Structure and Ideology in the Mathematics Curriculum [1]

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1. Introduction

Curriculum theorists routinely conceptualise the school curriculum in terms of its ideological role, particularly in terms of its function in socialising children into their ambient society. Many of these have adopted a generalist approach, seeing the curriculum-as-a-whole as an object of study, couching their analyses in terms of critiques of the curriculum in general rather than the specificities of curricular form and content. Nevertheless, the early nineteen-seventies spawned a number of studies which, under the banner of the "New Sociology of Education", attempted a reappraisal of the subject matter and underlying assumptions of particular disciplines such as music, science, and literacy—though little if any work was done in mathematics. The intention behind much of this effort was to clarify how, for example, musical, scientific, or literary meanings are constructed within the context of schooling

In this paper I explore whether such an analysis makes sense in terms of the mathematics curriculum. In doing so, the question of meaning will, once again, be central. I will try to explore what meanings children derive from the mathematics they encounter: inevitably this will lead me to consider meanings which impinge from outside the realm of mathematics itself—ideological meanings.

My plan is as follows. After a brief introduction to the notion of ideology, I focus on one curricular area which has received more attention than most, that of music. I then examine the similarities and differences of the analysis applied to mathematics. In order to concretise the analysis, I then focus attention on a particular mathematical topic—that of proof—which is considered as an example of the phenomena I am trying to unravel. Finally, I try to provide some theoretical framework within which to make sense of the mathematics curriculum, and the way in which knowledge is constructed within it

2. What is ideology

It is far from clear why mathematics and ideology are related in any way. Mathematics deals with the aesthetic and the theoretical. Ideology deals with the political and the pragmatic. So let me begin by stating that I view ideology as the body of ideas through which we see and with which we construct our reality. It makes the world intelligible, and people's behaviour predictable, understandable.

To claim that mathematics is a social construction, rather than, say, a representation of reality, is no longer as contentious as it once was. But the (school) mathematics curriculum too is socially constructed, and has, for example, changed its character many times in the last century or so since compulsory education was introduced into the "developed" world. In both cases—mathematics and maths (I use the latter term here to save distinguishing continually between mathematics and school mathematics)—there is an overwhelming temptation to view the subject matter as given, inevitable, *natural*.

From where does this apparent "naturalness" arise? There are many possible answers to such a question, but two polarised extremes are evident. One would argue that both mathematics and maths are as they are because of their inherent structures; that both are, in a sense, preordained—given that mathematics provides an idealised means of describing and predicting the material world, and that maths is simply a reflection of this. While such a view has some credence *vis-à-vis* mathematics, it is more difficult to sustain with regard to maths. The other extreme would view both mathematics and maths as essentially arbitrary, creations of people acting independently and autonomously, without any constraints imposed from the material reality of which they form a part

It is clear that there is scope for a range of intermediate positions, and a full examination would take us too far into philosophical domains. As far as maths is concerned, I begin from the premise that the curriculum, if not arbitrary, is socially constructed, and that a valid task for researchers in the field is to *de*naturalise the form and content of what is taught.

And so to ideology. Ideology flows from the social relations which exist within a society (which is not to say it is *determined* by them) and correspondingly functions to help maintain (or destroy) those relations. At any particular time, it may be that particular ideologies do or do not contribute to the dominance of a particular class or group. Nevertheless, it is not necessary that such social groups fashion ideologies explicitly (although, of course, in times of wars and other crises this may indeed be the case). On the contrary, there is a tendency for ideologies to become "common sense", applied *without* explicit intention and, most importantly, there is an accompanying tendency to see the surface reality of this as their unalterable bases and causes.

It should already be clear that I am using the word "ideology" in a sense dissociated from its strictly pejorative sense which, as Barrett [1991] points out, is a common everyday connotation of the word. It is not difficult to see how the word *ideology* acquired its pejorative connotations; indeed, if it is true that an essential element of the way ideology works is that it merges into the background—tending to make reality seem unmediated and natural—then it follows that only the most obvious (or

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repressive) attempts to influence ways of seeing and thinking will be apparent. When we say that a political despot uses ideology as a weapon to mould peoples' thinking, we are saying *both* that peoples' ideas are (deliberately) influenced *and* that this process is evident to us as observers because we can stand outside it. We may notice the ways that the media attempts to influence our opinions, but only if we disagree with the views being proposed. And we have much less difficulty branding as "ideological" (in the pejorative sense) ideas and beliefs that belong to history, at times and in places from which we are removed.

Of course, there are institutions which play a more or less explicit role in the fashioning of ideologies: an obvious example might be organised religion. Schools, too, may be thought of as contributing to ideologies: it would indeed be surprising if institutions explicitly concerned with influencing children's thinking did not play a role in fashioning the belief systems within which people make sense of their social and physical world. But institutions do not necessarily need to have a physical embodiment before they have some role in generating ideologies: established ways of seeing and thinking occur everywhere and mediate how we "read" them—in our appreciation of music (a subject to which I am about to turn) as much as our understanding of, say, infinity. With a notion of ideology stripped of its pejorative connotations of covert manipulation, mystification and obfuscation, it becomes a little more plausible that they way we conceptualise the mathematics curriculum—no less than the way we think about art or literature—is itself ideological.

There is a considerable literature on the ways in which schools in general contribute to ideological production (a useful starting point is Giroux [1983]). Common to almost all approaches is the view that schools are sites of social reproduction; that it is at school (but not *only* at school) that children learn how to function in the social niche they are likely to occupy in adult life (see Noss [1989; 1990] for a discussion of some of these issues from a mathematical perspective; also, Mellin-Olsen's seminal [1987] contribution).

The problem is to try to tease out the elements of schooling which contribute to their socialising function. Is it the structures of the school, by stressing forms of knowing and behaving which are alien to all but the most privileged, which are responsible for the social reproductive role of organised education? Or is it curricular content, which is responsible for instilling the specific values which are required by the society of which the pupil will form a part? Within the sociological literature, both positions have received support, as Whitty [1985] points out: "In one case the class structure was seen to be sustained because working class pupils *failed* to learn what the school defined as significant, while in the other case the process depended on what they *did* learn in school—that is to accept (and if possible respect) the status quo." [p. 20]

Thus in order to examine the notion of ideology more concretely, I need to look at how *meanings* are constructed within a domain, and how they are related to those developed within an educational setting. To do so, I temporarily turn from mathematics, to the field of music and musical education.

3. The construction of musical meaning

Music is a field which has received considerable and rather detailed attention, dating back to the work of Theodore Adorno (see, for example, Adorno [1976]), a member of the "Frankfurt School" of philosophers and sociologists interested in probing questions of ideological reproduction; how ideas and cultures are generated and sustained within societies. His work has generated a fascinating field of enquiry in relation to the teaching and learning of music in school (see, for example, Vulliamy [1976]); more recent work has been undertaken by Lucy Green [1988], and it is Green's work that I want to examine in some detail.

Green's project is to understand the "little social system, or musical world" which constitute musical ideologies, practices and products. These form

> "a network of functions both mental and material, supporting and legitimating one another, yet also fragmented, divided and oppositional. This social system does not survive autonomously, but is reproduced through a reciprocal relationship with the wider social system, of which it is only a part, divided musical practices being perpetuated materially, their divisions legitimated and maintained ideologically." Green [1988] p 11

The central question is to try to understand the ways in which musical meanings are understood: how people make sense of music in relation to their appreciation of and participation in their social and material world. For example, what are we to make of the fact that music *appears* to be a timeless essence, and that meaning is eternal, asocial and ahistorical, derived from meanings which "genetically reside in music" [*ibid* p.7]. It seems as if music's structures appear ready-made, inevitably shaped and distinguishable from, say, mere sound. Yet the self-evident fact that musical structures and appreciation differ in time and across cultures, gives an obvious clue as to the social derivations of musical meaning. The social system may not survive "autonomously", but what precisely is the nature of the relationship between it and the wider social system of which it forms a part?

In searching for the sources of musical meaning, Green finds it useful to distinguish between *inherent* and *delineated* musical meanings. She argues that

> "Individual temporal musical experience arises directly from musical materials that inhere in music and create meanings between themselves, for consciousness, through time" [*ibid* p 25]

These meanings are inherent, *intrinsic*, to musical material. They have both social and historical dimensions, but are nonetheless ultimately traceable to the structural facets of musical activities, and the ways they are experienced by people. Individual musical experience arises from musical materials that exist within music itself, and constitute the *parameters* of music. It is these experiences of material and their meanings that Green refers to as "inherent" musical meaning.

In contrast, she classifies as *delineated* meanings those

> "Images, associations, memories, queries, problems and beliefs inspired in us by music and musical meanings that, rather than inhering in musical materials and pointing only

to themselves, point outwards from music towards its role as a social product, thus giving it meaning as such for us." (*ibid* p.28]

These associations of different kinds of music with different subcultures (and even social classes, recording charts, opera as high culture, film scores—these are all different ways in which musical associations and beliefs are developed and sustained. By occupying a niche within this complex web of social relations, music delineates ideologies in relation to musical meaning and beyond into social ideologies in general.

Central to Green's case is the description of the ways in which music's function *as a commodity* tends to override its structure As such, the inherent meanings of "great" music are treated as if they were beyond the comprehension of all but the initiated, "...despite the fact that we create, develop and realise them, that we produce them collectively and divisively through history." [p. 86] Thus this is a musical variant of *commodity fetishism*, the process which Marx [1967] describes as the usurpation of an object's *use-value* (why we want and use it) by its *exchange value* (how we acquire and value it).[2] In the process of fetishisation, delineated meanings come to supplant the inherent meanings and values of the commodity; and in just this way, argues Green, "the delineations appear to be the only real and unalienated qualities of music, the only means by which we can grasp music at all." [*ibid* p. 86] As Green points out, the marketing strategy of the "pop video" is a complete expression of this supplanting of one kind of meaning by another.

An important corollary is the surface appearance that music arises naturally out of its own materials, and the obscuring of the reality of the social and historical roots of musical "laws". The structures of music and the relationships between them are autonomous in the sense that they have an internal logic. But this autonomy takes place only in a dialectical relationship with the conventional systematisation of musical materials.

When we turn to *school* music, the interplay between inherent and delineated meanings is, once again, a central issue. For Green, the implications of her analysis for education are clear-cut: "The classroom, rather than helping to separate delineated from inherent meanings, tends to add to their ideological confusion, or fetishisation." [p. 141] In other words, a central educational problem is to decouple the intrinsic meanings which inhere in music from images and associations coming from outside—e.g. from advertising, image-building, and commercialisation In order to gain success at music within school, pupils have to accept the delineated meanings which are offered by the textbooks and the examination boards, meanings which lie entirely outside and meanings which stem from the music "itself" The result for the pupil of failing to accept and identify with these delineations is that musical "experience" will be irrevocably "blocked".

For pupils trying to make sense of opera, the delineated meanings of high culture may present insuperable barriers to its appreciation. Conversely, even music which emanates from children's own experiences can be smothered by the importation of delineated meanings which are quite inappropriate to the context of the music itself. For example, Green provides a hilarious example based on her observation of a teacher "teaching" Reggae to a group of children using chime bars and glockenspiels, and failing even to spark recognition that the music "was" actually Reggae. She points out that the music was present in the classroom only as "a distorted image of its lost meaning", and that in this case, it is useless to pretend that the music was in "*any style, other than a special classroom style*" (emphasis added).

The message is that music has meanings in itself, meanings which inhere within it; and it is these meanings which need to be the focus of educational attention, at the expense of the delineated meanings which accompany them. For Green, it is the failure to achieve a balance between delineated and inherent meanings which results in many children failing to construct any viable meanings whatsoever from the music they experience in school.

4. Meanings and mathematics

Green's analysis is fascinating, not least because it provides us with an example of a scholar of the artistic/aesthetic arguing for the primacy of objectivity and inherence, when we may be more used to students of the mathematical/scientific arguing for cultural relativity and delineation. In order to study the extent of application of Green's argument in a mathematical context, I shall try to recast Green's argument in mathematical terms.

Mathematics—like music—is based on raw material: shapes, number, mathematisable situations. Of course, the concept of shape (rectangle as opposed to door, 4 as opposed to 4 cups) is already a mathematisation, a first link in the signifying chain which characterises mathematical activity. In creating mathematics, these objects are brought into relationship with each other, and these relationships themselves become (eventually) the raw material of further mathematics.

Thus, mathematicians are not free agents. The rules of the game by which mathematical objects may be manipulated and brought into relation with each other, are not arbitrary. The idea that mathematics is an arbitrary game is far from the truth: mathematicians care very much about the meanings which their games convey, even if they sometimes deny it Of course, it is true that new games are created, each with new variants of the rules. But the rules (and the rules about the creation of games, etc.) are built into the structure of what it means to do mathematics (as opposed to, say, literary criticism or history)—they are *inherent* meanings.

Of course, there is a crucial distinction between mathematics and music, in terms of the ways in which the two fields enter into popular culture. The difference is that the inherent meanings of mathematics (beyond the most elementary) are simply not "consumed" explicitly by any except mathematicians. Unlike music, where there is a *tendency* for the delineated meanings to usurp the inherent meanings, in mathematics there are, in the culture at large, *no* inherent meanings to undermine. The raw materials of mathematics are simply not available for debate, discussion and appreciation in the way in which they are for music.

Yet the difference is not as straightforward as it seems. For mathematics is, in fact, ubiquitous; it *is* consumed by all, but *implicitly*. As the introduction of new technology proceeds apace, this mathematics is increasingly hidden from view, concealed in technological artifacts (see Noss [1991] for elaboration of this position; see also Chevallard [1989]). As I write this sentence 30,000 feet in the air, those inherent meanings are embedded all around: in the mathematical modelling of the plane's wing structure, the computer navigation system, the portable computer on which I am typing. Yet this mathematics is almost entirely inaccessible, not available for me, the consumer, to participate in, listen to, join with. Mathematics may, as Skovsmose [1992] puts it, "format" society, but almost all must remain entirely alienated from the meanings which lie dormant within it.

Recognition of this ubiquity of mathematics brings to the surface a convergence between the cases of mathematics and music. For in mathematics, the process of fetishism which Green outlines—in which inherent meanings are continually supplanted by delineated meanings—has reached its ultimate expression. Everybody has a view about the way in which "computers" enter our lives. Statistics proliferate and are used for a variety of ends, political, as well as economic. Mathematics itself is often invoked as a source of certainty and power and equally, as a mechanism for replacing judgement by calculation [Weizenbaum, 1984]. In terms of the meanings of mathematics within popular culture, there is nothing but delineation.

At certain times, however, it seems as if the inherent meanings of mathematics suddenly emerge explicitly. Consider, for example, the recent explosion of interest in non-linear dynamical systems, in which there has been a large variety of serious applications which have caught the professional popular imagination. Yet even here, an industry has been generated; there is now a bizarre industry of sociologists, political scientists, and post-modernists of all kinds, all borrowing some version of what they perceive as "chaos" in the behaviour of human systems (a prevalent example is the belief that the "chaos" of, say, European politics is somehow connected with the "chaos" of non-linear systems). This example illustrates nicely the delicate balance between inherent and delineated mathematical meanings in a case where the inherent meanings have some visibility on the popular stage. The appearance of scientific theories as *only* consisting of inherent meanings, is shown to be illusory: in a society where almost anything can become a commodity, scientific ideas cannot remain immune.

The above analysis has provided some theoretical framework within which to consider the mathematical meanings more closely. In the following section, I will focus first on mathematics and then on maths through a concrete example, that of proof.

5. An exemplar for mathematics and maths

The meanings which mathematicians attach to proof appear to be entirely inherent—proof is seen as *the* essence of mathematics, a timeless essence which underlies the structure of the discipline. In fact, the picture is not altogether as simple as it seems. I will give some examples.

Rigorous proof has not always characterised mathematics. Restivo [1992] shows how the idea re-emerged some time in the early nineteenth century, and how the strength of its appeal in the last eight hundred years has ebbed and flowed in mathematical circles largely as a device for coping with different forms of competition between mathematicians. Restivo argues that the form and content of competition *between mathematicians* are a reflection of the economic and social contexts in which it arises: the notion of proof emerges in particular economic and political circumstances. Sohn-Rethel [1978] goes further in proposing, in the case of Greek mathematics, that the emergence of Euclid's elements arose "for the sole purpose of proving that geometry as a deductive thought structure was committed to nothing but itself". [p. 103] He argues that the Greeks drew a "dividing line" between mental and manual labour, and that this resulted directly from the development of a generalised coinage for monetary exchange. For Sohn-Rethel then, the delineated meanings of axiomatisation and proof arose directly as a result of social and economic forces at work within Hellenistic culture.

A further delineated meaning associated with the idea of proof, is the belief that mathematical proof is *generally* superior to everyday justification. It stems from the wider view that mathematical thinking is superior to practical thinking, a view deeply embedded in Western culture; it forms part of the ideology of what it means to think abstractly, perhaps even what it means to think. Scribner [1991] argues that "practical thinking is instrumental to the achievement of larger goals, while theoretical thinking is non-instrumental and "complete in itself" " [Scribner, in Saxe, p. 22] This distinction is helpful, but it has implications for mathematical meaning, since it belies a tendency to see practical thinking as more "restricted", designed to accomplish everyday goals which are somehow less elaborated than those of mathematics.

While I am happy to acknowledge the difference between mathematical and everyday behaviours, I am less confident that they can be organised into a general hierarchy. The difficulty with such a view is that it implies that persons acting in and on their environment are "only doing"—that practice is completely divorced from intellectual activity. And as I shall argue below in the context of maths, it is just this belief in the separation between practical and intellectual activity which has provided a cornerstone for an edifice of delineated meanings which pervade mathematical education. Mathematics has often been mobilised as the example, *par excellence*, of the superiority of theoretical over practical activity, and the notion of proof has played an important part in this process. Quite the reverse proposition is asserted by the Italian philosopher Antonio Gramsci who argues that:

> "There is no human activity from which all intellectual intervention can be excluded—*homo faber* cannot be separated from *homo sapiens*. Also, every man, outside his own job, develops some intellectual activity. ..and so *contributes towards maintaining or changing a conception of the world*, that is, towards encouraging new modes of thought." Gramsci [1957] p. 121 (emphasis in original)

What is at issue here is not so much the distinction between theoretical and practical—they are clearly different—but the attempt to elaborate precisely the nature of that distinction. As far as proof is concerned, if I justify to someone else why I have decided not to take a raincoat to work (there are no clouds in the sky and it is July), I am not operating solely at a *practical* level: indeed, I am bringing to bear a wealth of synthesised observations, constructing an implicit theory about rainfall, and at the very least, building (theoretically) on a substantial base of empirical observation.

A nice example of the difficulty of uncoupling the delineated and inherent meanings of proof is provided by Uri Leron [1983], who proposed that while the normal "linear" style of proof conforming to accepted mathematical practice is ideally placed for securing the validity of proofs, it is ill-suited to the second crucial role—that of communication. Leron argues that it is possible to construct "structured" proofs (by analogy with structured programming in computer science) which are arranged in stand-alone "modules" each of which conveys a single main idea of the proof. One interesting side-effect of this proposal is that asides, explanations and other informal comments can be included naturally in the "space" between modules. Leron argues that "the structural approach brings closer the human process and the formal-deductive one." [p. 179]

Leron (personal communication) recounts that —with some major exceptions—his proposal met with a cool reception from mathematicians who felt that their intellectual *raison d'être* was under threat—*and perhaps, the inherent meanings attached to proof itself*. In fact, by juxtaposing the two styles of proof, Leron showed convincingly that nothing need be lost in terms of inherent mathematical meaning. What would be lost is some of the delineated meanings—the very meanings which, it turns out, are mainly invisible to mathematicians themselves.

The complex interweaving of inherent and delineated meanings is similarly evident when we turn to maths, where the delineated meanings have not remained static—particularly in relation to proof as it has appeared in the mathematics syllabus of the UK classroom over the last four decades. In the nineteen-fifties, rather young children were often schooled in the axioms and proof of Euclid (an approach still adopted in many countries). The traditional statement-and-proof format of Euclidean geometry seemed to offer a route into the *rigour* which characterises mathematical discourse.

In discussing his classification of proof strategies exhibited by children, Nicolas Balacheff [1988] argues that "the practice of proof involves the commitment to a problem solving approach which is no longer one of effectiveness of practical requirements but one of rigour, a theoretical requirement". Balacheff has nicely caught the essential tension between everyday and mathematical thinking: the rules which govern behaviour in everyday life (where effectiveness is an appropriate criterion) are different from those which obtain in mathematics. They are different practices, different discourses, and Balachef's distinction correctly locates the central issue as being one of rigour. Rigour is an undefined concept, but it nicely sums up the privileging of fragile chains of reasoning as well as the pursuit of the general within *mathematical* discourse.

However, it is this notion of rigour which has often been used as a vehicle to import delineated meanings of *general* intellectual superiority into mathematical thought. In addition, these delineations have, in turn, often been exploited as a means to assist the mathematics curriculum to fulfill a variety of social functions within society. For example, the central role which proof was called upon to play in the post-war curriculum seems more akin to that previously played by Latin and Greek: the issue was much more to do with the socialisation of (certain classes) of future generations, than the utility of corresponding or conversing with an ancient Roman or Athenian. Similarly the emphasis on the axioms and theorems of Euclid may have had more to say in terms of conveying messages about the certainties and immutability of the world older than the wish to inculcate a feeling for mathematical truth: the traditional concern of the Euclidean curriculum was centrally focused on how to set out the page, how to refer to "properties", where to write Q.E.D. and very little about gaining insight or understanding into the geometry of the plane—once again, delineated meanings gained supremacy. I think it is not too far-fetched to argue that the reproduction of school proofs obscured rather than illuminated meaning; the proofs did not generate insight, they were a substitute for it: proof as educational commodity.

Since the nineteen-sixties, there has been a marked swing in the delineated meanings accompanying the notion of proof in the UK: proof is, for example, more or less absent from the National Curriculum. This is not quite true at the upper end of the school curriculum, but it is certainly the case that the compulsory mathematics syllabus can be successfully negotiated without *explicitly* encountering the idea at all. The banishment of formal proof from the syllabus has been seen by many as a litmus test of "progressive" mathematics curricula—paradoxically, perhaps, many mathematics educators have been happy to overlay the inherent meanings of proof with delineations of inert and sterile pedagogy. This situation is somewhat surprising. After all, for many practising mathematicians, mathematics *is* proof; and the rhetoric of some recent mathematics education literature has stressed the value of viewing children as young mathematicians rather than simply telling them about mathematics.

Thus there is a contradictory pair of delineated meanings for the notion of proof in school. First, there is a belief that proof is a *generally* superior way to reason, not just an ingredient which distinguishes between mathematics and other intellectual and practical endeavours. Second, some educators are convinced that proof in the curriculum is a barrier to investigative and creative activity, and that it is inherently (sic) opposed to the spirit of exploration and investigation which has permeated the UK mathematics curriculum for much of the recent past.

6. What does the maths curriculum mean?

I have tried to show how inherent meanings within maths are overlayed by delineated meanings which come from outside the boundaries of mathematics. This raises a crucial question: Where do these meanings come from? In this final section, I try to confront this issue, and attempt to draw some conclusions for the curriculum.

At one extreme, school knowledge might be seen as arbitrary, or at least contingent only on the whim and fancy of (say) politicians or educationalists. In this scenario, curricular content is essentially irrelevant, and reduced to an empty form, a position argued most forcefully by Ivan Illich:

> "It does not matter what the teacher teaches so long as the pupil has to attend hundreds of hours of age-specific assemblies to engage in a routine decreased by the curriculum and is graded according to his ability to submit to it." Illich [1973] pp. 61-2

This view meshes with that of Harry Braverman, a political economist whose seminal book *Labour and monopoly capital* provides a captivating analysis of modern working practices. In it, Braverman paints a picture of the social functioning of schools, which he argues play a critical role in the organisation of capitalist societies. For Braverman, schools serve to fill a vacuum created by the demise of traditional socialising influences (the family, community, etc.) And in seeking to fill this vacuum, he argues that "schools have themselves become that vacuum, increasingly emptied of content and reduced to little more than their own form". [Braverman, 1974, p. 440]

Writing as a political economist rather than as an educationalist, Braverman shows convincingly how this process has followed the demographic and social changes in the nature of Western economies, driven by the thirst for more competitive and intensive production techniques, and fuelled by technology. The essence of Braverman's argument is that twentieth-century society [3] has witnessed a gradual deskilling of the work process, a "deskilling" not just of factory production lines, but of office-workers, clerks and white-collar workers in general.

In the two decades since the publication of Braverman's book, the mushrooming of information technology into every area of social life has only exacerbated the process he outlined. A sizeable proportion of those in work in the "developed" world, have been reduced to little more than human appendages to a computer-system; shop assistants no longer need to calculate change, bank clerks need know nothing about banking, waiters and waitresses no longer work out bills, engineering is reduced to following blueprints: even computer programming, heralded only a short time ago as creating a need for a newly creative, mathematically-trained workforce, has become, in the hands of the large companies who employ programmers, largely a routinised and alienating activity. As technology invades all aspects of daily life, people actually need less—not more— mathematics (see Noss [1991] for an elaboration of this argument).

Viewed from Braverman's perspective, the content of the curriculum is very much a secondary, increasingly unimportant, concern. This is indeed a position which has been adopted by those more particularly concerned with education, in particular, the celebrated analysis of Bowles and Gintis [1976], who argued that there was a "correspondence" between the needs of society's economic base, and the practices of the educational superstructure. Following Braverman, they focused their attention on the ways in which the educational system corresponded with the economic one, even borrowing Marx's metaphor in referring to the "social relations of education", and arguing that as far as the socialisation of future generations to populate the production process was concerned "the actual *content* of the curriculum has little role to play in this process." [Gintis and Bowles, 1988, p. 28]

Somewhat paradoxically, Gintis and Bowles have also argued (still from a strictly deterministic perspective) that the social relations of production *directly* affect the content of what is taught. So for example, in considering the rationale for the "back to basics movement" they identify in the nineteen-eighties, they argue that

> "... so called "back to basics", while having little rationale in terms or either pedagogical or technological reasons, may be understood in part as a response to the failure of correspondence between schools and capitalist production brought about by the dynamics of the accumulation process confronting the inertia of the educational structures." [Gintis and Bowles, 1988, p. 20]

Put bluntly, their case is that "back to basics" represents a more-or-less conscious attempt to pull the structure of the curriculum into line with the changed priorities of industry and commerce.

Bowles and Gintis have rightly been taken to task for viewing the curriculum as essentially irrelevant, and certainly for seeing it as directly driven by economic forces. I think—and so, latterly, do Bowles and Gintis [1988]—that it is most useful to conceive of the curriculum as a site of struggle in which pupils, teachers, parents, as well as voices from industrial, commercial, and other settings have at various times competed in various ways and with varying relative strengths to assert their priorities. What is important is to note that the structure and content of the mathematics curriculum is only partially determined by mathematicians or mathematics educators themselves: that they are not the free agents which they would like to believe. All those who compete within the educational sites are themselves immersed in social practices and imbued with assumptions which emanate from ideological as well as educational or mathematical settings.

There is an alternative view, one which stresses *not* how the curriculum is *determined* by economic forces, but how the specificities of the curriculum *function* within those forces. From this perspective, the curriculum is *neither* free from *nor* determined by the economic and political space in which it operates: it makes more sense to ask how mathematical ideas fit with society, how they encourage particular ways of seeing, particular ideologies.

It might be argued, as do Lewis and Gagel [19992] for the case of literacy, that the *usefulness* of mathematical content is central to its social functioning. They point to the ways in which the reading of newspapers or manuals, for example, have become essential in the developed world for the everyday functioning of those involved in the labour process. Whatever the truth of this assertion, it is clearly limited in its application to mathematics; I have already pointed out that almost nobody uses the content of school mathematics in their daily lives beyond the most elementary level. [4] While we might fruitfully analyse the mathematics curriculum in terms of the intellectual power of mathematical thinking—not to mention its aesthetic properties—there is little to be made on the basis of its use alone.

This has not always been a problem in quite the same way. After all, when the mathematics curriculum was made up exclusively of shopkeeper arithmetic, and taught to future shopkeepers, its meaning—even if it was essentially trivial meaning—was not hard to come by. But for thirty-five years [5] at least, the relationships between what mathematics is taught, to whom, and why, have been considerably more complex.

Consider, for example, how the nineteen-fifties vision of UK social life—nicely mirrored in the order and certainties of Euclidean proof—gave way to the turbulent and anti-authoritarian sixties with Euclid still in place. In the UK, the new orthodoxy was legitimised by, among others, the Plowden report (set up by a Conservative Minister of Education), which argued that a child brought up in a school which "lays special stress on individual discovery, on first hand experience and on opportunities for creative work", has "some hope of becoming a balanced and mature adult and of being able to live in, to contribute to, *and to look critically at the society of which he* [sic] *forms a part*". [Plowden, 1967, para 505; emphasis added] Viewed in this light, the demise of Euclid owes much more to a sea-change in attitudes towards society and the social function of its schools, which briefly took hold in the nineteen-sixties and seventies, than to a wish to change the kinds of mathematics learned by children

Proof did not disappear from the mathematics education scene *because* the social order of the nineteen-fifties gave way to the questioning and social upheaval of the nineteen-sixties; but the *social* process allowed space for those arguing against the empty forms of proving (and calculating) which characterised school maths. The "back-to-basics" tenor of the UK National Curricula (see Dowling and Noss [1991] for a critique of the England and Wales National Curriculum from a mathematical perspective) was not *caused* by the economic recession, but the recession played its role in silencing progressive voices in favour of those who believe that a more orderly, routinised, and dull educational system would offer a more reasonable training for post-school life. *It changed the balance between competing ideologies.* As I pointed out in Noss [1991] the topic of long division has been enshrined by law in the curriculum of England and Wales at precisely that point in human development when the ubiquity of the calculator and the computer has made that skill completely redundant. Would not a generation schooled in the repetitive, routine, and mathematically useless skills of long division have some qualities which the societies of the recession-laden nineties would value? (There is no need for a conspiratorial view here: but see Bassey [1992] for a tongue-in-check but chillingly viable conspiracy theory.)

The foregoing analysis leads us to a view of the mathematics curriculum as an intersection of competing and often implicit demands and interests which are reflected in what Green refers to as the inherent and delineated meanings co-produced by mathematicians, mathematics teachers and pupils. At base, the analogy with music allows us to see the generation of mathematical meaning as emerging from a dialectic between inherent and delineated meanings. The workings of this dialectic are played out in many ways, not least in the tension between form and content; between, say, the empty ritual of the form of mathematical proving, and the neglected meanings which adhere from the structure of mathematical proof. Mathematical ways of thinking, formal proof, symbolic rigour are not surface realities, ways of expressing, representations of pure essences; and neither do they sum up what mathematics is, they do not *themselves* constitute mathematical activity.

My conclusion is that mathematical thinking in general, and the notion of proof in particular, offers an alternative aesthetic, a way for people to understand, appreciate, and act upon their world. Mathematical thinking is a (non-unique) form of *theoretical thinking* and, as Otte [1990] points out, "Theoretical thinking presupposes a *variability* in the distance between the knowledge level and the objective reality about which the knowledge speaks "[p. 39] Conceived of in this way, mathematics becomes a way of accessing formal, theoretical thinking, rather than being seen as the epitome of formal thought itself. And why is "theoretical" thinking important? Because a theoretical solution to a problem may serve in the solution of other problems, perhaps whole classes or problems. Pragmatic solutions to problems are often sensible and functional: they are always context specific. But an *exclusive* adoption of this stance leads to something being lost: in gaining the possibility of an immediate answer, one runs the risk of losing access to questions such as "Would this work in other situations? What is the logic behind this? Why is this true? More generally, there is a failure to see the general in the particular. It would be reasonable to assert that a person who asks such questions (to him/herself or to others) would constitute an educated, aware person, capable of trying to understand critically local and global situations in which they are involved, and perhaps, to seek to influence them.

There are some who argue that proving should not be seen as central to mathematical learning. For example, Hanna [1983], in her scholarly analysis of proof and rigour, correctly locates the emptiness of much school-proving, the reduction of proof to the "empty form" which Braverman would, perhaps, recognise, but draws the conclusion that proof should be demoted in the curriculum, or at least, lose its status as an "all-encompassing methodology."[p. viii] One may sympathise with Hanna's perspective, not least when it is realised that she is writing from a North American context where undue emphasis on the empty forms of Euclid-style proving has emptied the curriculum of much of its potential content. But we might also point to the UK experience in which just such a turning away from proof and rigour occurred in the nineteen-seventies towards (at least in theory) "process skills", such as conjecturing and generalising. The instructions to "explore" and "investigate" have become obligatory in UK worksheets and textbooks. The difficulty, as Tahta [1988] points out, is that it is not always clear what the purpose of these activities is, still less what it is that makes them *mathematical*. More crucially, the emphasis on finding and exploring patterns exists at the expense of discovering their underlying relationships: "The idea that mathematics is essentially about seeing why something is true, not just the *what* that it true, seems missing" [Pimm, 1988, p. 63] Delineated meanings have almost entirely supplanted the inherent meanings which are what gives the notion of proof its power, its sense, its rationale for existence in the curriculum.

Perhaps, therefore, we have at least the framework for understanding the mechanisms at work on the curriculum. Braverman and Bowles & Gintis are not altogether unhelpful—it is just that the level of analysis is too broad, too wide-angled. The point is that as the visibility of mathematics in everyday life has decreased—as its use value has (for almost everyone) disappeared—other roles have been asserted for maths (as a social filter, as a mechanism of social control etc.) And alongside this process, the meanings which might have inhered within the mathematics curriculum have correspondingly slid into the background, to be replaced by mere ghosts of the meanings which give mathematics its intellectual power, delineated meanings which reinforce the role of maths as a commodity, as other than knowledge and content

The above analysis provides us with some way of making sense of a variety of readily cognisable phenomena. To take a topical (UK) example, we might begin to unveil the source of the confusion which surrounds the popular view of mathematics. Why do calls for "back to basics" resonate with parents, many (or most) of whom found the study of just those basics boring, difficult, and useless? Why does the replacement of repetitive "sums" by the use of calculators generate such indignation? A possible answer is that what we see in such situations is a desire for meaning, an implicit recognition that the delineated meanings which form the basic diet of the curriculum are simply not enough. To be sure the alternative is no better—far from it! But that is not the point. If mathematics educators want to argue successfully for non-trivial mathematics as a viable and important intellectual tool, we had better attend to the meanings we offer children, and the meanings they take away from their mathematical experience. We had better try to reassert the inherent meanings of the subject at the expense of the delineated meanings which all-too-often replace them.

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Notes

[1] This paper is an expanded and elaborated version of that due to appear in: Biehler R., Scholz R.W., Strässer R. & Winkelmann B. (Eds) *Mathematical didactics as a scientific discipline*. Kluwer.

[2] For Marx, the classical example of this fetishism was labour power—he uses the idea to describe the way in which a working person's "value' as a human being is progressively replaced by his/her "value" as an object of exchange.

[3] Braverman's analysis is based on the United States

[4] This appears to apply even to mathematicians: Ormell [1991] estimates that only about one percent of the body of known mathematics is recognisably useful.

[5] In the UK, at least.

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