SITUATED MATHEMATICAL RESEARCH: THE INTERACTION OF ACADEMIC AND NON-ACADEMIC PRACTICES

MIQUEL ALBERTÍ PALMER

One of the main purposes of mathematical research on non-academic practices is to identify mathematical knowledge. However, once this goal is achieved, we are left with the question of what to do with what has been found (Alangui & Barton, 2002). In this article, I aim to offer an answer to this question that is grounded in the actual practice of mathematical research. I suggest that knowledge from non-academic practices should not be left to anecdotic oblivion, but should be treated with academic respect. This entails analysis of its implications for academic knowledge and for academic education – a process that might open a new socio-cultural interactive space between academic and non-academic practitioners.

The practice studied here is Torajan architectural ornamentation – that is, the carved designs on the wooden façades of familiar houses and rice-barns of the Toraja people from Sulawesi (Indonesia). As I will show, new mathematical knowledge was developed as I grappled with a geometrical problem. However, this new knowledge did not arise through mathematical thinking that was isolated from the practice. Rather, isolated thinking led me to an error. It was only when I approached the problem in terms of the practice's technology and procedures that I was able to appreciate it fully and solve it. Hence, this work supports theories of situated knowledge development, since the development of (new) mathematical knowledge only happened after I situated the research in context of practice.

My solution can be regarded as contributing to both academic and non-academic knowledge. It improves the woodcarvers' solution from an academic point of view, but is also welcomed by woodcarvers themselves because of its practical uses. These qualities make it culturally respectful. After all, academic research contributes to both academic and non-academic practitioners' learning of mathematics.

The problem at hand is the construction of an unusual figure in such practice: a fivefold regular pentagram. In contrast to other geometrical problems solved by Torajan woodcarvers through the application of quantitative and systematic procedures, the pentagram's lack of precision is due to the absence of a rigorous quantitative tracing method. To explain the way Torajan woodcarvers face such a problem, I begin by developing the notion of a *situated mathematical interpretation* (SMI). From there I analyze how woodcarvers' method of dividing a segment into equal parts can be adapted to draw regular pentagrams, which I describe as a *situated mathematical application* (SMA). Finally, I summarize the whole process as *situated mathematical research* (SMR), framing the closing part of the discussion with my experience of communicating the new solution to Torajan woodcarvers.

Identifying mathematics in non-academic practices

Mathematical research on non-academic practices has been of prominent interest among mathematics educators since D'Ambrosio's (1985) introduction of Ethnomathematics. Two principal aims have oriented the emergence of the field. One is the search for vernacular contexts where relations with western academic mathematical knowledge serve as starting point for education – which, for example, was the focus of Gerdes' (1996) discussion of *hidden* or *glassed* mathematical knowledge. The other aim has to do with mathematical cognition and on vernacular mathematical knowledge developed in non-academic contexts. This emphasis is foregrounded in the theory of situated cognition as developed by Lave (1981) and in examinations of the mediating role played by cultural tools in mathematical cognition (Abreu, 2000; Nunes, Schliemann & Carraher, 1993, Säljö, 1996; Saxe, 1982; Scribner, 2002). Put differently, a focus on tools and their uses is a hallmark of situated theories of cognition (Resnick, Pontecorvo & Säljö, 1997).

Careful and detailed research of this kind must take into account not only a practice itself, but practitioners' ideas, procedures, language and technology as well. It is from this perspective that I developed the notion of situated mathematical interpretation (SMI). [1] Practitioners' explanations of their work are essential elements of SMI, which is organized around a three-level structured practice: finished-work (practice's product), work-in-progress (making of process) and explained-work (authors' justifications).

A critical point in practice is the way that specific tools constrain or enable the resolution of a mathematical task (Abreu, 2000). Moreover, in practice, thought leads to action, and so the analysis of external operations contributes to the development of cognitive models of inner operations (Scribner, 2002). Therefore, the rigour and precision present in a finished work, the technology used and the way the work is used become signs of mathematical thinking. Considered together, these elements contribute to the development of a mathematical thinking model, the veracity, validity and fidelity of which can only be confirmed or denied by the authors of the practice themselves.

SMI approaches practice in a scientific manner. Its methods are based on mathematical interpretations (MI) – that is, mathematical models – which must be validated at all three structural levels of practice to be declared an SMI. [1] It was through SMI that a non-Euclidean method to divide a segment into equal parts was identified in Torajan practice of architectural ornamentation. I will develop SMA and SMR as theoretical concepts after consideration of the actual practices among Torajan woodcarvers.

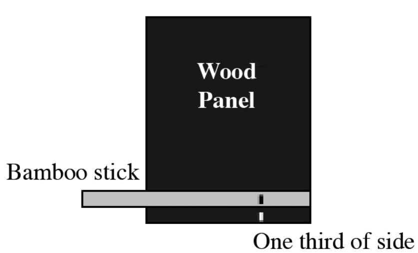
Torajan architectural ornamentation practices

Traditional architecture is a prominent aspect of Torajan culture. The family house (*Tongkonan*) and its smaller scale replica, the rice barn (*Alang-alang*) are wood buildings made without nails. The Tongkonan has a very important role in Torajan life as an axis of family kinship (Waterson, 1990) and a symbol of Torajan life and religious beliefs (Kis-Jovak *et al.,* 1988). The façades of both the house and the rice-barn are decorated with sophisticated carvings. In this oral culture with no written language, the carved designs serve as a type of a recording system through which the Torajan express and preserve varied and valued aspects of their society, culture and worldviews (Kis-Jovak *et al.,* 1988; Sandarupa, 1986).

The carvings are called *Passura*', meaning 'which represents to'. Designs are geometric and carved directly onto the wooden façades of houses and rice barns. Most are traced on grids of parallel and perpendicular straight lines. Others are made on geometric sketches. Grids are drawn using either a pencil and a straight bamboo stick or a pair of compasses. The main mathematical problem encountered by woodcarvers arises in the tracing of grids – namely, the division of the side of a wood panel into equal parts. Included among Euclid's Elements (I: proposition 10; VI: propositions 9, 10), this is a geometric problem that Torajan woodcarvers solve in a different way.

Academic and non-academic methods to divide a segment into equal parts

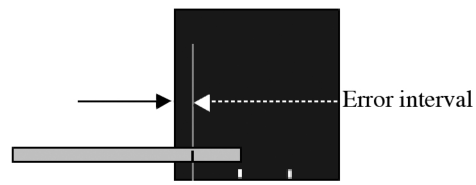
The educational systems in both Spain (my home country) and Indonesia (where the woodcarvers' live) follow the academically popular Euclidean solution, requiring a ruler and carpenter's square. The SMI of Torajan woodcarvers' procedure is called the *Kira-kira method.* It can be applied using a bamboo stick or a pair of compasses. Both tools are used to mark and transfer distances from one place to another, but not for measuring distances. By way of example, imagine that the side of a rectangular wooden panel has to be partitioned into three equal parts. The Torajan process is as follows:



i) Adjust the end of the bamboo stick to the side of the panel. Visually estimate one-third of the side and mark a tick on the bamboo and a tick on the panel side that correspond to that estimation.

ii) Move the stick to the left until its right end falls on the ticked mark on the wooden panel. Then tick another mark on the panel that aligns with the mark on the stick.

iii) Repeat the previous step. If the mark on the stick falls on the left side of the panel, the division is correct.



iv) If not, the error must be corrected. To do this, estimate (visually) one third of the error and mark the stick accordingly. Then repeat the whole process with the new mark on the stick.

This process moves exponentially towards the solution. A carver will rarely take more than a few seconds to finish a division of two or three parts following this method. (The solution is deemed 'correct' when the error becomes invisible.)

SMI of the Torajan construction of a regular fivefold symmetry star

The most common motifs of Torajan designs are spirals and circles. By contrast, stars are very rare and used only as ornamental accents. In such cases the star is typically very small and does not serve as the motif for a one- or two-dimensional design.

In figure 1, a pentagram, called *bintang lima* (five star) by woodcarvers, appears in the space enclosed between two concentric circles. To develop an SMI of this pentagram, I began by looking at a finished work. My first mathematical interpretation was that the star was drawn following proposition 11 of book IV of the *Elements* (Euclides, 1991) – a process that involve inscription of a five-sided regular polygon inside a circle and connecting its five vertices with consecutive straight segments. This is the procedure currently taught in secondary schools in both Spain and Indonesia.

After confirming the plausibility of my first mathematical interpretation, it was necessary to check its appropriateness by observing woodcarvers at work (work-in-progress). The following is a description of the artisans' method. It is important to note that neither the method nor the observation were silent. The two carvers discussed the process with each other and with me. Their method is summarized as follows (and illustrated in figs. 2–8):

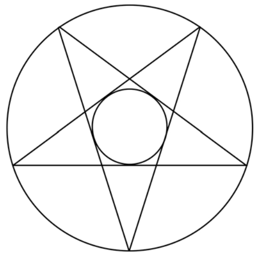


Figure 1. Bintang lima, the Torajan regular pentagram

i) Draw a square and its vertical median with pencil and bamboo stick straightedge.

ii) Inscribe a circle in the square with centre at the midpoint of the vertical median and radius the distance from this centre to the midpoint of one side (using pencil, stick and bamboo compass).

iii) Draw a concentric circumference of smaller radius (a rough guess, based on experience).

iv) Starting from the lower of the two points of intersection of the vertical median and the inscribed circle, draw a series of consecutive segments tangent to the small inner circle, each of which terminates where it intersects the large circle, returning to the starting point to complete a regular pentagram.

Based on this observation, my first mathematical interpretation has little to do with the woodcarvers' actual method. Moreover, the critical element in their procedure – namely the relation between the two concentric circles – was not readily apparent. Carvers' methods were tied to some kind of visual approximation, through which they seemed to start the process by seeking a specific angle of reference between the stick and the vertical median (see figs. 2 & 3).

Another plausible interpretation is that the inner circle was drawn according to some previously determined (correct) relation with the large circle, which I would visually estimate as 1:3 (fig. 4). However, this ratio is only an intuitive projection. At the time, I did not know the correct mathematical relation. I had never previously encountered this sort of mathematical problem.

Torajan woodcarvers treat this problem as an approximation. But, in contrast to the treatment given to the error that arises when applying the Kira-kira method to divide a segment into equal parts, here the error was either not controlled or not rigorously corrected. When the process was finished,

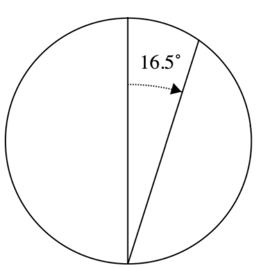


Figure 2. Torajan estimation of the regular pentagram's angle

it was clear that the star was not good enough (figs. 5–6). The carvers then slightly adjusted the tangents to the inner circumference to improve the result (figs. 7–8).

In brief, then, my second mathematical interpretation comprises the four-step procedure described above, along with the unknown approximations of the initial angle. Developed after interpellation, it became the SMI of the pentagram.

Note that the estimate of an angle is always perfect at its origin, where both segments meet in a point (fig. 9). But as the radius grows, so does the error, if any, at the vertex. Mistakes become more readily appreciated as the star becomes larger. In figure 9, the Torajan visual estimation (33°) for the regular pentagram's angle (36°) from the preceding images has been added to a regular pentagram. In conclusion, the estimate of any angle can be quite precise and, hence, useful for small figures. However, this is not the case for large ones, because the finest estimate becomes increasingly inaccurate as we move away from the vertex. In fact, carved pentagrams are small – never bigger than a compact disc. The images here show a very large one. Its size permits us to see errors that are not readily perceptible in a coin-sized pentagram.

Three questions arise, one mathematical, one situated, and one ethical:

a) *Mathematical problem:* What is ratio between two concentric circles enclosing a regular pentagram?

b) *Situated application problem:* Is it possible to apply the *Kira-kira* method to draw a regular pentagram?

c) *Ethical problem:* If it is so, should the researcher let Torajan woodcarvers know about it?

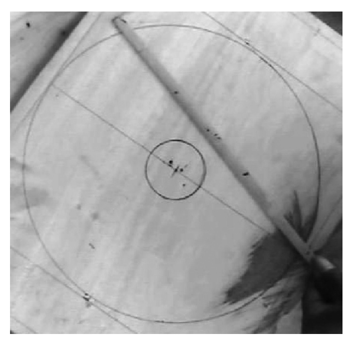


Figure 3. Visual estimate of the starting angle

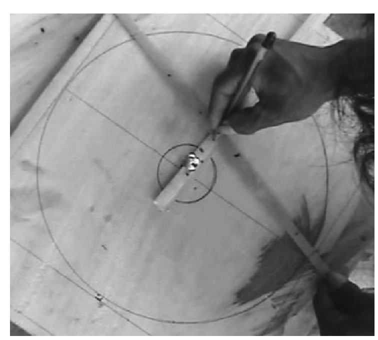


Figure 4. What determines the radius of the inner circle?

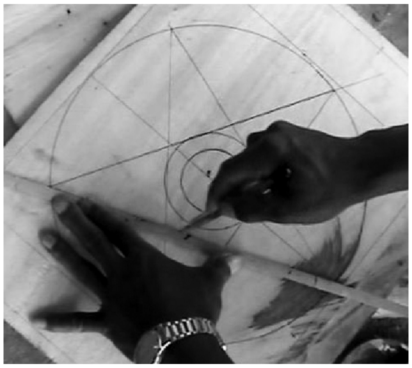


Figure 5. No tangency; result incorrect

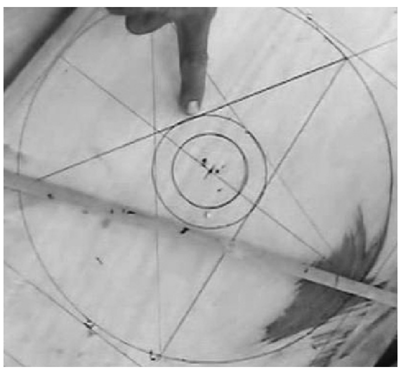


Figure 6. Mistake caused by imprecise tangency

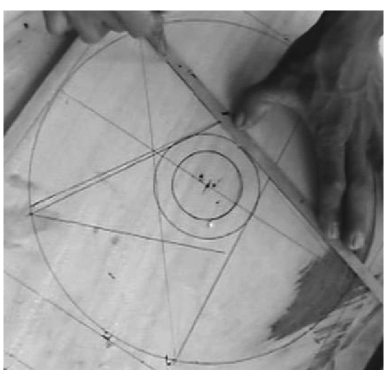


Figure 7. All tangents must be revised

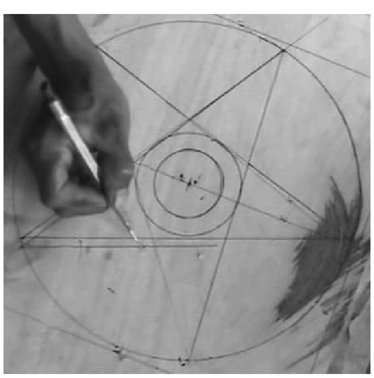


Figure 8. Good solution! Old tangents erased

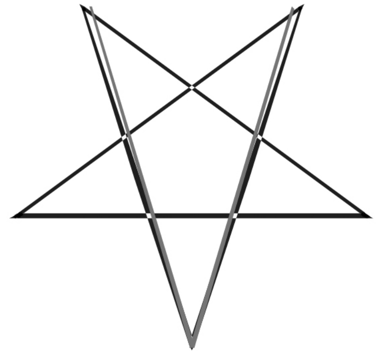


Figure 9. Torajan visual estimation (dotted lines) of regular pentagram's angle

Mathematical problem: generalization of the Torajan construction

Let me address the mathematical problem by stating its generalization: *What ratio (r:R) holds between two circles enclosing a n-fold symmetry star?*

I use the word *enclosing* to refer to the way that the star is inscribed in the outer circle and circumscribed on (tangent to) the inner circle (see fig. 1). Trigonometry offers a solution:

$$\frac{r}{R} = cos\left(\frac{2\pi}{N}\right)$$

For the case of $n = 5$, $r = R \cdot \cos(72^\circ) = 0.31R$. This result could lead to a third mathematical interpretation. As the ratio 1:3 seems a good estimation of 0.31, it can be argued that 1:3 is the ratio visually estimated between the two circumferences. Unfortunately, the actual practice reveals that a good numerical estimate does not necessarily lead to a good geometric estimate.

In theory, this method can be extended to the construction of all regular polygons. However, it would be necessary to know all ratios between both circumferences to obtain correct solutions, which is not practical. Moreover, trigonometry also reveals that the angle at each of the five vertices of a regular pentagram is 36˚. Hence, the Torajan artisans were 8.33% short.

Academic extension of the non-academic *Kira-kira* method

I now move the to situated application problem: *Is it possible to apply the Kira-kira method to draw a regular pentagram?* Five equidistant points on the circumference of a circle are needed to draw a regular pentagram. The pentagram is produced by connecting those points alternatively. The problem of determining the five points is equivalent to the problem of dividing the circumference of the circle (as a line) into five equal parts. The *Kira-kira* method properly divides a segment into equal parts.

The simplest case – namely, the division of a circumference into two equal parts – requires one to locate the midpoint, and Torajan woodcarvers have two sets of tools to accomplish the task. One is composed of a pencil and bamboo straightedge stick; the other comprises a pencil and a pair of compasses. As the bamboo stick straightedge offers the more obvious solution, I start with it.

Kira-kira **division of a circumference into two equal parts**

Let c_1 be the first visual estimate taken on the stick corresponding to the half of the circumference. Put the stick on the circumference, ensuring that c_1 and the stick's end fall on the circumference (fig. 10).

Mark their homologous points X and Y on the circumference (fig. 11). Placing the stick's end on Y, move the stick until c_1 falls on the circumference. Mark this point as Z (fig. 11).

As Z does not fall on X, it is evident that estimate c_1 is incorrect. How is such an error corrected? According to the Kira-kira method, the midpoint M_{ZX} of the left circular arc ZX must be estimated and its chord added to former estimate c_1 to obtain the estimate c_2. Unfortunately, and obviously, if this circular chord is added to c_1, estimate c_2 will be considerably longer than the desired segment. How to proceed?

It is helpful here to notice that, in actuality, we have not been working with arcs, but with their circular chords. These are the lengths taken on the stick. The resulting error must be corrected with the midpoint of the left arc(ZX), but not adding its chord to former estimate c_1. The second estimate should be the circular chord of arc(YM_{ZX}), which is not the sum of chords YZ + ZM_{ZX}, but the circular chord of the sum of their arcs:

$$\text{arc}(YZ) + \text{arc}(ZM_{ZX}) = \text{arc}(YM_{ZX})$$

Now fix the stick's end on Y and mark as c_2, the point at which the stick meets M_{ZX} (fig. 12).

We have come to an implicit use of a property concerning

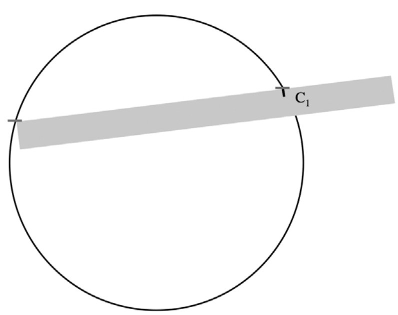


Figure 10. First estimate of a circumference midpoint

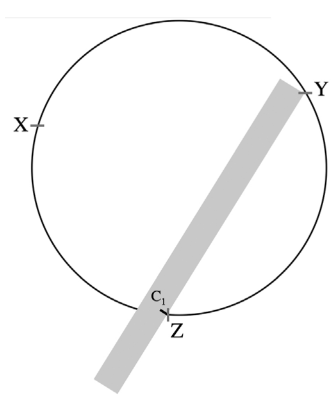


Figure 11. Second step

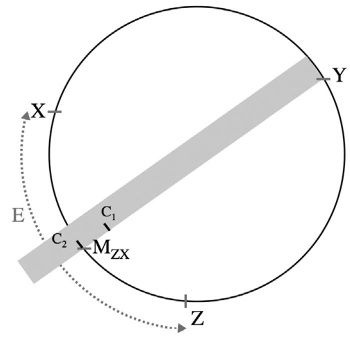


Figure 12. Improving the first estimate c_1 by c_2.

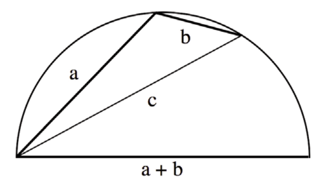


Figure 13. The arc of a sum of chords is larger than the sum of their arcs

to circular arcs and chords. The arc of a sum of circular chords is larger than the sum of their arcs, as might be justified using the triangular inequality (fig. 13). But equality actually holds if we change the way chords are added. If we consider chords *a*, *b* and *c* (fig. 13) as vectors, then vector *c* is the sum of vectors *a* and *b*. The property is true for non-consecutive chords. The vector sum makes evident that the arc of a vector sum of chords is the sum of their corresponding arcs.

Kira-kira **division of a circumference into *n* > 2 equal parts**

The process starts with a visual estimate, *c*, of the *n*th fraction of the circumference line, which is marked on the stick and repeated consecutively on the circumference to get n equidistant points P_1, P_2, …, P_n. The arc error *E* must be corrected by visual estimate, $E1_{1/n}$, of its *n*th fraction. Should a correction to an arc error be either inadequate or excessive, a new estimate c′ is taken. In such a situation, c′ is always the circular chord of arc $P_{n-1}E_{1/n}$, provided $E_{1/n}$ is taken from P_n (see fig. 14).

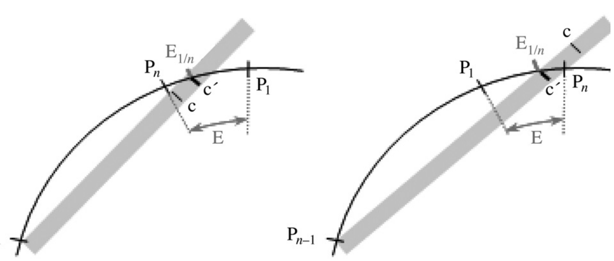


Figure 14. Correcting inadequate (left) and excessive (right) errors

Therefore, the *Kira-kira* method extends to circular arcs with all estimates taken as the circular chord of the vector sum or difference of two arcs, one being the former estimate, the other one corresponding to the *n*th fraction of the error arc:

$$c_{k+1} = chord[arc(c_k) + arc(E/n)]$$

Figures 15, 16 and 17 show an application of this method to divide a circumference into five equal parts using Word-Drawing tools instead of a bamboo stick. All estimates of the fifth of the error arc have been done visually. The first application (fig. 15) leads to a large error.

Kira-kira compass division of a circumference into *n* equal parts

Instead of pencil and bamboo stick, we can apply the *Kira-kira* method with a pair of compasses. With this approach, estimates cannot be written down and errors must be corrected in the air. Nevertheless, a pair of compasses can be quite practical because it is not necessary to shift attention to the circular chords, as one must when using the stick. With compasses, the focus remains on the arcs of the circumference. To make a new estimate, one merely opens or closes the compasses slightly until matching the sum of the former arc and the new error fraction.

The researcher's dilemma: academic education of non-academic practitioners

I now turn to the third question, addressing matters of ethics. Torajan artisans do not follow a circular *Kira-kira* version to create a fivefold-symmetry star. And thus my dilemma: should I, as an outside researcher, communicate to them such a possibility? Among the issues that rush to mind are those related to the colonization of foreign countries and to the imposition of (external/foreign) mathematical (ways of) thinking on people who probably do not care about it and who are already able to do their jobs effectively and efficiently without access to such thinking.

There is something else to consider. The formal education of many Torajan artisans is limited to elementary school. Some do not know how to write. From a western perspective, a primary goal of education must be toward democratic citizenship, and the school's role is typically defined in terms of offering education to all citizens. Arguably, educational researchers have an obligation to offer to non-academic practitioners everything that the academy was created to offer – that is, in my case, mathematics education and collaboration to improve practices. In doing this, the researcher returns in kind to those who have permitted investigation and, in the process, have supported the development of academic knowledge.

Following this line of thought, perhaps there would be multiple benefits to formal education, including: (i) offering academic meaning to mathematical thinking in non-academic contexts; (ii) formalizing non-academic knowledge; (iii) helping to improve practice; and (iv) providing non-academic practitioners with access to mathematics.

One and one half years passed between my recording of the Torajan woodcarvers' construction of the pentagram (January 2007) and communicating the circular *Kira-kira* solution to them (August 2008). They listened and shared

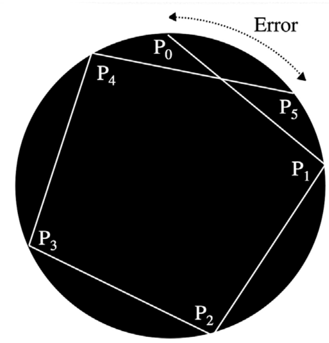


Figure 15. First Kira-kira *iteration to divide a circumference into 5 equal parts*

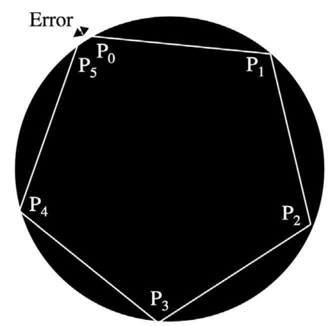


Figure 16. Second Kira-kira *iteration: error substantially reduced*

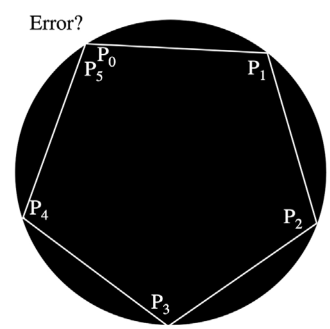


Figure 17. Third Kira-kira *iteration: error vanishes*

with me a practical application made on a wooden panel. During my exposition, they began to anticipate the results, thus demonstrating that they clearly understood and saw its close relation with the segment's *Kira-kira* procedure. Of course, it remains to be seen if they will alter the procedure they have been using to draw a regular pentagram. Such change cannot be forced.

When I returned home, I was unsettled, wondering if I might have interrupted the development of their vernacular mathematics by communicating my solution. Perhaps I had lost an opportunity to see what kind of solution Torajan artisans would have eventually developed? Was my attitude a consequence of the pleasure I derived at having come across a solution, or was it perhaps rooted in a typical western haste? Mathematical research should, perhaps, be more patient, mindful of avoiding the sorts of developmental interruptions it has imposed in past times.

Concluding remarks

Vernacular and academic approaches to a new problem

Torajan artisans do not use the *Kira-kira* method to divide the circumference into equal parts nor to draw the regular pentagonal star. However, formal mathematical analysis shows that, if properly adapted, the *Kira-kira* method is useful to:

i) divide a circumference and a circle into equal parts;

ii) draw a regular pentagonal star;

iii) inscribe regular polygons into a circle.

The extension of the *Kira-kira* method to the circle has to be made *mutatis mutandi,* reproducing into the circle precisely the procedure followed on a segment – that is, treating the circumference as if it were straight or, alternatively, considering the straight line as a particular case of circumference.

Säljö (1996) sees learning in modern society as a matter of mastering artifacts that embody rich and potent conceptual constructions. According to Abreu (2000), it follows that problem solving potential is affected by the tools physically present in the environment. This is something I have experienced myself as a researcher. The *Kira-kira* method and the bamboo stick tool have actually affected my abilities to solve problems. Only after mastering this tool and after learning the *Kira-kira* method was I able to develop a new approach to the problem of dividing the circle into equal parts and to realize that, within such a problem, the straight line is a particular case of a circle.

It is important to note that, in contrast to a ruler, a bamboo stick is not a tool by itself. It is just a broken branch in the jungle. What transform a stick into a tool are the way it is used and the need it addresses. Torajan artisans use it to solve a geometric problem, which entails mathematical thinking. Thanks to it, I, a research mathematician, found a way to construct regular polygons that is neither Euclidean nor similar to the Indonesian academic method (see fig. 18).

My method is completely different from that of the local artisans. Torajan woodcarvers saw the question in terms of proportional similarity. They wanted to reproduce an enlarged version of previously constructed regular pentagrams. They had no formal geometrical references and did not know any calculation-based approach that would generate a rigorous result. So they used bamboo stick and compass to inscribe a circle in a square, producing a figure that was not useful and resulting in a failed attempt. If they finally arrived at a good-enough version of the pentagram, it was because they focused their attention on angles, on the pentagram's vertical axis of symmetry and on trial-and-error. Facing a truly new problem seems to constrain thinking in the sense that new problems imply new methods and, perhaps, new tools. Regular pentagrams are odd singularities in Torajan woodcarving that Torajan artisans have not yet mastered. A fissure had been opened in their closed system of mathematical resolutions.

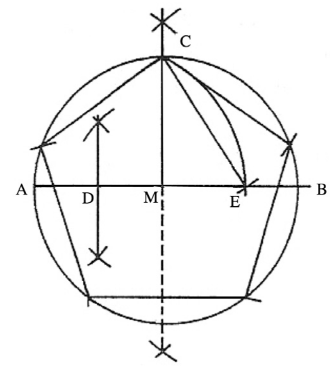


Figure 18. Construction of the regular pentagon (Negoro & Harahap, 1998, p. 476)

This system is a closed one; it solves the whole set of problems that it was developed to address. When a new problem is introduced, as in the case explored here, Torajan artisans do not use or develop their own processes of resolution. They do not extend their knowledge with the aim of covering the new situation. Instead, they behave as pupils often behave in mathematics classrooms, from the assumption that their former knowledge cannot be good enough to face new problems.

I would argue that what I have reported so far shows is a clear demonstration that things do not necessarily go that way. Stretching the *Kira-kira* method to the circular arcs expands the Torajan mathematical system S to another one, S'. The new problem that S could not solve is now solved by S'. This is the sense in which mathematical knowledge usually develops. Closure is not the rigid geometric surface of a mathematical resolution system, but its topologically soft conceptual border.

Epistemological questions concerning ethno mathematical research

Powell and Frankenstein (1997) raise several questions related to researching mathematical knowledge outside academic contexts. The first is around the common belief that "mathematics found in different cultural processes and artifacts are different mathematics or different manifestations of a unique universal mathematics" (p. 321). Based on what I have presented, I would argue that the notion of a unique universal mathematics, a platonic myth, should be regarded as a consequence rather than an axiom. Neither the *Kira-kira* method to divide a segment into equal parts nor its extension applied to draw regular polygons fits within the conception of western academic mathematics and its commonly assumed universality.

The second issue raised by Powell and Frankenstein is that, perhaps, "the value we bestow on local tradition is solely due to connections we see between it and academic mathematics" (p. 321–322). At this point it should be clear that the value bestowed here to ethnomathematical knowledge comes out from two perspectives. One is academic: it is useful for solving relevant mathematical problems; the other is cultural: as local traditions develop solutions for the problems encountered and social and cultural interactions support the development of other solutions.

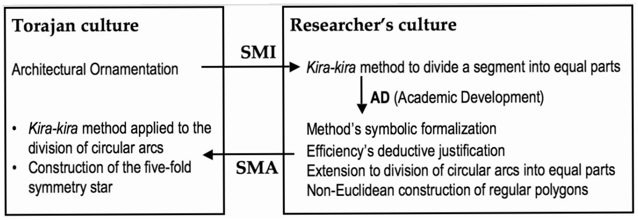


Figure 19. Interaction between Torajan non-academic practice and academic research

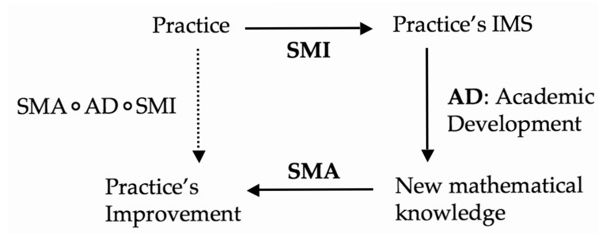


Figure 20. Academic role

Powell and Frankenstein's third point is that we can "miss truly different maths because we examine different cultural traditions through the lenses of academic maths" (p. 322). I have attempted to show here how taking off my academic lenses enabled me to see other mathematics. How could non-academic mathematical knowledge be identified (as happened with the *Kira-kira* method) by western research mathematicians who are not able to position themselves in relation to a new problem? The abilities to think openly and to situate oneself in a different cultural context might depend on *forgetting* some of what was learned in school.

Social and cultural interaction between academic and non-academic mathematics

Summarizing the interaction process described in this article, non-academic mathematical knowledge was applied to solve a traditional academic problem. The mathematically novel solution was returned to its non-academic origin to solve a practical problem. On the way, it inspired new academic mathematical knowledge. The process is summarized in fig. 19.

And so, returning to my opening question, what should we do with the mathematics identified? Speaking from the example developed here, I would suggest the following process:

a) Formalize it by expressing it in academic mathematical language (equations and formulas).

b) Extend its scope of application to include different mathematical problems from those it was developed to solve – be they new applications in academic or non-academic situations, practical or theoretical, formal or informal.

c) Bring it back to the practice from which it was taken to determine if the extension is of practical use and if it contributes to the improvement of the practice.

d) Move beyond the walls of academia and challenge others' misconceptions by offering readier access to mathematics education.

I close with a brief comment on the role of school in relation to non-academic practices. In this frame, the role of formal education is linked to its fundamental goals (fig. 20), which include mathematical enculturation, mathematical knowledge development, and the improvement of practice.

Note

[1] See Albertí, M. (2007) *Interpretación matemática situada de una práctica artesanal,* unpublished PhD dissertation, Departament de Didàctica de les Matemàtiques i les Ciències Experimentals, Tesienxarxa, Universitat Autònoma de Barcelona, SP.

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