USING TEMPORAL RANGE TO THEORIZE EARLY NUMBER TEACHING IN SOUTH AFRICA

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In writing about her year-long examination of her own mathematics teaching and children's learning in a US third-grade classroom, Ball (1993) describes her practice in the following way:

> With my ears to the ground, listening to my students, my eyes are focused on the mathematical horizon. (p. 376)

Reflecting on this teaching, she concludes:

> My understandings and assumptions about nine-year-olds equipped me to make decisions about mathematical representation and activity that served their opportunities to learn. Similarly, my notions about mathematics allowed me to hear in the students' ideas the overtures to important understandings and insights. (pp. 394-395)

Two interconnected temporal dimensions are referred to in these statements. Firstly, there is a mathematical temporality that relates to mathematical ideas, their precursors and horizons. Secondly, there is a mathematical learning temporality in which what students say and do at particular moments in time provides the "ground" on which future learning can be built. In Ball's statements, skillful teacher mediation involves working at the junctures between these two temporal dimensions in the classroom. I say junctures in the plural because different learners' existing understandings meet the trajectory of the mathematical idea being dealt with at different points. Skillful teacher mediation needs to recognize these different points, and work to move these points forward in the context of the classroom.

My interest in temporality stems from previous findings about primary school mathematics teaching in South Africa, including several studies of my own. A stubbornly persistent finding across this relatively small body of literature relates to the ongoing presence of concrete counting approaches to number problems amongst students well into the intermediate phase (stipulated student age 9-13 years). Studies looking at mathematics teaching in this context have used a number of theoretical frames, directing attention towards different features of teaching activity. In this article, I argue that the different features of teaching highlighted across these studies can be re-interpreted in relation to the two temporal dimensions identified above. I conclude that a common feature of teaching across these studies is an ongoing emphasis on the "present" in mathematical terms, with the focus on producing the correct answer to the immediate example at hand. A concomitant effect of a lack of attention to temporal range in mathematical terms in the foundation phase years (stipulated student age 6-8) is an acceptance of concrete counting methods. These methods can produce correct answers, especially in the lower number ranges that are foregrounded in the foundation phase curriculum. Thus in terms of the temporality of mathematical learning, students remain saddled with highly rudimentary strategies for working with number.

In order to make this case, the article is arranged as follows. I begin with a brief outline of theoretical models of progression in early number learning. In these models, moving on from concrete counting is necessary if students are to progress towards more complex mathematics. Temporal range across mathematics and mathematical learning are intricately intertwined in these models, with students' current strategies underpinning teachers' selections of "good" follow-up tasks, explanations or demonstrations. I then introduce the small body of literature dealing with the prevalence of concrete counting approaches in South Africa, and note both their findings relating to teaching, and the interpretations and implications that the various theorizations point to. These findings are then re-interpreted in terms of the two dimensions of temporal range. As a result of this re-analysis, I make the case that ongoing work with the "present" in mathematical terms, without attention to temporal range in this dimension, leads to a situation where concrete counting, as the prototypical approach to working with number, comes to be viewed as acceptable.

Theoretical models of progression in early number learning

Summarizing literature on early number learning, Askew and Brown (2003) describe a well established trajectory within children's strategies for working with early number problems. For addition, they describe this sequence as: "count all, count on from the first number, count on from the larger number, use known facts and derive number facts" (p. 6). Thompson (2008) provides a similar trajectory in the context of subtraction. Movement along this trajectory makes work with addition and subtraction problems in higher number ranges possible, since the concrete counting processes used completely and more partially in count all and count on processes respectively become inefficient and error prone as the number range increases. Gray (2008) has described the "compressions" of number that underlie this trajectory, with reifications of counting processes into abstract number objects (Sfard 2008).

Sfard (2008) describes "objectification" as the process underlying the growth of mathematics more generally. The reification of processes into objects allows for the building of new mathematical objects through further operational activity upon them. Temporal range in mathematical terms is therefore viewed in terms of building new process layers on previous processes that have been compressed into objects. Temporal range in mathematical learning terms relates to working discursively to compress operational processes into abstract objects. In the context of early number learning, an inability to work with abstract notions of number rather than concrete counting-based notions of number makes it very difficult to move beyond low number ranges effectively. South African evidence of primary mathematics learning (outlined in the next section) suggests that the use of counting, process-based notions of number is widespread.

The South African context of learning and performance

In primary mathematics in South Africa, the national picture is bleak: the national mean percentage mark in 2012 stood at 41% in the Grade 3 numeracy test and at 27% in the Grade 6 Mathematics test (DBE, 2012). Underlying this weak performance, multiple sources of evidence point to the ongoing prevalence of unit counting, and repeated addition and subtraction strategies well into the late intermediate phase. Schollar (2008) graphically illustrates this with data drawn from learner work examples in the Primary Mathematics Research project, and notes the following (based on a national sample of 7028 learners):

> The data indicated that 79.5% of Grade 5 and 60.3% of Grade 7 children still rely on simple unit counting to solve problems to one degree or another, while 38.1% and 11.5% respectively, of them rely exclusively on this method. (p. iii)

Schollar contrasts this prevalence of concrete "counting-based" strategies with the more abstract notions of number as object that underlie what he refers to as calculation-based strategies. Similar patterns of results were noted across foundation phase in Cranfield *et al.*'s (2005) study in the Cape Town area. The broad problem of low performance and the more specific problem of inefficient ways of working with number are viewed as playing an important part in the gap that exists by the end of foundation phase (and continues into the intermediate phase) between the prescribed curriculum and the performance of the majority of South African learners (DBE, 2011; 2012).

A small body of more recent work in South Africa has turned attention to the teaching and artifact use situations within which concrete counting strategies are located. Whilst small, this literature theorizes this prevalence from a range of different viewpoints, each focusing on different aspects related to the problem. The purpose of this article is to re-interpret these findings in terms of temporal range.

Literature on the teaching of number in South Africa: summaries and re-analyses

In this section, key findings and theorizations from six studies on pedagogies associated with concrete counting are presented alongside re-analyses in terms of temporal range using the two dimensions of mathematics and mathematical learning. Implications from these re-analyses are discussed in the concluding section.

Reeves and Muller (2005)

Reeves and Muller (2005) noted that evidence on content coverage within the enacted curriculum in the intermediate phase pointed to coverage that, whilst placing emphasis on the teaching of number topics, often focused on topics drawn from previous, rather than current grades. Using the notion of "opportunity to learn", they investigated the enacted curriculum in relation to the prescribed curriculum in a sample of disadvantaged Grade 5 and 6 classrooms (assuming, in a context of underperformance, that some coverage of prior grade content would be needed). Looking at samples of learner workbooks, they noted poor pacing and gaps in coverage:

> The descriptive data also show that learners are spending more time on subtopics that they were expected to have covered in earlier grades than they do on subtopics at the level expected for their grade. Data reveal evidence of slow curricular pacing across grades 5 and 6. In other words, the study shows evidence of slow across grade curricular pacing and that learners are studying topics lower than grade level expectations. (Reeves & Muller, 2005, p. 124)

For Reeves and Muller, the implications of poor pacing and coverage within teaching included the need for revisions of the curriculum:

> policy documents such as curriculum frameworks and guidelines in South Africa may need to provide schools and teachers with a concrete picture of the entire trajectory of each learning phase (across grade framing over pacing) and more in the way of guidance in relation to the pace they should maintain in order to cover the grade level expectations. Teachers appear to need greater signaling as to how much time learners should be given to work on topics or subtopics. (p. 126)

The curriculum introduced in the Foundations for Learning (FFL) campaign (DoE, 2008) included the specification of termly "milestones" and the subsequent Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011) included weekly coverage schedules. Both addressed some of Reeves and Muller's criticisms, providing explicit specification on the sequencing and pacing of the curriculum.

Looking at poorly paced and sequenced teaching in terms of temporal range indicates a lack of awareness of the nature of mathematical progression or, perhaps, a reluctance to push forwards into more complex mathematical terrain within teaching. The call for curricula that make mathematical trajectories more visible and more explicit—that have since been introduced—represents a response in which temporal range in mathematical terms is much more explicitly specified than was the case at the time of Reeves and

Muller's data collection. Detail on a specific vision of mathematical progression is highly visible in both documents mentioned above. It is important to note that within this standardized specification of progression is a belief that an explicit statement of the progression, sequence and pace of mathematics—essentially a statement of mathematical pasts and futures *a priori* of the learner—is desirable. One arm of Ball's vision is highly visible in these curricula: a mathematical temporal range. However, in its standardization of the timeframes for this progression, the need to "hear" and address multiple student inputs at different points on this trajectory is largely sidelined. There is therefore little room for attention to temporal range in mathematical learning terms.

Hoadley (2007)

Zooming in on teaching in the foundation phase specifically, a small number of studies have focused on different aspects of the nature of the teaching activities within which concrete counting is produced. Hoadley's (2007) study provides a comparative analysis of four teachers' Grade 3 numeracy teaching in working class schools with four counterparts in middle class schools. Looking across 31 lessons, Hoadley drew the following conclusions, based on the Bernsteinian distinction between concrete unit counting—viewed as "everyday" and "context dependent" knowledge—and "school" or "context independent" knowledge (see Bernstein, 1971; 1975):

> In the working-class context the trajectory from one form of knowledge to the other, was ill-defined, largely due to the subordination of school learning to everyday meanings and practices (often embodied in a "theme"). The potential for acquiring the school code, and in particular the specialized knowledge of mathematics, is seriously undermined in the pedagogy found in this context. (Hoadley, 2007, p. 704)

Dowling (1998) elaborates on the process of moving from everyday domains towards the esoteric domain of mathematics in terms of "specializing" strategies. These strategies allow for moves from specific actions to general procedures or principles. Teaching that presents only context-dependent messages without any move to generality is described as exhibiting "localizing" strategies. Hoadley shows that in the working class context, half of all the tasks presented by teachers worked with localizing strategies. This contrasted with an equivalent figure of 2% of tasks presented with localizing strategies in the middle class schools.

The analyses of orientations to meaning in Hoadley's study, based on Bernsteinian concepts, tends to view localization in spatial terms. Restricted, rather than elaborated access to meanings is viewed in relation to class-based punctuations in social space. Dowling's (1998) notion of "strategies" introduces a temporal frame into this localization, working from the assumption that mathematics teaching has to begin in "everyday" domains that are familiar to students, and proceed through specialization into the esoteric domain. Hoadley (2007) suggests two kinds of specialization moves: specializing-principling and specializing-proceduralizing. Specializing-principling strategies are described as those involving:

> a rule-based or rule-governed performance on the part of the learner, where novel applications of knowledge, operations, or skills are required, or in oral work, reasoning, justifications, or explanations. Here the understandings, competences, or reasoning behind the mathematics are required for the construction/production of legitimate texts for evaluation. (p. 686)

In contrast, specializing-proceduralizing strategies are those where:

> the focus is on the procedure required for the construction/production of legitimate texts for evaluation. These strategies entail that learners practice a concept or perform an operation without necessarily engaging in the principles for the generation of texts. [...] Specializing-proceduralizing strategies are often visible in tasks that merely provide learners with a set of instructions to carry out; where a trajectory may be discerned, but more general principles are not made explicit and are unlikely to emerge at a later stage. (pp. 686-687)

Whilst Hoadley's findings pointed to a greater incidence of specializing-proceduralizing strategies in both contexts, there was a much greater prevalence of localization in the spatial and temporal senses for the working class students. Everyday orientations to meaning were privileged and specialization moves were limited.

Localizing strategies are related to everyday, or commonsense, rather than more specialized disciplinary knowledge, and according to Hoadley, are "not designed to produce a cognitive shift" (p. 686). Thus, in their formulation, they explicitly deny the "mathematical future" that Ball (1993) recognizes as critical to good mathematics teaching. Specializing-principling strategies can focus on "novel applications" or more general principles. Specializing-proceduralizing strategies can focus on the progression built into the practice of more sophisticated operations. Both types of specializing strategies therefore include elements of a "mathematical future" in their enactment. The pedagogic practice seen in Hoadley's study can therefore be re-interpreted as an ongoing acceptance of students' current ways of working with number, described by Hoadley as "the most rudimentary of counting schemes" (p. 700), with limited opening up of a mathematical future for working class children.

Ensor *et al.* (2009)

Expanding the focus on teaching to eighteen disadvantaged classrooms across foundation phase, Ensor *et al.* (2009) noted the prevalence of concrete representations of number across the three grades. They viewed this prevalence as an insufficient degree of shift towards the more abstract symbolic representations required by the intermediate phase mathematics curriculum. Ensor *et al.* break down the notion of specialization into:

- specialization of number content (progression identified in terms of tasks focused on "counting", "calculating by counting" and "calculating");
- specialization of forms of representation of number (progression interpreted as movement from more

> concrete representations towards symbolic and syntactical representations of number).

They analysed the proportions of lesson time devoted to each category. In relation to specialization of content, they made the following observations:

> 89 per cent of total pedagogic time was spent on counting or counting-by-calculating. [...] there was relatively little pressure, in Grade 2 and 3 in particular, towards calculating without reliance on counting. In two Grade 3 classes, for example, learners were asked to solve word problems involving addition and subtraction and were given counters to assist them to do this. [...] while some degree of specialisation of number content occurred across the three grades, the amount of time spent on calculating was very low, and occurred only in Grade 3. Very little attempt was made by teachers to encourage calculation without counting in the lower grades. (p. 20)

In relation to specialization of forms of representation, their analysis showed the ready availability of concrete apparatus for counting across all three grades, which they noted "has a negative impact on the conceptual level of the number work offered to students and the use of time" (p. 21).

As noted already, a temporal dimension underpins Dowling's notion of specialization, and this dimension is thus present in Ensor *et al.*'s findings of a lack of sufficient specialization of number content and representational forms in the foundation phase. As in Hoadley's work, the conceptualization of specialization is in terms of mathematics, but Ensor *et al.*'s study provides explicit elaboration of progression in relation to number strategies and adds attention to the nature of early number representations. Temporality in mathematical terms for these two aspects of early number is therefore delineated in this theorization. Teaching, in allowing children to use counters and work with counting-based strategies, once again anchors learning in the past and present: in prototypical strategies with concrete representations, with limited encouragement to move to more sophisticated strategies.

Venkat and Askew (2012)

Venkat and Askew (2012) further elaborate on foundation phase (Grade 2) teachers' work with representations of number. In this study, attention was on the teachers' use of 100 squares and abaci. Both of these structured representations of number built into apparatus are becoming more commonly available in South African classrooms. Our findings indicated that the demonstration of unit counting strategies within teaching continues in the presence of apparatus designed to make the base ten structure highly visible, and to support the visualization of 10s-based grouped counting strategies. Essentially the artifacts present models of number that are based on the decimal system, but the ten-based structure does not feature in the models of counting presented by teachers. Instead, unit counting models are demonstrated.

Potential progression from counting to calculation (involving grouped counts using tens as benchmark) is disrupted in teaching that presents only more rudimentary counting forms. Analyzing the interplay between teaching and apparatus use in temporal terms, there is a blocking of the "mathematical future" through teaching that delivers correct answers through prototypical models of counting. Whilst Ensor *et al.*'s (2009) study critiques the ongoing provision of concrete counting resources, in our study, we note that unit counting continues to be promoted with the use of resources inlaid with more symbolically structured numerical forms, *i.e.,* in the context of the use of more specialized forms of representation.

Throughout the studies discussed above, the focus is solely on teaching, with critique directed towards the absence of a mathematical future that depends on the modeling of more advanced strategies. These critiques are about teachers' relationships with mathematics and its progression, rather than with the junctures between mathematical learning and a mathematical trajectory. The high levels of specification of content, sequencing and pacing in recent South African primary mathematics curricula (DoE, 2008; DBE, 2011) can be viewed in this context as attempts to make mathematical temporality highly visible and accessible to teachers.

Venkat and Naidoo (2012)

Whilst maintaining a focus on teaching, the two studies dealt with next make more reference to students' responses in classrooms, allowing for greater insight into the appropriateness of teaching in relation to learning as well as in relation to mathematics. The notion of localization *vs.* specialization continues to figure in these studies. As above, I address findings and theorizations before re-interpreting them in relation to temporality.

Venkat and Naidoo's (2012) study uses systemic functional linguistics (Halliday & Hasan 1985) and variation theory (Marton, Runesson & Tsui, 2004) to provide analysis of a Grade 2 number lesson focused on different ways of making the number 16, through addition, subtraction and repeated addition. Across the three episodes with these foci, we note the teacher's recurring reference to empirical verification for the answers to sums suggested by students: children are asked to check the correctness of the sums and differences offered by counting on their abaci. There is no "backward reference" in the lesson to the processes used to produce previous answers, and almost no backward reference to the results that have already been established. For example, in one episode of the lesson, students establish through counting on their abaci that 8 + 8 gives 16. Following the offering of 10 + 6 (which is also empirically verified), 9 + 9 is offered as a sum making 16. In this instance, as in all the instances across the episode, answers are established through counting, rather than through consideration of the relationship between 8 + 8 and 9 + 9. We describe this absence of "deriving" new facts from established facts in terms of "extreme localization":

> a scenario in which not just episodes, but individual examples within episodes are played out in what appear as *ahistorical* ways. What we mean by this is that each time a new example enters the scene, the past appears

> to vanish (including the methods and answers that might have been generated in the very recent past). Our analysis indicates that neither teacher talk, nor the juxtaposition of examples, encourages links to prior learning. In this scenario, the only recourse is to empirical verification, to the concrete counting that has been identified at both the level of learner strategies (Schollar, 2008) and of teacher advocacy (Ensor, Hoadley, Jacklin, Kühne *et al.,* 2009). (Venkat & Naidoo, 2012, p. 32)

Establishing deductive relationships between examples can be viewed as one way of specializing tasks through moving beyond the immediate task context. The problem pointed to here, in terms of the lack of reference to previously established facts, is the disappearance of the past, in mathematical terms. The teaching focus consistently remains in the present, with attention restricted to the specific task that is being worked on.

We noted a further tying to the immediate task context in the extensive reliance on repetition, rather than elaboration of objects or strategies. Elaboration of ideas can occur through the use of either linguistic synonyms or through alternative mathematical representations. Both of these devices allow connections to representations that go beyond the immediate example or set of examples. Dowling's (1998) notion of localization continues to hold in this formulation, given that strategies and representations always remain tied to the context of their use rather than being viewed as more general. Indeed, the only generality that exists is concrete counting as the means for working out answers. Given the lack of backward reference in teacher talk, coupled with random generation of examples, the possibilities for specializing into processes and concepts that have generality are severely restricted.

Some learners' responses suggested that they were able to work at the recalled fact level in terms of producing answers to suggested offers. One learner, for example, is heard immediately calling out the answer to 9 + 7 as 16, whilst other learners are able to produce repeated addition partitions of 16 quickly. For these children, the need to verify answers through counting would seem to represent a "pulling back" into concrete counting in ways that seem unhelpful. In this teaching, there are indications of lack of awareness of mathematical progression in relation to specific ideas, of the various points along this progression that learners' current understandings connect with, and of how they might be moved forward. Teacher responses suggest instances where regardless of individual student input, teaching resumes at the start of the counting trajectory, with concrete counting.

Common across the studies discussed so far are theories that are built on the idea of connection, progression and generality, contrasted with empirical data on teaching that appears to maintain a stubborn focus on the immediacy of the present. Whilst the work of Reeves and Muller (2005), Hoadley (2007), Ensor *et al.* (2009) and Venkat and Askew (2012) point to a lack of attention to mathematical futures in their empirical data, the Venkat and Naidoo study also notes the lack of drawing upon what has been established mathematically in the past. In this interpretation, concrete counting can be seen as a product of pedagogic attention to the mathematical "present" in temporal terms: a "Groundhog Day" scenario, in which learners' existing concrete strategies are accepted, affirmed, and in some cases, insisted upon. Limited efficiencies into calculation by counting are sometimes built, but they are insufficient to build the abstract number concepts that need to be established by the end of foundation phase.

Askew, Venkat and Mathews (2012)

A focus on teaching leaves in the background attention to the second arm that Ball (1993) points to as important for good teaching: attention to "learning pasts", that is, to connecting with, diagnosing and scaffolding from learners' existing understandings. Askew, Venkat and Mathews' (2012) study provides commentary on the majority of children's inability (in their analysis of a Grade 2 class) to provide correct answers to a missing addend task very similar to a task that had just been completed in whole class activity format. A further feature of this study that is not represented in the other studies discussed in this article is an attempt by the teacher (albeit largely unsuccessfully) at a specialization step. A micro-analysis of a teacher's work with a missing addend problem is presented and discussed. In an episode where the teacher has explained that the problem is to find out what number needs to be added to 2 to give 7, lack of response from the class is followed by this direction and interaction:

T: Make seven with your fingers

Everyone holds up seven fingers (following what the teacher does as five on one hand and two on the other)

T: Now hide two and which number are you left with? Make your seven first and hide two. Which number can we add with this two to make seven?

L: Eight

T: No, we made seven and hide two and what is left? The number that we will add with two to give us seven?

L6: Five

With most children now giving the right answer, the switch to subtraction is demonstrated again for working out what needs to be added to 3 to make 7. It was noted though, that within individual working on a task asking for missing addends for 11, most children were unable to independently work out answers either using fingers or cubes.

Noting the teacher's mediating operations nested within actions in dealing with missing addends, we saw that the direct object (getting the answer) could be achieved in an imitative format with the mediating action shown. However, this mediation left unaddressed both getting the answer in more independent work settings and the indirect object of understandings and capabilities related to task completion:

> If neither objects, nor shifts in objects, are established through coherent mediation in the classroom, it

becomes hard for novices to appropriate the operations and actions needed to, at the most basic level, produce correct answers independently—as what they have to draw upon are experiences of disconnected actions that have to simply be taken on trust. (Askew, Venkat & Mathews, 2012, p. 34)

The teacher's mediation in this lesson suggests an orientation towards the direct object—producing the answer to the task at hand, in order to proceed to the next task. The authors note that in this teacher's case, there is awareness that subtraction can be used to solve missing addend problems. Carpenter *et al.* (1999) have noted that this transformation requires an abstract understanding of the relationship between the quantities represented in the task. In this lesson, a correct generalized strategy is used, which suggests some understanding of mathematical temporality in relation to the topic, given that the transformation can be represented as a specialization move. However, this specialization strategy is presented without attention to students' existing understandings, and the need to establish this step as reasonable. Thus, temporality in terms of the need to work with students' understandings is *visibly* absent here, in contrast to its simple absence in the theorizations within the previous studies. The teacher's attention towards the direct object without simultaneous attention to the indirect object points to a localization that produces the answer to the immediate problem through the provision of a disconnected procedure. This kind of localization has been widely criticized in the mathematics education literature (for example, see Artigue, 2011).

Discussion

The findings I have presented in this article indicate that more attention is devoted to progression in mathematical terms (to mathematical pasts and futures) than to the need to connect with learning pasts. My analysis suggests that even before learning trajectories are taken into account, many of the shortcomings seen in South African foundation teachers' handling of number can be related to poor understandings of progression in number. The ongoing acceptance of concrete counting can, therefore, be conceptualized as a mathematical focus on the present, without attention to mathematical futures related to number topics. This finding underscores the ongoing usefulness of Dowling's (1998) notion of specialization, which, like the South African curriculum move to specification of progression and pacing, can be applied to pedagogic texts without a need to take into account temporality in terms of students' prior understandings.

It may be the case that an understanding of mathematical temporality—of the past, present and future of mathematical topic strands—is necessary as a vantage point from which to interpret student questions and responses. The quotations from Ball (1993) that opened this article suggest so. However, whilst mathematical temporality may well be primary within teaching, the Askew *et al.* (2012) study attests to the fact that this temporality is necessary, but not sufficient. If understandings of temporality in terms of students' understandings of number cannot be linked with a mathematical temporality within teaching and then mediated, possibilities for learning in well-connected and solidly founded ways continue to be disrupted. This point of warning has been made before in other countries in the context of the introduction of tightly prescribed primary mathematics curriculum content and sequencing (Brown *et al.,* 1998).

The re-interpretation of South African research on number teaching in terms of temporal range points to two issues that traverse the findings relating to concrete counting. Firstly there is an emphasis on the "present" in mathematical terms, with inadequate or absent attention to mathematical pasts and futures. Secondly, across the instances of localization and the attempts at specialization when students were unable to provide a correct answer through counting, teaching often either accepts, or produces, the answer to the immediate problem without attention to the broader understandings and longer term efficiencies needed for autonomous student work with similar and related problems. This production allows lessons to progress without any need for learning to progress within them. The more general theorization of teaching in terms of these two temporal dimensions therefore appears to be useful for looking at foundation phase teaching practices in ways that encapsulate multiple, but overlapping, shortcomings through a common lens.

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From the archives

Editor's note: *The following remarks are extracted from an article by John Mason (1980), published in* FLM1(2). *They, along with the rest of his article, strike me as having some relevance to the writing published in the current issue.*

Bruner found it useful to distinguish three modes of internal representation which seem to describe stages in children's thinking. When asked a question, children seem to make use of the following internal representations:

Enactive:	able to respond only by recourse to previous practical experience. The classic example is a number question for which the child turns to a balance and physically performs the required acts. Here the response is by the musculature.
Iconic:	able to respond by recourse to mental images of physical objects or to an inner sense of pattern or structure. In the case of numbers having a balance in sight, or a drawing, can assist the work by extending the mental screen. Icons need no articulation because within a culture they need no definition.
Symbolic:	able to respond by using abstract symbols whose meaning must be articulated or defined. In the case of number, 3+4=7 now has meaning, and no recourse to the balance or balance image is needed.

Because Bruner was looking at stages in children's development, giving a slightly different perspective to Piaget's work, people seem to have identified

Enactive:	with physical toys
Iconic:	with drawings and pictures
Symbolic:	with words and letters

or, worse,

Enactive:	with primary school
Iconic:	with middle school
Symbolic:	with upper school

and missed the essential qualities which I describe as

Enactive:	confidently manipulable
Iconic:	having a sense or image of
Symbolic:	having an articulation of.

Notice too that symbolic expression must ultimately become enactive if the idea is to be built upon or become a component in a more complex idea. Thus to a pre-school child 1, 2, 3 are truly symbolic, having little or no meaning. With time and extensive encounters a sense of one-ness and two-ness develops which underpins the symbols and provides a source of meaning when 1, 2 and 3 are encountered in a new context. To proceed with arithmetic it is essential that 1, 2, 3 become enactive elements, become friends. If they remain as unfriendly symbols then arithmetic must be a source of great mystery.

Reference

Mason, J. (1980) When is a symbol symbolic? *For the Learning of Mathematics* **1**(2) 8-12.