On 'How to Make Our Ideas Clear': a Pragmaticist Critique of Explication in the Mathematics Classroom [1]

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Two years ago, at Easter, I was busy organizing a conference for primary school teachers. A local school - the only one with an amphitheatre large enough to accommodate all the teachers - offered to host the event While the principal gave me a tour, he happened to describe another in-service session on mathematics education that had taken place recently in that same amphitheatre. Participating teachers, he told me, observed a tutor teaching a class, which, for the purposes of the session, had been moved to the amphitheatre. "The invited teacher did a good job", he added

Later on, I stood alone in the amphitheatre, trying to imagine the scene: the tutor teaching the class, while the teachers looked on My musings brought to mind two of Thomas Eakins' most famous paintings (*The Gross Clinic* and *The Agnew Clinic*), along with Ray Carney's (1998) essay entitled 'When mind is a verb: Thomas Eakins and the doing of thinking'. In the paintings reproduced below, the master surgeons, with the help of their assistants, operate on a patient while quite a number of students observe the operation The master surgeons are depicted in a state of apparent withdrawal from the act of operating, still holding their surgical blades, with facial expressions revealing intense thinking.

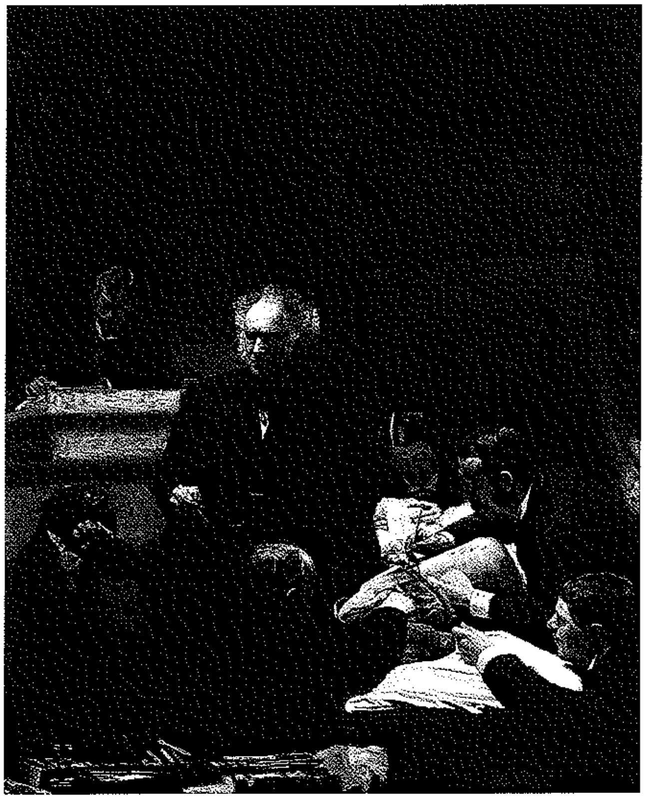

Figure 1: 'The Gross Clinic' by Thomas Eakins (1875)

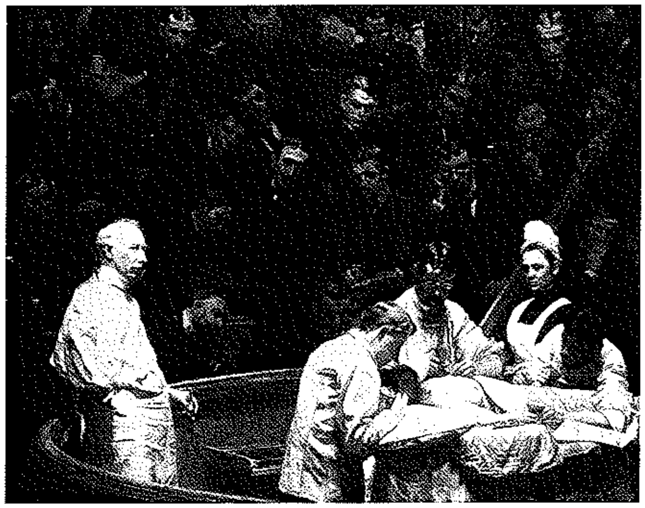

Figure 2: 'The Agnew Clinic' by Thomas Eakins (1889)

Eakins' paintings, Carney suggests, dramatize an attempt to supplement a philosophy of 'being' with a philosophy of 'doing' Gross and Agnew are successful in erasing:

> the distinction between manual and mental dexterity, insofar as their scalpel-wielding hands are clearly energized and mobilized by their thoughts as much as their thoughts are disciplined and instructed by their practical performances. (p. 383)

They both have a pair of "knowing hands", as Pimm (1995, p. 27) would say, in contrast to the dozens of other arms and hands of all the rest of the participants in the paintings who appear to be failing to blend hand and mind The arms and hands of the patient's mother, for instance, express her emotions but remain uncontrolled while those of the surgical assistants are dextrous but unimaginative Both are examples of incomplete amalgamation of action and thought

Ostensibly, paradigmatic instruction follows a pragmatic logic since the master surgeon/teacher must translate ideas into action. The instructor stands as an example of "heroic mind" (Emerson, 1837/1983, p. 60), in the sense that he succeeds in bridging the gap between theory and practice. Carney's 'pragmatist' analysis, drawing out the links between Eakins' paintings and the teachings of his contemporaries James and Dewey, emphasizes the fact that thinking consists in action The pragmatist approach for William James (1907) provided a method that directs us to "concreteness and adequacy, towards facts, towards action and towards power" (p. 51). Thus, in part, the seductiveness of paradigmatic instruction in training programs for pre- and in-service teachers in Greece can be explained [2]

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What a pragmatist analysis overlooks, though, is the fact that both the paintings and the in-service session on mathematics education I described at the beginning are learning experiences as well as experiences of learning about learning. Besides the presentation of content, 'novices' are taught to observe the center of activity, the 'experts', from a distance. In the case of the in-service session in mathematics education, paradigmatic teaching contributed not only to the formation of methods for teaching the content of mathematics, but also, even if not explicitly stated, to the formation of the essence of belief concerning how the learning and teaching of mathematics might be approached in the classroom.

Instruction in the mathematics classroom often involves a paradigmatic explication of material. Children are asked to follow along with the reasoning of an 'experienced' adult who actually acts on the part of the children. Knowledge and understanding of the material by the children are measured by the ability of the children to catch up with the reasoning of the teacher.

In this article, I focus on the legacy of paradigmatic instruction in the Greek mathematics classroom, arguing that it is based in false understanding of pragmatic logic as simply the 'translation' of ideas into action. Elaborating on Peirce's *pragmatic maxim*, I differentiate a 'pragmaticist' [3] framework for mathematics education from a paradigmatic one, which places an 'expert' at the center of activity. Peirce's pragmatic maxim locates the utmost clearness of ideas in our conception of the effects of all the practical bearings we conceive the object of our conception to have (W 3, p. 266). The maxim of pragmatism for Peirce is not oriented towards practical reactions and sensations, as was James', but to a mode of being, a habit of action or a habit of thought (Thayer, 1981).

I will support my argument with a discursive analysis of two short interactional sequences between the teacher and children in a sixth-grade class in a primary school in Greece. In this article, I also intend to draw attention to certain linguistic phenomena that characterize social encounters and, therefore, are partly responsible for framing the interactional space of that encounter.

From paradigms to the production of a 'practionary'

> Well, that's what we do first. We open the little trunk where we keep the materials for the circle, we empty it, and what do we have? We have a center, a diameter, a curve. [...] Everything I might need I take out. Same thing as if we wanted to install a faucet. I go and take the toolbox. (Sixth-grade teacher addressing his class, Volos, Nov. 2000)

The phrasing of Peirce's *pragmatic maxim* can easily mislead. In his famous 1878 essay 'How to make our ideas clear' and in his later attempts to clarify the meaning of the maxim, Peirce identifies three grades of clearness of ideas:

> A clear idea is defined as one which is so apprehended that it will be recognized wherever it is met with, and so that no other will be mistaken for it. (W 3, p. 258)

Let us take the idea 'red', for instance. In order to obtain the first grade of clearness of the idea 'red', we should learn to recognize the color 'red' under any disguise by means of extensive familiarity of instances of the idea. We can easily distinguish 'red' from 'blue' or 'black', even if we might need to think twice for certain instances of 'red' that differ in lightness, saturation or hue.

In the second grade of clearness:

> A distinct idea is defined as one which contains nothing which is not clear. This is technical language; by the *contents* of an idea logicians understand whatever is contained in its definition. So that an idea is *distinctly* apprehended, according to them, when we can give a precise definition of it, in abstract terms. (W 3, p. 258; *italics in original*)

In a technical definition, then, we may describe 'red' as:

> the hue of the long-wave end of the visible spectrum, evoked in the human observer by radiant energy with wavelengths of approximately 630 to 750 nanometers. (American Heritage Dictionary, 1992, p. 1512)

Despite the importance of definitions, Peirce maintained that, "nothing new can ever be learned by analyzing definitions" (1878/1991, p. 162-163). Indeed, if one's knowledge of 'red' concerned familiarity with only a number of instances of the idea and its technical definition, one would be very weakly equipped to do anything interesting with it. To approach a 'living' comprehension of the idea and complete our knowledge of its nature is:

> to discover and recognize just what general habits of conduct a belief in the truth of the concept (of any conceivable subject, and under any conceivable circumstances) would reasonably develop; that is what habits would ultimately result from a sufficient consideration of such truth. (Peirce, 1908/1991, p. 275)

This third grade of clearness of the idea 'red' would include, for instance, the intimate knowledge of an artist, a house painter, a textile designer, a historian or a politician (communism, exile, war).

In a similar sense, the idea 'regular pentagon' would be clear enough if we were familiar with the basic characteristics of the idea (sides, angles) and felt confident in distinguishing it in a wide range of instances. Our comprehension of 'regular pentagon' becomes distinct as soon as we are able to comprehend and acknowledge the need for a definition of the idea:

> a regular pentagon is a closed, convex, planar figure that is bounded by five line segments of the same length.

The third grade of clearness of the idea 'regular pentagon' would also require the intimate knowledge that the Pythagoreans had of the idea, or that of an inhabitant of Flatland (Abbott 1882/1992) or of other non-Euclidean lands. [4]

As we find linguistic definitions of ideas in standard dictionaries, one might be tempted to consider the possibility of composing a kind of super-dictionary of praxis – a *practionary* (Parker, 1998) – that would furnish the reader with descriptions of all the conceivable practical effects a thing could have in experience, which would satisfy Peirce's third stage in the clarity of ideas. The possessor of such a practionary could acquire a clearer understanding of an idea simply by emptying the appropriate 'little trunk' - to use the metaphor of the teacher whose pedagogical style I will describe below.

For Peirce, though, the pragmatic maxim is not to be used as an alternative way to define ideas. Knowledge of the practical effects of an idea can only be achieved through personal hands-on experience with an idea or a thing (Parker, 1998; Peirce, 1878/1991) At this juncture, a pragmatist and not a pragmaticist reading of Peirce's maxim could lead to a literal translation of the adjective 'hands-on' and to a belief that the meaning of an idea consists in action. On the contrary, the meaning of an idea:

> consists in our *concept* of what our conduct *would* be upon *conceivable* occasions. (CP 8, § 208, *italics in original*)

Peirce's pragmaticism, according to Fitzgerald (1966), is based on the premise that the meaning of a concept or a sign consists in the habit to which it gives rise in the interpreter, either a habit of action or a habit of thought. Clearly, this habit will guide future actions, being forever on trial in the light of upcoming experiences.

Peirce's discussion of the three grades of clearness of ideas and the notion of *practionary* are all too relevant for mathematics education. A number of researchers have stressed, for instance, the fallibility of the assumption that concepts are acquired mainly through their definitions Sfard (2000) notes that, in mathematics, the introduction of a new signifier is accompanied by a definition. In practice:

> Definitions certainly help in establishing meaning of signifiers, but they do not tell the whole story. (p. 75)

The definition of an idea is an exercise of power over the idea: its utterance conveys an appropriation of complete knowledge of the idea Ohtani (1996) characterized the telling of mathematics definitions as a method of moral persuasion, loaded with a number of social functions. Characteristic of the irreducibility of mathematical concepts to strings of words, and of the resemblances to Peirce's grades of clearness of ideas, are the notions of 'concept image' (Tall and Vinner, 1981; Vinner, 1991, 1983) and 'figural concept' (Fischbein, 1993). Even within the deductive theory of mathematics education, signs are depleted of meaning, if considered without reference to their anthropological *milieu* (Cobb, 1990)

Even though research suggests otherwise (Hewitt, 1997; Pimm, 1991; Voigt, 1994), teaching is still considered, or at least approached, as an act of explication. The processes of symbol manipulation present in traditional school mathematics are often linked to a mechanistic instruction of 'basic' skills and ideas Teaching as symbol manipulation does not require the agency of a human being, in the sense that it leaves no room for creativity, initiative or talent, for estimation or surprise (Romberg and Kaput, 1999)

In his critique of the 'explicative order', Jacques Rancière (1991) describes explication as a series of reasonings used in order to explain a series of reasonings that already exist within the material being taught. If a student cannot understand the first series of reasonings, why should we assume that he or she would understand the teacher's reasonings. And, if the teacher's reasonings themselves need to be explained, we can see how, in Rancière's words"

> the logic of explication calls for the principle of a regression *ad infinitum*. (p. 4)

Over-reliance on the act of explication inevitably places emphasis on what Roman Jakobson (1960) defined as the poetic function of language, a focus "on the message for its own sake" (p. 356). Indeed, what speakers say is always evaluated according to aesthetic rules, in other words, for its efficacy in 'moving' an audience

The master-teacher of the in-service session described above had to employ a series of reasonings in order to explicate to the students the material that he himself had chosen. The outcome of the series of series of reasonings, including those of the attending teachers, judged by the standards of the explicator, created the illusion of an instance of successful mathematics teaching The ability of the master-teacher to 'move' the audience of teachers made this session a successful instance of paradigmatic teaching - judging from the story that was told to me by the school's principal.

I suggest that teaching, conceived as an act of explication [5], represents an attempt to produce a *practionary*, a false substitute for Peirce's third step in the clarity of ideas. Reaching or, to be more precise, approaching this third stage is, thus, trivialized and reduced to cramming practices or a series of reasonings stored in a 'little trunk' The appropriation of knowledge by the students is measured by their ability to follow and/or reproduce these practices, even if this requires in many cases an act of collusion by all the engaging parties (McDermott and Tylbor, 1995).

Analyzing instances of explication

I now turn to the description and analysis of an episode during a mathematics session with a sixth-grade class in Volos, Greece The educational system in Greece is nationally standardized Mathematics syllabi are prescriptive in the sense that they allocate a specific number of hours to the teaching of each area. Instruction in the classroom is restricted to the textbook provided by the state with teachers usually teaching 'from the front'.

With its 140,000 inhabitants, Volos is the fifth largest city in Greece and the third largest port The primary school that I visited is in a lower middle-class area far from the center of the city. My almost daily visits to the school began in October 2000, though I was familiar with the school from a project I had carried out there two years before It was then that I first met with the children who were now in this year's sixth-grade class.

They had a different teacher, though I was not surprised to find a male teacher; reflecting gender hierarchies and stereotypes: the few men among primary school teachers tend to be overwhelmingly represented in the higher grades of primary school. My presence in the classroom was limited to observing and audiotaping the mathematics sessions. I was conscious of the fact, though, that even as a bystander I was co-constructing the course of events (Goffman, 1981)

Linguistic anthropologists view language as a set of practices:

> [practices] which play an essential role in mediating the ideational and material aspects of human existence and, hence, in bringing about particular ways of being-in-the-world (Duranti, 1997, pp. 4-5)

Delineating types of organization that characterize the interactional space of a mathematics classroom highlights how particular ways of 'being-in-the-world' are formed, sustained and altered.

The extract that follows comes from a session on geometry. The class is discussing the concept of perimeter, having already described the characteristic features of a square. Perimeter is a concept formally introduced in the fourth grade, so the current lesson served as an opportunity for the teacher to refresh the children's memory of the concept. In the transcript that follows, I interpolate brief explanatory comments and theoretical elaborations. (A description of the transcription symbols is provided in the appendix at the end of this article.)

(The class has already discussed equal angles and equal, parallel and perpendicular sides in a square. The teacher had drawn a square EZHK on the board.)

1 Teacher: **NOW**. (3.0) We are looking for (2.0) the perimeter of the square EZHK. WHAT INFORMATION do we have for this unknown quantity you see here?

2 Child 1: Sir!

3 Teacher: Yes, Elena

4 Child 1: We know the length of all sides.

5 Teacher: Well **LENGTH** (0.5) TOTAL LENGTH (0.5) of all sides. Splendid!
[
6 Child 2: Sir, it also has equal sides

7 Teacher: = What we have to do is take this information, what we know (0.5) and apply it here in this certain (0.5) square.
[ ]
8 Child 3: Sir! Sir!

9 Teacher: = TOTAL length (0.5) of **ALL** (0.5) sides. What aspect do you think I get from this information that I can match up here?

The teacher takes his turn to speak as soon as he receives the first response (turn 4) from the class. He restates what Child 1 has just said, adding the phrase "total length", an expression that frames the concept of perimeter (turn 5). At the same time, he speaks louder in an assertive tone, underlining the significance of what he has just mentioned for the desired answer. As soon as Child 2 chips in with another piece of information (turn 6), the teacher interrupts the process once again to restate the task and suggest how to proceed (turn 7). Child 2's comment concerning the equality of sides is ignored. The teacher refuses to give to Child 3 permission to speak (turn 8). He repeats and emphasizes the words "total" and "all", using them as cues that would evoke further information about the concept of perimeter.

Child 3's hesitant and faintly uttered responses in turns 10 and 12 below indicate the bafflement of the class as to what the aim of the process is.

10 Child 3: _Perimeter? Perimeter? _ That the::

11 Teacher: It says (0.5) **length** (0.5) **of** (0.5) all (0.5) **sides**

12 Child 3: We know how many: _are all the sides:: *((almost like asking))*

13 Teacher: BRAVO! How many are all the sides. = Where are they? Here (1.0) We write them down again (12.0) *((teacher writes on the board))*

I made the BEST OUT (0.5) of the element that says (0.5) SIDES. Did~I~know? = I~did~know = I~took~them~and (1.0) I~placed~them~there. = **WHAT~ELSE~DOES~THIS~INFORMATION~TELL~US?** = It~says~sides~only? = It~says~perimeter (1.0) **TOTAL** (1.0) **length**. = A :::: that's right! Addition.

14 Child 4: Total (1.0) Total (0.5) with addition

Sensing the hesitancy of the class, the teacher interjects another explication in turn 11. This time, he pauses briefly between words, implying that "this is it!" Child 3 strives to follow the teacher's line of argument. Even though her response is expressed in almost a questioning mode (turn 12), the teacher congratulates her and proceeds with yet another explication of the argument so far (turn 13). Next, he asks the children to think of other characteristics of the square that would be relevant in finding its perimeter. Child 4 (turn 14) interprets correctly the cues given by the teacher and takes the process a step further. Notice that in turn 13 there are no "transition-relevant points", i.e. moments when a change of speaker may take place (Sacks, Schegloff and Jefferson, 1974). Besides speaking too fast, the teacher leaves no interval between the end of a prior unit of speech and the next piece of talk.

15 Child 5: Sir, we have done these before.

16 Teacher: Everything's fine?

17 Child: Yes.

18 Teacher: **What else** did we discover today or better did we mention again? = WE KNEW IT BUT WE SEE IT AGAIN (0.5) FOR THE SIDES:

19 Child 6: That these *((pointing towards the board))* are:: opposite to one another

20 Child 4: That these:: are:: parallel and do not meet.
[
21 Teacher: They are parallel. Yes.

This aspect (3 0) can it be used? (6.0) At~the~point~where~we~are~now? = In order to see let's say =

22 Child 4: The parallels?

23 Teacher: **YES**. That they are parallel (2 0) does it interest us in finding the total length?

24 Child 6: Eh sir::

25 Child 4: Ho-? How much are all the parallels? *((she wants to say how long))*

26 Child 7: **SIR, the parallels are equal!** *((an unsuccessful blend of concepts by the child))*

27 Teacher: A ::::! *We have another element* = Look here. = That sides: =

28 Child 6: Are equal.

In turn 16, having just ignored Child 5's remark, the teacher seeks reassurance that the class comprehends. More information is needed in order to calculate the perimeter, though. Children 6 and 4 suggest that the missing link is the fact that the sides of a square are parallel to one another (turns 19 and 20). The teacher interrupts (turn 21) making an 'implicit' statement that the concept of "parallels" is not the information missing. Child 4's question in turn 22, though, indicates that the class is not following the process as presented by the teacher. Children appear to be choosing information at random from the 'little trunk'. The teacher attempts again to lead the argument away from the information "parallel sides" (turn 23). Child 4, with the help of Child 7, seems to be making the required connection (turns 25 and 26).

29 Teacher: **They are** (2 0) equal Therefore
[
30 Child 4: We can multiply.

31 Teacher: **BRAVO!** We move to the property that says that multiplication~is~a~**SHORT**:? (1 5)

32 Class: Addition.
[
33 Teacher: Addition When~the~terms~ARE:? (3 5)

34 Class: Same.

35 Teacher: **THE SAME**. That's righ- = Instead of saying 4~and~4~and~4~and~4~I~say 4 times 4. = Or 3~and~3~and~3~and~3 (0.5) I~say 4 times (1 0) 3. Is that so? If here then (1 0) instead of saying side EZHK I put a general name (2.0) α *((he writes α on all sides of the square))* for the sides (0.5) of the square ZH α and KH α (0.5) and this (1 0) and the other (1 0) = *((pointing to the figure on the board))*

36 Child 4: **4~TIMES** α (4.0)

37 Teacher: With a 4 times α then I define:

38 Child 4: The::: the:: perimeter!

39 Teacher: **BRAVO!** I define the magnitude that is called perimeter (1 5) of a square.

I had the impression that the scene was approaching a summary, since Child 6 in turn 28 realized that the information missing from the puzzle was that sides are equal. Child 4 in turn 30 points to the connection between the concept 'perimeter of a square' and the information 'equal sides'. In turns 31, 33 and 35, the teacher uses another series of explications to unfold the relation of multiplication to the concept of perimeter as the sum of the lengths of all sides of a shape. The episode concludes with me as observer wondering about the class's actual degree of engagement. Child 4's stumbled response in turn 38 is indicative of this ambivalent participation in the making of meaning.

Explication as action? A pragmaticist's reflections

In order to look closer into the production of a *practionary* by the teacher, I will draw once more from Peirce's philosophy, particularly his views on the process of semiosis. I will present a short outline of his terms in order to make his comments on the practionary more accessible. At the heart of Peirce's semiotics are three universal categories: Firstness, Secondness and Thirdness. These three categories:

> belong to every phenomenon, one being perhaps more prominent in one aspect of that phenomenon than another but all of them belonging to every phenomenon. (CP 5, § 43)

The category of Firstness is characterized by the absence of all conception and reference to anything else. Every phenomenon will exhibit:

> qualities of feeling or sensation each of which is something positive and *sui generis*, being such as it is quite regardless of how or what anything else is. (CP 7, § 625)

Imagine that a first-grader, with not much prior experience with Cuisenaire rods, has in front of him or her a construction that represents all the trains of length four. [6] This will be a rectangular construction with pieces of purple, green, red and white rods. The shape of the construction, the color, the material and the length of the pieces that comprise the construction, all are qualities of the phenomenon at hand. The child may experience feelings of pleasure from the rather colorful and, in parts, symmetrical construction, perhaps combined with bafflement as to what is required from this activity. This, then, will be an instance of the category of Firstness.

The moment the first-grader stops and thinks of the *qualities* of the phenomenon, of how they are felt together or in contrast to other qualities, is the moment he or she leaves the territory of Firstness and moves towards Secondness. The color of a piece is contrasted with its length, the choice of colors and lengths in one line is contrasted with those in another line, taking also into consideration that all trains have the same total length of that of the purple rod. The perception of the differences or relationships that appear through this comparison or blending together signals the emergence of the *connectives* of the phenomenon.

The qualities of the phenomenon, along with the connectives that bring their connection and contrasts to the fore, comprise the brute fact of the experienced phenomenon, the *percept* in Peirce's terms. In order for the first grader to experience this phenomenon as meaningful, a perceptual judgement has to be attached to the percept. "Each row represents a way to make 4", "The construction represents all the different ways that I can make 4", "3+1 is different from 1+3", and so on. Statements of this kind are expressions of Thirdness:

> the mode of being which *consists* in the fact that future facts of Secondness will take on a determinate general character. (CP 1, § 26, *italics in original*)

Thirdness, then, is the category that involves mediation, representation, habit, memory and signs (Spinks, 1991), so that the first-grader will be able to make sense of the construction of the trains of length five or build a construction of all the trains of length six.

Signs are a phenomenon of Thirdness. Very roughly stated:

> A sign, or *representamen*, is something which stands to somebody for something in some respect or capacity. (CP 2, § 228)

A sign is essentially triadic. It has the capacity to establish a triadic relation among a representamen (an instance of the sign), an object of representation and an *interpretant*, that is the idea that it produces or modifies in the interpreter. Signs function as manifestations of phenomena. Therefore, a blending of the signs' triple quality relates and the universal categories of phenomena produce further categories of signs. The most well-known, and fundamental according to Peirce, trichotomy concerns how the sign represents its object to the interpreter. The types of signs in this classification are the icon, the index and the symbol, a division based on the prominence of Firstness, Secondness or Thirdness respectively. An *icon* partakes in the characteristics of the object it represents, resembling its object in some respect (e.g. diagrams, photos). An *index* is an existent thing that points our attention to the object it represents without describing it (e.g. a scale - as pointing to equality, smoke). An index denotes nothing unless the interpreter is familiar with the connection with the denoted thing. A *symbol* is a consequence of habit and relates to its object solely by virtue of convention (e.g. ∇ABΓ, a triangular traffic sign, words - see Fitzgerald, 1966; Greenlee, 1973; Parker, 1998; Spinks, 1991).

To return to the episode I analyzed in the previous section, apart from the square he drew on the blackboard in the beginning of the episode, the teacher involves no icons in his teaching. On the contrary, he made constant use of indices. Pronouns, the letters EZHK attached to the square and the length α of the side of the square are all indices forcing attention to the subject of the discourse, the idea at hand. Body language, the changes in the pitch, amplitude and volume of the teacher's voice, the controlled breaking of words or units of speech carefully point up important turns of phrase or other ideas implicated in his reasoning (e.g. see turns 5, 31 and 35) are also indices that are intended to point the children's thought at particular experiences.

These other experiences entail a walk through a number of qualities, drawn at random from the 'little trunk' belonging to the idea 'square', whose relation to the object of thought is not straightforward. A meaningful proposition should involve signs of each of the members of the trichotomy. Fitzgerald (1966) would have had serious doubts as to whether anything worth mentioning concerning the subject of the discourse was communicated on this occasion, since the children appear unable to associate any icons with the idea discussed.

Evident then is the teacher's attempt to rush towards the category of Thirdness. He values highly expressions that implicate Thirdness in the line of reasoning (e.g. see turn 5), while instances of Secondness are reduced in an excess use of idiosyncratic indices that point to a retrieval of facts from memory. In that sense, the children were simply guessing at a riddle, being made to manipulate terms, definitions and memorizations. To approach the third degree of clearness of ideas is to spend time experiencing the intimate knowledge that only instances of Secondness can provide.

I still remain puzzled as to what children were learning from this exchange. Peirce's pragmatism is founded in the theory of interpretants (Fitzgerald, 1966; Greenlee, 1973). For something to be a sign, it must produce an effect in an interpreter, an effect of the category of Thirdness. The teacher's intention was to produce in the children's minds, through a right interpretation of the signs he used, the idea of the 'perimeter of a square' (the immediate interpretant). This right interpretation is a mere possibility though; nothing guarantees that it will ever be realized in the desired manner.

The totality of direct effects produced in the children's minds by the teacher's use of signs in this episode is the dynamical interpretant of these signs. These effects involved feelings of self-sufficiency, e.g. that they understand or not (see turns 6, 10, 24, 32 and 38 - the emotional interpretant), the effort to act, either physically or mentally (see turns 14, 19 and 26 - the energetic interpretant) and a formation of a habit or a rule of action, even though we see no indication of it in the episode (as condensed in the teacher's words in turns 35 and 39 - the logical interpretant). Likewise, we have no indications of the final interpretant, the effect that would be produced in the children's minds after a sufficient development of thought and the idea 'perimeter of a regular shape'.

Concluding remarks

Participation in any social scene requires a minimum consensus on what is getting done (Gumperz, 1992; McDermott and Tylbor, 1995). On a micro-perspective level, teacher and children appear to be discussing already familiar concepts in an attempt to deepen or widen their understanding of the concepts (see turns 15 and 18). Despite this observation, though, the fact that children's responses, such as those in turn 15, are overlooked indicates that the teacher ranks the correct use of information and the pious following of a process higher than understanding the concepts. On a macro-perspective level, children are disciplined into following the series of reasonings used by the teacher in explicating the process of finding the perimeter of a square with side α. [7]

As McDermott and Tylbor suggest, "it is possible to live lies without having to tell them" (1995, p 221). Judging by Child 4's response in turn 36, I remain puzzled as to whether the teacher actually moved through this series of reasoning alone or not A fair amount of collusion was involved in this process of explication, both by the teacher and the children, in the sense that a particular state of affairs was sought that could be accepted as an indication of understanding and, therefore, of closure and successful or satisfactory teaching (see turns 5, 12, 13, 16-17, 30-32, 33-35 and 39).

The confusion of the class concerning the task at hand, along with their hesitant decision to follow the teacher's explication of the task, may suggest a form of resistance to the way of 'being-in-the-mathematics-classroom' proposed by the teacher. One cannot force reasoning to follow a specific order of steps. What the teacher does is lay out his own interpretation of what 'being-in-the-mathematics-classroom' might mean. Knowledge and understanding are then measured by the degree to which the students collude with the teacher's mode of explication. Analyzing modes of explication can show subtle ways that states of 'being-in-the-mathematics-classroom' are sustained and reproduced. A discursive analysis of conversational exchanges in the classroom may be one tool for understanding this process.

Notes

[1] This paper is an expanded version of a presentation given at the 25th Conference of the Psychology of Mathematics Education Group in Utrecht, July, 2001

[2] Readers should not perceive paradigmatic (nowadays the term virtual is preferred instead) instruction as prototypical of current in-service training programs in Greece, even though it was characteristic in the past. Nevertheless, the mentality it reflects still prevails in beliefs about learning and teaching, albeit in more subtle ways I return to this issue towards the end of my article

[3] Peirce introduced 'pragmatism' in 1877-98 as a doctrine in logic intended to furnish the analysis of concepts In 1897, James shifted its focus from an analysis of the meanings of ideas to the analysis of their value or moral uses In 1905, Peirce announced the birth of a new word, Pragmaticism, a word "ugly enough to be safe from kidnappers" (CP 5 § 414), to refer to his original definition of pragmatism

[4] The similarities between Peirce's stages of clearness of ideas and van Hiele's levels of development of geometric thinking are clear

[5] Readers may wonder about the choice of the word 'explication' instead of explanation To explicate means to "unfold", hence to explain, to make clear the meaning or sense of, especially a piece of writing or an idea To explain is to 'flatten', hence spread out, to make plain or comprehensible, to make something clear or easy to understand by describing or giving information about it (Partridge, 1983). As will become evident in the next section, acts of explication involve devotion to a complete unfolding of an idea in order to reach a clear presentation of its constituent parts - an unfolding in the sense of following a preferred order without allowing for any detours

[6] This is a variation of a typical activity with Cuisenaire Rods, that of finding the lengths of two-car trains Children have to place two or more cars together to match the length of a certain rod

[7] At this point, we should stress the targetting of the task itself. The teacher - and the schoolbook - was aiming at the definition of the perimeter of a regular shape in terms of a short addition (e g. $\alpha + \alpha + \alpha + \alpha = 4\alpha$)

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Appendix: Transcription features

Boldface indicates emphasis signalled by changes in pitch and/or amplitude.

A **left bracket**, connecting talk on separate lines, marks points at which one speaker's talk overlap the talk of another.

A **right bracket** marks the place where the overlap ends

Colons indicate that the sound just before the colon has been noticeably lengthened

A **dash** marks a sudden breaking-off of a particular sound

A **degree sign** indicates that the talk following it is spoken with noticeably lower volume.

A **tilde** indicates that there is no interval between the two connected words.

An **equals sign** is used to indicate that there is no interval between the end of a prior unit of speech and the next piece of talk

Capitals indicate increased volume

Numbers in parentheses mark silence in seconds

Text in **italics** between **double parentheses** marks the author's comments.