Ethnomathematics and its Practice

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Alan Bishop [1988] stresses, rightly I think, the difference between **M** and **m**. **M** is Mathematics as it is known and developed in a scientific discipline in or out of universities. In other words, it is the mathematical knowledge of the mathematician and of the highly trained scholar in physics or engineering. On the other hand, **m** is the set of skills and procedures for counting, measuring, and the like, that a group or an ordinary individual knows and uses in life. There is probably some overlap between **M** and **m**, but apart from that the relationship between them is far from clear. When confronted with the task of teaching mathematics, the problem only sharpens: should we take the view that the **M** of the mathematicians is what everybody should learn to acquire, or should we develop the **m** of the culture of our subjects? Are both choices as exclusive as they often seem to be? I will offer some suggestions on these questions, using examples from field work with Navajo Indians (in the USA) and with Turkish immigrants (in Belgium).

Ethnomathematics

Partly because of the growing crisis in mathematics teaching, and partly because of major problems in the formalist or "absolutist" views in the study of the foundations of mathematics, a naturalist theory of mathematical knowledge (and its correlate pedagogy) gains respect in our days. The "absolutist" view on mathematics (formalism, logicism, and constructivism: see Ernest [1991]) holds that the knowledge of **M** is *a priori* and indubitable. They claim that mathematically true statements are always deduced from *a priori* or given propositions. Empirical reality does not enter at all into this system of thought, according to this school. Hence the teaching of mathematics should follow the "logical structure" of **M** and thus initiate the pupil in the deductive and basically decontextualized knowledge of the mathematical field. Ideally, one should start by teaching the concepts and procedures of the "ground floor" or foundations of the mathematical "building" (e.g., set theory) and logically proceed to teach the next "store" along the way. The philosophy of the New Math (in programmed instruction or otherwise) is the most radical elaboration of this point of view. The concepts and procedures of **m** are considered to be "bad mathematics," false procedures, or at best prescientific aspects of **M**, in this perspective. They should be eradicated or at least corrected through education.

An opposing view on the nature of **M** is emerging in our time. Because of the failure of the absolutist program (through Gödel's theorem on which I will not dwell here) and because of internal contradictions in the absolutist views (see Ernest [1991]), and because of the growing disaster of teaching programs based on this philosophy of mathematics, naturalistic views on the nature of mathematical knowledge have emerged. One such view is that mathematics is "essentially a symbolic technology" [Bishop, 1988: 18] growing on skills or "environmental activities" that are cultural in nature. A concomitant view is captured under the label of ethnomathematics, as I understand it. Generally speaking, the proponents of any naturalistic view hold that mathematical knowledge (including the insights or intuitions of **M**) is founded within the cultural context of the knower. Knowledge in general, and mathematical knowledge in particular, is contextual and cultural in nature [Pinxten, 1992]. A consequence of this position is that **M**, in a nontrivial sense, builds on and is somehow guided by the concepts and intuitions of **m**; empirical facts, elements of faith, even the action procedures of one's own culture, form the genuinely contextual foundation of **M**. The rest of the paper will be concerned with this perspective.

An obvious question that arises from such a view of mathematics is: what does **m** look like? The bulk of ethnomathematics, as I see it, seeks an answer to this question. One way to approach this question is given in Bishop's notion of six environmental activities. Throughout the world, in all cultures we know of, people engage in a series of activities that provide the basis of "formal" or mathematical knowing. Bishop [1988, 1988b] identifies six types of activities: counting, locating (in space), measuring, designing, playing, and explaining. Different cultures exhibit and develop these activities to a different extent. The study of these activities will allow us to get an insight into the mathematical procedures and into the ways people are trained in them in any particular culture.

Over the years different authors have in fact documented one or the other of these activities in several cultures. A magnificent early work is Zaslavsky [1973] on counting in Africa; also Needham [1965, 3] on algebra and geomancy in ancient China. Lancy [1983] gives precious information about Papuan skills and traditions, and my own work concentrated on the spatial knowledge and intuitive geometry of the Navajo Indians [Pinxten *et al.*, 1983, 1987]. The most comprehensive work to date is the great book by Ascher [1991]. It is obvious that Ascher is a mathematician and not a social scientist (like Lancy or me). Her approach basically starts from deep insights into the field of **M**. From this disciplinary background she selects those concepts and skills in "our" mathematical experience that seem most relevant and interprets the variegated activities and skills of other cultures in these terms. Thus, the notion of number (and not just the activity of counting) is investigated as an implicit or explicit concept in various cultural phenomena. Going beyond describing the superficial level of terms and actions which express numerical relations within a cultural group, Ascher develops a (deep structural) analysis which shows how numbers are operating in a strict way in the phenomena. Similarly for graphs: the complex drawing of "geometric" figures in the sand is analysed in a systematic way as an exemplification of the use of graphs. Spatial behavior, geometric decorations, the logic of kinship relations, and other cultural phenomena are thus described and explained through the mathematical concepts and procedures that are employed or expressed in them. In my opinion the importance of this work is twofold. In the first place Ascher produces a complement to social scientific research on these topics: rather than describing the phenomena in terms of the culture or the psychology of the subjects, she goes the other way and describes the cultural and psychological data as arising from ways of dealing with the mathematical problems she recognizes. This complements the "soft" approaches of the anthropologist and the psychologist in an important way. In the second place I identify in her work a solid bridge between **m** and **M**. Indeed, where social scientists have described in detail at least part of the field of **m** (the mathematical knowledge of the common man in a particular culture), Ascher shows through her reinterpretation of these data in terms of **M** (the discipline of mathematics) how **m** and **M** can be linked with each other. An explicit and conscious link can now be built—that is, an educational program can be devised to take pupils from their **m** to **M**. From this perspective the broad scope of D'Ambrosio's proposals [1987] is taken an important step forward: the program can actually be implemented now. It is at this point that my present work leads me to make a small modification.

Field data: Navajo Indians and Turkish immigrants

The field work, leading up to a curriculum book for geometry teaching among Navajo Indians, was carried out in 1976-77 and 1981 (with my wife, Ingrid van Dooren: Pinxten *et al* [1987]). The collection of data of Turkish immigrant spatial knowledge is largely the work of Marijke Huvenne, my collaborator on a recent project of immigrant studies in my Department.

The Navajo case

In the course of our research on the spatial knowledge of Navajo Indians we were struck by several findings. The Navajo experience the world they live in as profoundly dynamic: nothing is still, there is always movement and change. Their world is one of events and processes, whereas ours is first of all one of situations and objects. In language we find a correlate to this way of dealing with the world: the Navajo language expresses almost everything in verb forms, whereas the Indo-european languages distinguish between noun and verb forms. In Navajo the verb "to go" conjugates in a vast multitude of ways (over 300,000 is a rough estimate), and at the same time a genuine correlate of the verb "to be" is missing. The many aspects (rather than tenses) in the grammar of the language describe a variety of ways actions are begun, stopped, repeated, and so on. A consequence of this way of speaking and thinking of the world is that the part-whole logic is missing in the Navajo perspective. In the Western tradition this logic is basic to our formal thinking: at a conscious or a subconscious level we conceive of the world of experience as a whole, a whole which can be divided into a myriad of parts. In the same way, we may solve a problem by splitting it up into parts which can safely be solved first. The "sum" of all the solutions then yields the solution to the larger problem. In mathematics teaching this basic structure of our world view is nearly ubiquitous: set theory distinguishes between a set (the whole) and its elements (parts), with particular operations defined over each: geometry says that a line is the sum of an endless series of points, and so on. In the dynamic world view of the Navajo this part-whole reasoning does not occur. In the second place, the Navajo spatial and (intuitive) geometric notions are nearly always co-defined in terms of movement: a border or edge is not a line but rather a barrier which causes some modification of an action (e.g., stop, transgress, etc.); distances are conceptualized in terms of walking, or moving around with reference to landscape markers, and so on. In the curriculum we developed for intuitive geometry it was our aim to take these cultural foci fully into account. Because it is my conviction that the naturalistic perspective on mathematics education is the most sensible one, the curriculum explicitly started from the "world as experienced by Navajo children." Thus, we selected several contexts from the preschool experience of the Navajo and elaborated these to gradually reach more sophisticated geometric knowledge on the basis of them. The difference in nuance, rather than in nature, with ethnomathematical programs so far is that we proposed to develop, train, and elaborate the native categories as far as possible. This could take several years of schooling in the Navajo vernacular working with Navajo concepts of space and intuitive geometry. Only after that time, and on the basis of the insights developed along the way from their own perspective on space, can the step be made to Euclidean or "Western" geometry and mathematics proper. Short of that, we are convinced, mathematics education will be abortive for most children and adults in this culture because these insights are lacking in a learning process that disregards the Navajo world view and assumes that the world is present in a Western (or so-called scientific) perspective in everybody's experience. Visualization is a crucial element in this educational process from the native point of view, more important than its conscious exploration and sophistication in the direction of geometry and mathematics proper (see Pinxten *et al.* [1987] for more details). The link between **m** and **M** is not automatic, in my view, and should be allowed to develop in native terms first. An example will show what I mean.

One of the contexts of experience we proposed is that of the hooghan (or traditional building). The hooghan is a cosmological scale model, exhibiting Navajo concepts of up-down, the cardinal directions, propositions, moving space, and so on. The child "knows" this from visits to the hooghan, or from living in one. In the classroom the children are invited to explore mentally all the relevant aspects of the hooghan and to reconstruct it. A group works together to "build" a hooghan out of waste material. In the process the orientation and the proportions are explored. As a second step a scale model is constructed. The visualization processes are more focussed here and group agreements on size, proportions, and the technical terms for concepts are reached. Finally, graphic representations of the hooghan are undertaken, leading to a substantially more "abstract" representation. The Navajo language and the native concepts are respected and actively developed throughout these lessons.

The Turkish case

Turkish immigrants in Belgium come, in large part, from rural areas in Turkey. Their language and its world view are to an important extent pre-Newtonian, although the distance between them and the European correlates is far less than that between Navajo and European worlds of experience. In our observations we found differences in the world of daily experiences: the tasks of the math curriculum are often too ethnocentric here (e.g., speaking of Western city life rather than Turkish rural life) causing misunderstanding in Turkish children. Moreover, the Turkish language offers some difficulties in the Western school context: the plural is not indicated in the noun but a generic noun often substitutes for ordinal numbers. Prefixes do not exist in Turkish, leading to difficulties with such "obvious" references as *on, under, beside,* etc., the table. These few examples should be sufficient to show the problem we are facing. We propose the following curriculum project to deal with these difficulties.

In mixed classes (immigrants and natives) the children are asked to describe the neighborhood. Notions such as center, border, far and near, and so on, are described in both Turkish and Dutch. Children are encouraged to collaborate across cultural borders on this task. The translation problems are left largely to the children to "solve" in practice. Building scale models and making graphic representations of the neighborhood allow them to gradually come to grips with common meanings and the differences in conceptualization and speech about such abstract notions as distance, size, proportion, and geometric figures (we are working in a city environment). Only after that first period of consciousness-building and the training of visualization does the jump to Euclidean geometry and to mathematics proper impose itself. Again, insights into the functionality and use of geometric notions and the relevance of precise conventions in speech and thought will grow on the children, laying the foundation for a better entry into mathematics education proper. The mixed classes will provide mutual understanding and a deeper grasp of the difficulties and possibilities of a multicultural setting for dealing with the world

Conclusion

Even this very succinct presentation of these cases may allow the reader to get a better understanding of the point I want to make in this paper. In general I wholeheartedly embrace the perspective of ethnomathematics as I have come to understand it. The one point I want to advocate here is missing in most of the treatises I know. That is to say, I claim that these action research studies show that children from another culture live with another world view than the one implicitly held by our curricula in mathematics education. My proposal is to make this conscious and explicit, and to actively train these (other) native insights of the subjects in order to reach a level of understanding and sophistication in them which is already largely available (subconsciously) in the preschool knowledge of the Western child entering the curriculum. The exploration of functional relations and spatial concepts in the vernacular, using the contexts of the experience of the children allow us to do this. Through this exploration, linguistic mapping, and the actual training of visualization, the child will reach the insights needed to come to grips with the (implicit) world view of **M**. Hence the move from **m** to **M**, and the use of **m** notions in the successful development of **M**, will have to pass through the conscious, systematic, and explicit exploration of the largely subconscious and ill-developed outlook in **m** in the preschool child of another culture. The question of whether we will opt for the exclusive format of **M** as we know it, or allow room for the development of alternatives or elaborations of it according to the different cultural knowledges we encounter, is not dealt with here. I leave it open

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