivated.

d) Principles and research ends are not fixed. Actually, both scientific praxis and the corresponding results contribute to transforming them over the long-term. For instance, after several years of research, it has become apparent for ATD that the *minimal unit of didactic analysis*, in reference to which didactic problems are formulated, does not consist of a single type of task (*e.g.,* to perform a division of natural numbers) but has to be enlarged to include quite a substantial amount of the context in which this type of task appears (Bosch & Gascon, 2005). In parallel with this, at the institutional level, this minimal unit covers, beyond the classroom and the teaching institutions, all the institutions taking part in the process of *didactic transposition* (Chevallard, 1991). Thus, in short, scientific praxis in ATD has contributed to creating the principle according to which *micro-didactic analysis*, in order to be fertile, should be included in a *macro-didactic analysis.*

Our point is that, in order to make progress in this dialogue about the role played by value judgements and norms in didactics, we need to consider the problem of the relations between research in didactics and teaching. More explicitly, we need to answer the following questions:

> Which are the teaching ends underlying value judgments and normative prescriptions in each approach?
>
> Which are the reasons to favor the teaching ends assumed by each approach in didactics?
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> Which is the link between those teaching ends and the principles in each approach?
>
> Are the teaching ends and the principles compatible among different approaches? If not, to what extent are the different approaches working inside the same discipline?

For our part, we suggest continuing the dialogue in the direction suggested by these questions. We would like to invite all members of the community of didactics of mathematics to give their own answers to these or related questions, in the form of communications sent to *For the Learning of Mathematics.* We will do so ourselves, in a subsequent issue, from the perspective of the Anthropological Theory of the Didactic.

We think it is indispensable to unearth, make explicit and expose to criticism the *non-questioned components* (principles, teaching ends) of each approach, and to compare them to those of other approaches. Only in this way can we make progress in a fruitful dialogue about the role played by value judgments and norms in didactics. This is an indispensable starting point for a real networking of different approaches.

Notes

[1] The complete answers, in the original languages and in translation, can be found at http://flm-journal.org. See the link by this article in the online table of contents for this issue.

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