A Theory of Intellectual Development

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PART III*

A FRAMEWORK FOR A REVISED PERSPECTIVE

From this revised perspective on human intellectual development, a set of issues emerges which have been accorded less than adequate examination within the Piagetian and Vygotskian perspectives.

1. Human development depends on the environment
2. The self is both autonomous and communal
3. Diversity and dissent are anticipated
4. Emotional intelligence is acknowledged
5. Abstraction is reconceptualized and placed in a dialectic
6. Learning is viewed as a reciprocal activity
7. Classrooms are studied as interactions among interactions

Human development depends on the environment

Subsuming the labor/production and reproduction metaphors under the evolutionary biology metaphor leads to the statement that in all phases of education, one must educate for a global society that includes living and nonliving things. It rejects the exclusive perspective of Vygotsky, inherited from Engels’ dialectic materialism, that views nature as subject to man’s dominance and mastery. It suggests that as we consider the materials of labor, we recognize that those materials are also the limited resources of the environment. It obliges us to consider how we change as the environment changes. It reminds us, as we seek to mold the environment to meet our needs, we need to respect that other living creatures share our dependence on this environment. Labor and production may be our gauge of progress, of movement towards the accomplishment of human goals, while reproduction brings into our awareness the cycles of human life, and the need to create an enduring and sustainable existence. As Stephen J. Gould [1987] has written, history is composed with two dichotomous views of time: “time’s arrow views history as an irreversible sequence of unrepeatable events”; while time’s cycle sees “apparent motions as parts of repeating cycles” in which “time has no direction” [p. 11].

The metaphors of labor/production and reproduction, like time’s arrow and time’s cycle, place human development in linear and cyclic progressions, but related by a dialectic rather than a dichotomy.

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Vygotsky in his theoretical work stresses the distinction between man and other animals as a standard of comparison for “higher” cognitive development. Placing his work within the setting of evolutionary biology allows one to examine human development not only in terms of our differences from other animals, but in terms of our similarities and common interests. It also encourages the consideration of a more diverse set of beliefs about the capabilities of others (animal, plant and inanimate objects) as regards language, thought, social behavior, and spirit. Fundamentally, embedding human development within evolutionary biology warns us against what Marilyn Frye [1983] has described as “the arrogant eye,” the view that “man is invited to subdue the earth and have dominion over every living thing on it.” With this view, man sees with arrogant eyes which organize everything seen with reference to themselves and their own interests [p. 66-7]. The placement of the reproductive metaphor alongside the labor/production metaphor further argues against the use of arrogant eyes to dismiss or diminish female models of development while elevating male ones [Gilligan, 1982; Brown and Gilligan, 1992].

The self is both autonomous and communal

Vygotsky described the development of self through the process of internalizing social norms. Wertsch [1985] describes this as the process by which a higher level process moves from the realm of the interpersonal to the intrapsychological. Essentially, Vygotsky argued that “it is necessary that everything internal in higher forms was external, that is, for others it was what it now is for oneself” [in Wertsch, p 62]. He saw the act of internalization as transforming: “it goes without saying that internalization transforms the process itself and changes its structure and functions.” According to Leont’ev, a student of Vygotsky, “consciousness is a product of society: it is produced. Thus the process of internalization is not the transfer of an external activity to a preexisting, internal “plane of consciousness”: it is the process in which this internal plane is formed” [ibid., p. 64]. Internalization is “the process of gaining control over external sign forms” [ibid., p. 65].

In the constructivist framework a construction of self is frequently developed in relation to objects. In a careful description of this development of self, von Glaserfeld [1978] distinguishes the construction of self as: 1) “part of one’s perceptual experience” and 2) “the locus of the perceptual (and other) experiences I am having” [p. 46]. The first construction entails creating a differentiation between one’s own body and other perceptual items. Watching my son learn to put his hand in his mouth provided a clear illustration that this is learned gradually. At age five months he began to like to suck on his hand. For days he...
worked on bringing his hand to his mouth. At first he would simply seem to find his hand in his mouth. Then he began to try to bring it there. He would see it in front of him, but he could not seem to bend his elbow intentionally to bring it to his mouth. This was puzzling, because the motion of bending the elbow was not a new one: he used it regularly in reaching for objects. He solved the challenge in an interesting fashion. If he wanted to bring his right hand to his mouth, he would reach across with his left hand and pull his right hand. His visual attention would remain on the right hand. His left hand was apparently not focused on, though it would bring the desired object, his right hand, to him. After another week he could bring the desired hand directly to his mouth without needing the other hand. The difficulty he had in bringing his arm to his mouth can be interpreted as evidence that it was not the physical action which challenged him, because this movement was not a new one, but doing it intentionally, reflexively controlling his own movements. A sense of self is created as the child constructs a permanent entity that coordinates sensory signals and gains control of physical movements, that is, gains motor control over visual items.

The second construction in von Glasersfeld [1978] is that of a self-regulating system: “the concept of an invariant that arises out of mutually or cyclically balancing changes may help us to approach the concept of self” [p. 60]. This invariance, he adds, is not a steady resistance “but the invariant is achieved... in a feedback loop, [where] we find the present act pitted against the immediate past, but itself already on the way to being compensated by the immediate future. The invariant... consists in one or more relationships—and relationships are not in things but between them. If the self... is a relational entity, it cannot have a locus in the world of experiential objects. It manifests itself in the continuity of our acts of differentiating and relating and in the intuitive certainty we have that our experience is truly ours” [p. 109].

The constructivist position presumes that a self develops through experiences in the physical world, and this is discussed much more extensively than the self that develops in relation to others. Kegan [1982], however, proposes a description which is compatible with von Glasersfeld but also draws on the feminist development of relations. He describes the evolution of the scheme of “object-relations” by first pointing out the etymology of the word, object, as object, to move or throw, which “together with ob is the motion or consequence of “thrown from” or thrown away from” [p. 76]. Constructing a scheme of object relations is, in the terminology of Kegan, “a motion, the motion of “throwing away from” of differentiation, which creates the object, and the motion of integration, which creates the object relation” [p. 81].

Kegan’s description of object relations complements and enriches that of von Glasersfeld. In it, he emphasizes that the construction of objects by the baby, up to eighteen months, might be better understood as the evolution of the baby-and-object-relation, and that together with this evolution of external objects is a loss and an anxiety for the child about its own organization, showing up as “separation anxiety.” “Emergence from embeddedness involves a kind of repudiation, an evolutionary recognition that what before was me is not-me” [p. 82]. Kegan’s description achieves a fundamental integration of cognition and affect, “because all objects are themselves the elaboration of an activity which is simultaneously cognitive and affective” [p. 83]. He defines affect as “essentially phenomenological, the felt experience of a motion (hence, e-motion)” [p. 81].

Kegan bridges the emotional aspects of psychoanalysis to the cognitive aspect of Piaget to declare a fundamental principle, basic to constructivism:

It is the greater coherence of its organization which is the presumed motive [White, 1959], a transorganic motive shared by all living things. A more cognitively-sounding translation of the motive is to say that the organism is moved to make meaning or to resolve discrepancy; but this would not be different than to say it is moved to preserve and enhance its integrity” [ibid., p. 84].

Kegan suggests that the combination of differentiation and integration yields a lifelong theme, which David Bakan called “the duality of human experience, the yearning for “communion” and “agency’’. The desire to preserve independence or autonomy is counter-balanced in turn by “the fear of being completely unseparate, of being swallowed up and taken over; and the fear of being totally separate, of being utterly alone, abandoned, and remote beyond recall” [ibid., p. 107]. This balance between the acts of differentiation and integration provides a theory which:

recognizes the equal dignity of each yearning, and in this respect offers a corrective to all present developmental frameworks which univocally define growth in terms of differentiation, separation, increasing autonomy and lose sight of the fact that adaptation is equally about integration, attachment, inclusion. The net effect of this myopia, as feminist scholars are now pointing out [Gilligan, 1978; Low, 1978], has been that differentiation (the stereotypically male overemphasis in this most human ambivalence) is favored with the language of growth and development, while integration (the stereotypically female overemphasis) gets spoken of in terms of dependency and immaturity. A model in pursuit of the psychological meaning and experience of evolution—intrinsically about differentiation and integration—is less easily bent to this prejudice [p. 108-9].

Placing the metaphors of labor and production with that of reproduction helps to remind us to balance the ideas of autonomy and of connection in our interpretation of models of cognitive development. It allows one to assert that greater coherence can be a goal of intellectual development as much as active control or manipulation.

3 Diversity and dissent are anticipated

As discussed earlier, Vygotskian theory seems to offer no way to explain and support invention, creativity, and dissent. Without a goal of autonomy one is unlikely to encourage students to invent new approaches or to challenge existing ones. Educating for a secure and sound development of one’s potential for acting and reflecting is
a basic quality in Piaget, but its articulation in Vygotsky is limited. As asserted by Piaget, in the revised theory, the individual is viewed as more than the internationalization of society's norms, as also a product of his/her own experiential path and unique activities of sense-making.

In order to acknowledge the importance of diversity a theory of intellectual development must assert the value of multiple views. The placement of the theory within a biological evolutionary framework allows for this in the form of bio-diversity. Having a broad variety of intellectual perspectives from which to select is arguably the best assurance of more viable conceptions. The selection procedure is the means of assessing the endurance of the diverse proposals. Hence, selection functions as the mechanism by which coherence is assured.

I criticized radical constructivist views for their failure to recognize a person's placement in many classifications, not only as members of a developmental age-group. In today's multicultural societies it seems imperative to recognize one's identity as involving membership of many different sociological groups. Each of these groups has its own identity, values, norms, and means of acting. A theory of intellectual development must be able to handle these multiple identities. To this end, it seems important to revise the view of autonomy expressed in radical constructivism. Such a revision would start by acknowledging the view that the self is constructed as a viable actor in accomplishing its purposes. It is, physically, the most immediate actor, in that we can control our physical actions. However, there is nothing in such a concept of autonomy that denies the possibility of creating a sense of identity within a dyad, a group, or a community. Surely a mother-child dyad possesses both the physical and the emotional attachments that give it an identity even before birth. A family, a marriage, a partnership, a working group, and a community, can allow the construction of identities in a similar fashion. Identities are not limited to solitary individuals.

That one both forms such relationships anew and emerges from such relationships commits one to the previously stated view of the self as communal and autonomous. Only, according to the revised view, the community is also assumed to have some autonomy in relation to other communities. This revision of the view of autonomy allows us to propose a role for dissent. Dissent anticipates the existence of competing views within a diverse population. Also, it does not simply assume that these views can be made compatible, or will merit equal acceptance. Some form of negotiation and resolution of conflict needs to be established as a part of knowledge development.

To see the force of asserting a means of dissent, consider the relationship between the metaphors of labor/production and reproduction. Over history, feminists have repeatedly documented instances where the social value of reproduction has been diminished and subjugated to labor and production. For example, Marilyn Frye [1983] describes the potential connections between the tool/material relation and systems of exploitation and oppression. The ax is used to transform or manipulate the tree to its telos. The ax retains its identity while the tree is rearranged to accommodate the ax's purpose: This is, arguably, the tool-material relationship that underlies the labor/production metaphor. Frye points out that when such manipulation is applied to animate objects by humans we see forms of exploitation. Exploitation may exceptionally involve killing, but it can also involve the manipulation of other beings through shaping and restriction such as harnesses, braces, shafts, and other paraphernalia, a process referred to as breaking or training. When the manipulation process is applied to exploit another person or persons, we see the development of systematic networks of forces and barriers that act to reduce, immobilize, mold and shape. Ultimately, Frye points out, enslavement results if the dis-integration of the Other results in attaching the victim's will, interest and intelligence to that of the exploiter [p. 57-60].

Within the framework of the reproduction and labor/production metaphors we see in Frye's description how the misuse of the production metaphor as a means of conceiving of human relations can result in oppression and enslavement. A means of dissent, as well as being a stronger application of the reproduction metaphor, is necessary in order to protect against such imbalances. Thus, dissent becomes a necessary construct to ensure the maintenance of the dialectic.

4 Emotional intelligence is acknowledged [7]

Bringing the reproductive metaphor into knowledge construction necessitates a reconsideration of the role of emotion in cognitive development. Vygotsky [1962] acknowledges the importance of a relationship between intellect and affect and wrote: "Their separation as subjects of study is a major weakness of traditional psychology since it makes the thought process appear as an autonomous flow of "thoughts thinking themselves," segregated from the fullness of life, from the personal needs and interests, the inclinations and impulses, of the thinker" [p. 8]. In fact he argues for locating the roots of language in emotion and gestures: "The preintellectual roots of child development have long been known. The child's babbling, crying, even his first words, are quite clear stages of speech development that have nothing to do with the development of thinking. These manifestations have been generally regarded as predominantly an emotional form of behavior. Not all of them, however, serve merely the function of release" [p. 42].

Much of the time, Vygotsky [1986] locates emotions in the biological realm and thus views them as having little connection with the processes involved in higher mental thought. He connects them to animal-like behavior. For example, he wrote: "In the sphere of emotions, where sensation and affect reign, neither understanding nor real communication is possible, but only affective contagion" [p 8]. Before the development of the higher level functions, he sees the role of emotions as potentially damaging to higher level thought: "The affective states producing abundant vocal reactions in chimpanzees are unfavorable to the functioning of the intellect. Kohler mentions repeatedly that in chimpanzees emotional reactions, particularly those of great intensity, rule out a simultaneous intellectual operation" [1962, p 40].

However, ultimately he does believe that an integration of affect and cognition is possible and desirable, but a successful integration for Vygotsky depends on having the
affect controlled by the intellect rather than having the intellect pushed about by affect. He wrote about:

the dialectical law, that in the course of development causes and effects change places. Once higher mental formations have emerged on the basis of certain dynamic preconditions, these formations themselves influence the processes that spawned them. Above all the interfunctional connections and relations among various processes, in particular intellect and affect change [in Wertsch, 1985, p. 190; my emphasis]

I would suggest that the treatment of emotions as primitive, as requiring release, and as needing to be controlled by the intellect, represents an example of how a sole emphasis on a productive model of cognitive development can serve to disenfranchise female development. In a reproductive model, the expressions of relationship developed through the tones of parent-child communication, including approval, pleasure, joy, warning, disapproval, and fear, would lead to the development of language, but the emotional character of these exchanges would not be stripped away nor be judged secondary to cognitive development. As Vygotsky predicted, one would seek the dialectic unity of emotional and cognitive thought, and both the intellect and consciousness would be viewed as involving both. Unlike in Vygotsky's theory, an equitable but different contribution would be presumed from each.

Salovey and Mayer [1990] propose the term, "emotional intelligence" which they define as "the subject of social intelligence that involves the ability to monitor one's own and others' feelings and emotions, to discriminate among them and to use this information to guide one's thinking and actions" [p 189]. They also define emotions as "organized responses, crossing the boundaries of many psychological subsystems, including the physiological, cognitive, motivational and experiential systems. Emotions typically arise in response to an event, either internal or external, that has a positively or negatively valenced meaning for the individual. Emotions can be distinguished from the closely related concept of mood in that emotions are shorter and generally more intense" [p 186]. Finally, Salovery and Mayer create a conceptualization of emotional intelligence which includes: the appraisal and expression of emotion, the regulation of emotion, and the utilization of emotion.

The introduction of emotional intelligence into discussions of mathematics education allows one to assert that both facilitating and debilitating emotions play a significant role in learning, and that the emotional qualities of classroom interactions will exert a significant influence on what is learned. For example, if young women view mathematics classes as prone to embarrassing public exposure, and unwelcome risk and competition, then one can see why young women often fail to persist in mathematics. Incorporating a facilitating view of emotions would allow one to recognize, for instance, that the tendency of young women to seek a deeper level of understanding, because of holding less instrumental views of mathematics, is a positive characteristic.

5 Abstraction is reconceptualized and placed in a dialectic. As indicated in the discussion of the limitations of the Vygotskian perspective, two interpretations of Vygotsky's theory can be proposed. In one, the dialectic between thought, as it evolves from practical intelligence, interplays with language to create a dialectic unity. In this case, both practical activity and facility with signs are necessary to create complex human behavior and consciousness.

Alternatively, one can assume an interplay between practical activity and sign use, but assign to sign use the governance of practical activity while neglecting the ways in which practical activity constrains and guides sign use. If this path is taken, then there is a privileging of abstraction to the detriment of practical activity. The result may be, in fact, a detachment of languaging from practical activity and the development of modes of thought that alienate humanity from everyday activity.

Two versions of this alienation can be witnessed in academic circles. One is the result of an over-emphasis on social interaction based in verbal exchanges without reference to other kinds of shared or individual activities. Thus the basis for knowledge is assumed to be solely an issue of human negotiation, influence, and decision-making, without regard for human action.

The second, criticized by feminist scholars, is the disembodiment of the mental from the physical. Cartesian dualism has been followed by a post-modernist tradition that seems to desire the elimination of the body from discussions of intellectual development altogether. Feminists have argued that this distancing from the physical, as a form of objectivity, has already made the female body an object of male manipulation. Leaving the physical out of discussions of knowledge allows oppressive practices to remain as part of the personal, unacknowledged practice of dominant cultures.

An alternative is to point out that with any practical activity systematicity develops once it has been undertaken repeatedly and in multiple forms [Ceci, 1990]. Musicians who do not use musical notations surely understand phrase, signature, cadence, and rhythm, even though they may not write their compositions down. Gamblers at the track possess deep insight into probability despite their lack of a standard notation. Sign use, the development of symbolic systems such as algebraic expression, models, musical notation, architectural plans, do serve as means of communicating and reflecting about such practical activities; but as with any representation, there is loss as well as gain.

For example, an ellipse can be described as a relation for which $x^2/a^2 + y^2/b^2 = 1$ and in doing so, describe all possible ellipses that are symmetric with respect to both coordinate axes. Alternatively, I can build a variety of tools which trace out ellipses. One which was in use by South African carpenters is a board with narrow perpendicular slots and a stick placed so that a nail at each end of the stick sits in each slot [Millroy, 1991]. A hole is drilled in the stick between the two nails and a pencil is placed in the hole to trace a curve. As one drags one nail up and down one of the slots, consequently dragging the other nail across the other slot, the pencil traces an ellipse. The action gives one experience of the variety of rates of change involved in the
description of the ellipse directly. What one “learns” is not particularly well captured by the equation listed above. Geometrically, in terms of the loci of points, the same outcome is achieved. Epistemologically, the outcomes differ. In carpentry, as in many other trades, mathematics is experienced in action, yet typically, when asked, the “experts” deny any mathematical competence.

My concern, as I expressed earlier, is that mathematics can easily become detached from its practical activity. And frequently when writers discuss mathematics the practical activity is systematically eliminated. For instance, Newton is well known for his contributions to mathematics; his genius is applauded widely. And yet he owes a debt to Hooke unacknowledged by history, for Hooke was the curator at the Royal Society for forty years and in this role, was required to produce each week a physical demonstration of scientific principles. According to V. I. Arnol’d [1990] many of Newton’s insights were formalizations of Hooke’s demonstrations, and yet only in Hooke’s Law do we currently credit Hooke’s achievements. The mathematics was regarded as the formalization of the results in symbolic form. These are not simply errors of historians but systematic attempts to hide the genesis of the ideas. Newton destroyed the descriptions of Hooke’s demonstrations to protect and enhance his own reputation and contributions. Within the framework for cognitive development just proposed such an elimination of the genesis of ideas would lessen rather than enhance the value and legitimacy of the contribution.

I am proposing that there are two historical roots of abstraction: one which is epistemological; the other is rooted in political oppression and elitism. The history of the term abstraction itself betrays these dual roots, as does its common usage. [8] “Abstract” derives from the Latin “tra-here” which means “draw”, and the prefix “ab” which means “away from.” That is, it means “withdrawn.” The word entered into common mathematical discussion in the 17th century in the work of Hobbes, Newton, and Leibnitz, but its primary use came from the priesthood. It was closely connected with “absolve” and “absolute”, which were originally past participles of “absolve”, meaning free from sin or imperfection or material consideration. Quoting the Oxford English Dictionary, from a religious tract of 1690, “The more abstract therefore we are from the body, the more fit we shall be both to behold, and to endure the rays of the Divine Light” [J. Norris, 1690, The Beatitudes].

One path of mathematics thus evolved as a highly valued form of penance—withdrawn from material considerations. This is the history of pure mathematics, where purity is the goal of the penitent. And mathematics is seen to “elevate the mind” and get at the “essence of the truth.” Mathematics was viewed as the discipline of the mind. It is mathematics in this sense that creates a society of priests, with implicit ties to monastic training. It is no wonder that the mathematics that evolved from this tradition led to the elitism and isolation that characterize the milieu of many mathematics departments. And it is ironic that as a number of us challenged “absolutism” in mathematics [Confrey, 1980; Ernest, 1989] we did not reexamine its relationship to abstraction.

What is the character of an abstraction that does not assume disembodiment and absolutism? I would propose three possible solution paths: the recognition of 1) a genuine dialectic between practical activity and sign use; 2) the value of multiple forms of representation; and 3) the role of action in the act of abstracting.

Taking the first path, I suggest that we establish a dialectic of grounded activity and systematic inquiry. Sign use is assumed to make systematic inquiry more accessible. Conceptual development then becomes the effective synthesis of these two activities, demonstrated by a sign use that captures the character of the grounded activity while creating its own systems of manipulation and movement; and grounded activities that predict or anticipate interesting aspects to be represented in symbolic forms. By portraying this as a genuine dialectic in which neither part is presumed to require more intelligence than the other, one communicates an important sense of the equity of different forms of human labor. One rejects increasing alienation of one’s physical being, an essential element of a reproductive perspective.

Secondly, I have elsewhere suggested the importance of establishing an epistemology of multiple representations [Confrey, in press c] in which one recognizes that all demonstrations, whether as grounded activity or as symbolic representation, require one to learn to “act” and “speak” in context, and it is the ability to move among representations that signals intellectual progress. The assumption in this viewpoint is that all representations involve showing and masking, thus one no longer seeks to find a single form, the Platonic ideal, but to contrast and compare the different uses of different representations. Accordingly, the history of mathematics becomes not only the story of the establishment of more and more encompassing generalities but also the acknowledgment of the value of distinctions within particulars. In this approach contextualization becomes as valuable as decontextualization.

Finally, I point out that though abstraction may mean withdrawing from the particular objects of a situation, the action that creates the concept is not removed. For instance, symmetry is not abstracted from the activity of folding but only from the particulars of the medium in which the folding takes place. Even such abstract concepts as cyclic groups, limits, or derivatives, all have significant ties to actions, and hence an understanding of abstraction may be revised to mean pulling away from the trappings of the construct while retaining ties to the action.

Consider the idea of averaging. Students experience multiple competing views of averaging [Mokros and Russell, 1992]. One concerns the most likely outcome, another concerns balance, and a third concerns equal distribution. Each has simple roots in human activity, based in actions related to “most”, “balance”, and “equal sharing.” Only the third is easily seen to generalize as the sum of the individual values divided by n. Developing the connections to the first two requires rather careful curricular development and activities in probability and the use of histograms. According to the portrayal of mathematics in our revised theory, conceptual development in averaging requires one to recognize the different forms of action, to
be able to represent the idea in multiple ways, and to coordinate the grounded activities with the systematic inquiry.

In this revised view the Piagetian idea of schemes may be useful. Schemes signal connections to one's goals, forms of actions, means of communication and of reflection. If schemes are then situated within the larger view of activity theory, one creates an understanding of reflective abstraction that makes the concept of pure mathematical abstraction obsolete.

Elimination of the oppressive view of abstraction demands that one disclose the ways in which keeping mathematics mystifying, secretive, and unapproachable, serves to preserve the status quo of a powerful elite. Allowing mathematics to continue to require students to disengage from their personal sources of experience and to learn a system of rituals that makes little sense to them but which will admit them to the ranks of the elite is one of the most effective ways of maintaining this oppression. A critical view of mathematics is essential to changing this current state of affairs.

6 Learning viewed as a reciprocal activity
In emphasizing the adult/child relationship in learning, Vygotsky recognized the intergenerational character of teaching and learning. Educating one's children is a powerful part of parenthood, and to ignore the role of such a force in institutions of education is unproductive. Furthermore, Vygotsky emphasized that learning should lead development, and that a failure to allow it to do so slows down and limits a child's potential.

If, as Vygotsky has posited, learning must lead development, and interactions with an adult or a more expert other must provide this leadership, then that more expert other must be doing more than "finding out where the child is at." S/He must strive to move the child forward. One way to do so is to admit the gap between expert and novice knowledge and allow the child to use the language of the expert and to perform the expert routines with the goal that eventually those performances and routines will be transformed from pseud-concepts to concepts.

The potential issue with the Vygotskian approach concerns the nature of the adult-child interaction. Most researchers in the Vygotskian tradition consider this a form of apprenticeship, a model closely connected with labor and production. One disadvantage of such a model, as I argued earlier, is that it suppresses the natural diversity in children. Now I wish to suggest that bringing in the reproductive metaphor in relation to adult-child and child-child interactions can help to overcome this limitation.

In studies of mother-child vs. father-child interactions researchers have documented that mothers tend to decenter towards the child's activities and goals and to use the child's goals as a means of educating. Fathers, in contrast, tend to hold firm to their original intentions in interactions and to coax the child to strive to accomplish their goals. Neither of these can be judged better, but an extreme version of either will limit a child's growth, leaving the child either without any forward progress or without any possibility of success. Analogously, by asserting both reproductive and productive metaphors in conceptual development, one asserts that both father type and mother type forms of interactions with children are desirable.

Just as Vygotskian theory fits more closely with the father-oriented view of parent-child interaction, constructivism can be related to the mother-oriented view. Constructivism postulates a gradual but continuous process of growth and transformation from a child's conceptual world to the acquisition of scientific knowledge. If there is one central maxim on which all constructivists agree it is "start where the child is at". And "starting where the child is at" is demonstrably challenging, for it requires significant effort to be put into trying to understand where the child's thinking is, that is, how the child views the problem. Researchers have documented that this strategy has a variety of positive instructional outcomes. It increases the child's self awareness, it allows the teacher to hear how the child views the problem, it focuses on task-based interactions, and it encourages the two to find bridges in language. This genuine interest in understanding the student's perspective is lacking in Vygotsky's writings, which predate many of Piaget's insights into the diversity in child thought.

Also missing in the Vygotskian position is the realization that from such interactions the teacher learns as well. The expert learns how the child sees things, and at the same time, the adult gains new mathematical insights. This view of adult learning, of changes in an adult's knowledge of content is missing in both Piaget and in Vygotsky. Only recently have constructivists come to acknowledge that they are not just discovering or articulating student schemes but learning genuine mathematics for themselves [Confrey, 1991]. I have used a distinction between "voice" and "perspective" to signal the two kinds of learning that result from a reciprocal interaction between a student and a teacher. Voice refers to the student's conceptions and perspective describes an experienced person's view of the material. I have suggested that in clinical and teaching interviews one seeks to model the student's voice through the perspective of a better informed knower; however, I have also pointed out the importance of using the student's voice as a way to reexamine, modify and strengthen one's perspective. Both voice and perspective contribute important epistemological content to the teaching-learning interaction [Confrey, in press b].

A fundamental result of this approach is the development of a deep respect and support for diversity. If one enters the educational enterprise with arrogance one's own views of knowledge quickly overpower the insights of the children. When the classroom norms are developed in such a way as to promote the exchange of student methods with mutual tolerance and respect, the children themselves become increasingly confident of their contributions and the system becomes self-reinforcing. In both peer relations and in adult-child interactions, the roles of expert, teacher, learner, and novice, are flexibly drawn. (This does not mean that the teacher ever becomes "the same" as the student, but only that she or he acknowledges opportunities to take on multiple roles.)

A reproductive view of human development in which cycles of interaction are expected, in which the student voice is solicited and valued, and in which authority does not come from the dispersal of knowledge but from the
creation of a knower, is a key quality of empowerment. Jean Baker Miller [1986] in A psychology of women discusses how the domination and subordination of women has limited the development of humanity. She recognizes, however, that there are times between two people when there is temporary inequality. She describes this relationship as follows:

The "superior" party presumably has more of some ability or value quality which she/he is supposed to impart to the "lesser" person. While these abilities vary with the particular relationship, they include emotional maturity, experience in the world, physical skills, a body of knowledge, or the techniques for acquiring certain kinds of knowledge. The superior person is supposed to engage with the lesser in such a way as to bring the lesser member up to full parity; that is, the child is helped to become the adult. Such is the overall task of this relationship. The lesser, the child, is to be given to, by the person who presumably has more to give. Although the lesser party often also gives much to the superior, these relationships are based in service to the lesser party. That is their raison d'être. It is clear, then, that the paramount goal is to end the relationship, that is, to end the relationship of inequality.

In this passage we see that Baker Miller recognizes the educational purpose of imbalance, acknowledges that the learning goes both ways, and identifies the service component in the act of educating.

For example, when a group of fourth grade students was asked to identify the prime numbers less than 100, three strategies were revealed. One student went through the process of crossing out every other number, every third number, every fifth number, and so on, until the primes were left. Another student displayed the odd numbers in groups of ten in such a way that all the odd numbers ending with 5 were in one column, and proceeded to look for number patterns. A third student used divisibility rules to eliminate all the multiples of 2 and 5, and when he did not know how to test for multiples of three he mentally broke the numbers into sums or differences and examined each term for any shared divisibility. For example, he saw that 51 was not prime by viewing it as 21 + 30 and pointing out that both 21 and 30 are multiples of three. Therefore, he argued, 51 was divisible by 3. Another student thought of it as the difference of 60 and 9.

The teacher set out with the instructional goal of having the students learn the rules for divisibility by 2, 3, 5, 9, 11 and to have them explore the patterns in a 100s table. She did not anticipate the solutions by sums and differences, however, and used this student proposal to revise her view of the patterns in the 100s table. She found a way to use all three methods successfully in the patterning exercise. One could mark off every nth number (method 1), could find the vertical and diagonal patterns in the multiples (method 2), could teach them to test for divisibility (method 3). The student's way of breaking into parts and testing each for divisibility led the teacher to explore the diagonals in the hundreds table more carefully. It was easy to see that the movement from one multiple of three to the next was "down one row and over three spaces" (an increase of ten and minus three), and one could also see a pattern of "down two rows and over one" (an increase of twenty plus one).

This is an example of a teacher learning from students' diverse methods and then using her knowledge to lead students to greater insight into the multiple forms of representations.

7 Classrooms studied as interactions among interactions

One of the most profound aspects of Vygotskian theory is the use of dialectic relations, such as the dialectic between thought and language. In the revised theory, I have proposed a dialectic between grounded activity and systematic inquiry. In the revised view, I claim explicitly that both types of activity guide each other.

Classrooms, according to this revised view, can now be described as places in which children engage in grounded activities and in systematic inquiry. Instead of suggesting, however, that grounded activities are essentially individual and systematic inquiry is essentially social, I propose an alternative description drawing on Vygotsky's concept of mediation. In this way one can view grounded activities as actions involving practical activity which are mediated by one's interactions with others. In contrast, systematic inquiry, which involves communication through the use of signs, can be viewed as social activity mediated by one's experience in grounded activity.

This proposal makes it clear that one's physical interactions with materials and tools are influenced by one's social interactions. Likewise one's use of signs is influenced by one's personal experience with grounded activities. Looking at the interactions between these two forms of mediated activity may yield some useful insights into how we might successful educate people in mathematics.

Radical constructivism, with its focus on the development of student conceptions and its biological descriptions, should admit this reproductive metaphor more easily. Piaget focused on the generative character of human thought. Reproduction plays a central role in evolutionary biology. Bruner and Bornstein [1989] in the introduction to a book entitled Interaction in human development wrote, "Wherever one looked, it seemed to us, there were forms of interaction that were important in their own right, forms of interaction whose nature was somehow not captured by being reduced to the role they played as influences on intra-individual growth factors" [p. 1]. And "Again and again, we found, research and theory point to specific interaction experiences and specific times in development affecting specific facets of growth in specific ways" [p. 12]. In this introduction they identify multiple forms of interaction: tutor-tutee, genetic and environmental interactions, individual and cultural interactions, and so on.

An interactional cycle

![An interactional cycle](image)

Figure 5
This draws our attention to the interplay between two factors. While the socio-cultural perspective leads us to focus heavily on the verbal interactions among students and teachers (discourse, dialogue, register, etc.), the cycle reminds us to consider, at the same time, what children are doing with the elements of the non-human environment. What materials are they working with, what mental operations might they be building? What actions are they taking? What constraints are they experiencing?

For example, when a group of children try to solve the problem: share 162 jelly beans among 3 children, a child who chooses to work with Dienes blocks might approach this by first trying to share the "flat" (10 by 10 square) and that child’s choice of materials and strategy differs considerably from another who chose to trade the flat to produce 16 longs (1 by 10 sticks) and two singles. However, once the language of "trades" is introduced into the classroom, the first child’s problematic may not yield a direct solution (how to split the 100 block into three parts), but the child will nevertheless obtain a solution to the problem. How s/he comes to understand this new solution may not mirror how the child understands the method s/he originally proposed. I believe it is an open question whether both methods are as meaningful, generative, and enduring. Understanding the interplay between these different forms of interaction is important to understanding the development of knowledge.

One could choose to describe this as an interplay between social and individual aspects; however, I chose not to make social vs. individual the basis of the interaction in order to retain the dialectic between the two theoretical metaphors at the root of this new theoretical approach. Thus one’s encounters with materials, environmental factors, and the natural and physical world reflect the Piagetian radical constructivist view of the world—our biological adaptation to the constraints we encounter; and our interaction with others reflects the way in which our biological, physical, material natures are shaped by our participation in—or, better—our immersion in a cultural perspective. At the same time we must recognize the likelihood that the child’s approach to this problem of sharing one hundred evolves because of our decimal-based number system, which has a biological basis in our ten fingers and toes and a cultural trajectory in terms of how that particular basis was developed and socially accepted.

Conclusions

There is no question that all scholars in mathematics education would benefit from a thorough study of both Vygotsky and Piaget. In this paper, I have sought to describe the perspectives of each concisely, hoping that such summaries will encourage further examination of their original work. Examples of how each theory deepens our insight into the classroom are provided, as well as outlines of each theory’s limitations. In offering these discussions my hope has been to do justice to the contributions of each theory while creating a tension between them.

In the final section of the paper, I have tried to move beyond the tension between the theories to create a bridge between them. My intent is not to minimize the very real difficulties in doing this. And rather than create a strictly logical dichotomy between the two theories (such as: one is individualistic and the other is social), I have chosen to select the evolutionary biology metaphor as primary, and then create a dialectic relationship between the substructures of labor/production and reproduction. The umbrella status of the evolutionary biology metaphor, however, is recognized as tentative and evolving in the light of its own historical/cultural character.

The final sections of the paper then articulate the cluster of ideas that emerge as significant in the production-reproduction dialectic. Many are revisions of issues identified as problematic in the two theories. Most revisions are tied closely to the changing circumstances we face in North America at the close of the twentieth century. For instance, I choose to make biological evolution the bridging construct and to recognize the importance of environmental concerns. Vygotsky’s recognition of how physical tools transform products is selected as preferable to Piaget’s non-contextual treatment of global constructs in part because of its importance in understanding the impact of technological tools in our time. Identifying and stressing the importance of diversity as a fundamental construct is a particular necessity given the multi-cultural nature of our country and the increasing international influences on all our lives. Feminist theory is called on to argue for a balance in the treatment of connection and autonomy, and for a view of adult-child interactions.

Finally, the paper has attempted to illustrate that in a climate of reform, we need theories to guide us. At the current time, in the United States, the National Standards for Curriculum and Evaluation are serving as a document for reform. Yet much of that reform threatens to be more a matter of producing the correct slogans than achieving lasting changes in practices. Theories, in so far as they create systems of thoughts, may be an important vehicle for creating genuine reform. If some of the ideas in this paper can contribute to enduring reform, it will have accomplished its purpose.

Notes

[7] I am particularly indebted to Elizabeth Rowe for discussions we’ve had on this topic and for articles she has provided, including her term paper on this topic.

[8] I am particularly indebted to David Dennis for discussions we’ve had on the topic of abstraction and for the historical investigations he conducted as a research assistant.

References


The best way of overcoming a difficult problem is to solve it in some particular easy cases. This gives much light into the general solution. By this way Sir Isaac Newton says he overcame the most difficult things.

David Gregory (1705)
(By abandoning the historical and psychological presuppositions of previous periods, and by devoting itself to an object it wanted to describe in an exact and precise fashion, linguistics found an example of rigor in the mathematical sciences, whose models and concepts it borrowed. For a time this mathematical rigor was believed to be absolute; no one considered that a mathematical model (like any formalist model, by the way), once applied to a signifying object, requires some justification and is only applicable because of the implicit justification the researcher gives it. The ideology one wanted to escape from thus turned up again in latent form in the semantic root of the model applied to the description of language.

In this way, by moving away from empiricism, the study of language should enable science to understand that its ‘discoveries’ depend upon the conceptual system that is applied to the object being studied and that these discoveries are more or less given ahead of time. In other words, linguistics believes that its discoveries of the properties of language are dependent upon the model used in the description, even upon the theory the model belongs to. This has resulted in a considerable interest in the innovation of theories and models, rather than a sustained investigation enabled by the use of a single model. Linguistics doesn’t describe language as much as make its own language. This turnabout, which seems paradoxical, has a double consequence. On the one hand, theoretical research in no way implies that language remains unknown, buried under the mass of forever-new models of linguistic operation. But on the other hand, the attention of scientific discourse is drawn to the very process of knowledge as a process of the construction of a model, which is overdetermined by a theoretical, or even ideological agency. In other words, the science of language is not oriented solely towards its object. *la langue*; it is also oriented towards its own discourse, its own foundations. Every discourse about language is thus required to think about its object *its* language, through the model it has chosen for itself, that is, through its own matrices. Without ending up in a relativism and an agnosticism that would deny the objectivity of all knowledge, such a means of proceeding forces linguistics (and every science that follows its path) to question itself about its own foundations, to become a science of its process, while remaining the science of an object.

Julia Kristeva
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