The Limits of Rationality*

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0. When I was invited to give an address to the Study Group, the title seemed to choose itself. I had in my mind traces of recent readings that had rubbed against each other and created a disturbance The period of almost a year between the invitation and the delivery appeared to offer a fine opportunity to do what needed to be done to arrive at a fresh, structured, survey of the territory As is usual with me, the opportunity somehow slipped by unseized. I bring only a few out-of-focus snapshots.

1. The phrase itself I take from the introduction to Herbert Simon's *The sciences of the artificial*. This particular occurrence of it has lodged with me, though the phrase — as against the context in which it is used — is unlikely to be original. Simon is talking about "artificial" phenomena which "are as they are only because of a system's being molded, by goals or purposes, to the environment in which it lives." [Simon, 1981, p ix] How is it possible, he asks, to make empirical propositions about systems "that, given different circumstances, might be quite other than they are?" [ibid., p. x]

> My writing . has sought to answer those questions by showing that the empirical content of the phenomena, the necessity that rises above the contingencies, stems from the inabilities of the behavioral system to adapt perfectly to its environment — from *the limits of rationality*, as I have called them [ibid., p x; my italics, D. W.]

Simon offers the image of an ant making its laborious way across rough ground. The track the ant makes is irregular and apparently unpredictable. Yet it is not a random walk for it takes the ant towards a particular goal. We can readily suppose that any very small animal starting at the same point and having the same destination may well follow a very similar path

> *An ant, viewed as a behaving system, is quite simple The apparent complexity of its behavior over time is largely a reflection of the complexity of the environment in which it finds itself.*" [ibid., p 64; author's italics]

Could we not hypothetically substitute the words "human being" for "ant"? Simon continues.

> A thinking human being is an adaptive system; man's goals define the interface between his inner and outer environments, including in the latter his memory store To the extent that he is effectively adaptive, his behavior will reflect characteristics largely of the outer environment (in the light of his goals) and will reveal only a few limiting properties of the inner environment — of the physiological machinery that enables a person to think. [ibid., p. 66]

To show that there are only a few "intrinsic" cognitive characteristics and that "all else in thinking and problem solving is artificial" [ibid , p. 66], Simon analyses a familiar cryptarithmetic problem. He finds that solvers differ mainly in their solution strategies and suggests that efficient strategies could easily be taught to those subjects who do not spontaneously produce them. The "limits of rationality" are not to be found here but in the general weakness of human short-term memory, a weakness that makes it necessary for human beings to adopt compensatory strategies

> Insofar as behavior is a function of learned technique rather than "innate" characteristics of human information-processing system, our knowledge of behavior must be regarded as sociological in nature rather than psychological — that is, revealing what human beings in fact learn when they grow up in a particular social environment. [ibid., p. 76]

As always in reading anything by Simon, I get the sense of an immensely powerful intellect sailing on towards the magnetic rather than the true North. The clarity, however, is bracing, the ideas challenging to many of my presuppositions I feel I am closer to grasping the nature and purpose of strategies in problem solving, for example; and the proposition that the complexity of behaviour arises from the complexity of the task and not the complexity of the organism working on the task becomes a hypothesis worth struggling to refute. But before I give in to the temptation to enlarge the first snapshot, let me change the slide

2. A different and more alarming view of "the limits of rationality" is captured in the following sentence from Léon Brunschwicg's paper, "Dual aspects of the philosophy of mathematics":

> . . . the preconceptions of an overly abstract and narrow definition transforms reason into a machine for fabricating irrationality. [Brunschwicg, 1971, p. 228]

Brunschwicg draws his theme from the Pythagoreans.

> When, by representing numbers by points, they showed that the successive addition of the odd

numbers furnished the law for the formation of squared umbers, they were extracting evidence of a perfect harmony . . between what is conceived in the mind and what is obvious to one's vision. [ibid., p. 225]

This "triumph of reason should have been decisive; it was immediately compromised by a twofold weakness in itself." [p. 226] On the one hand the Pythagoreans could not resist the temptation to push their luck, to go far too far. "Thus 5, the sum of the first even number, 2, and the first odd number, 3 (unity remained outside the series), would be the number for marriage because even is feminine and odd is masculine." [p. 226] And on the other hand, when the difficulty of incommensurability surfaced, the Pythagoreans turned their backs on rationality by banishing incommensurable magnitudes to a "beyond"

> They receive a command from their avenging gods to deliver to the fury of the tempest the sacrilegious member who had the audacity to divulge the mystery of incommensurability [ibid , p 227]

They implicitly — and the more dangerously because of the implicitness — decide that incommensurability will be "something that one does not dare to speak of" and so, Brunschwicg says, "the irrational threatens to obscure the whole philosophy of science " [p. 227]

From a rich and subtle paper I select another example

> Pascal and Leibniz seem to be working together to force open the doors of mathematical infinity But is this to be done by pushing beyond the normal resources of reason? Leibniz parts company with Pascal on this fundamental issue. He returns to the path of Cartesian analysis, while Descartes and Pascal find themselves united in their opposition to Leibniz's position that the deductive process is self-sufficient. The two of them have proclaimed the primacy of intuition, even though they otherwise give it a radically different meaning. [ibid., p. 232]

All three mathematicians reject the position that mathematics is a natural system reduced to its ultimate abstraction; for them "it is the fitting prelude to, and the relevant proof of, a spiritual doctrine wherein the truths of science and religion will lend each other mutual support." [ibid., p. 233] Not every mathematician, of course, chooses this same path.

Brunschwicg's general message is that there are fundamental characteristics of mathematical thought that underlie the disagreements among mathematicians abut the sovereignty of reason, and that undercut all dogmatisms that would place the limits of reason "here" or "there" Fortunately for mathematics "the manner of investigation has no bearing on the value of a discovery." [p. 234] As to this, I can't be sure; meanwhile I retain that particular image of the Pythagorean machine, reason gone mad, spewing forth irrationalities. The image resonates unnervingly

3. Less unnerving, but decidedly unsettling, is the drift of Dick Tahta's article, "In Calypso's arms" (*For the Learning of Mathematics*, 6, 1). Did mathematics originate in commerce or ritual?

> There was a time, for instance, when historians of mathematics would very confidently assert that mathematics began in the needs of highly organised social systems to calculate taxes and to keep inventories. In a less confident economic climate, there has begun to be some cautious speculations about other origins [Tahta, 1986, p 17]

We have no records to tell us unequivocally how mathematics began, and just as in other cases where we don't know the "facts", we construct "myths". Even the procedures and purposes of the high culture of Greek mathematics, about which we may feel we know a lot, remain essentially a matter for conjecture

> For the purist, there is almost nothing that can be said about the early classical period with any certainty. We know the names of a handful of individual mathematicians . (The) arithmetic tradition (of the Pythagoreans) is mainly interpreted from commentaries written *several centuries* later [ibid., p 18]

Tahta goes on:

> Such aspects have been mythologised to such an extent that it hardly seems relevant to question whether they describe what was the case. This is, however, to accept a view that "narrative" truth, or myth, is — in some situations — more important than historical truth; it is to accept willingly that myths grow by accretion, so that, for example, what people have thought about Greek mathematics may become part of the history of Greek mathematics [p. 18]

When alternative myths are available, as they are for the origins of deductive geometry, say, which shall we choose? There is no real possibility of settling the question objectively. "It is, I claim, a question of preferred myth " [21] Some myths may work better than others, especially for pedagogical purposes, and it is sensible to choose, openly and knowingly, those myths that are most powerful and helpful Historians will naturally disapprove but

> the continuing reflexive generation of the account mathematics gives of its own history is too important to be left solely to historians — or to mathematicians. Teaching is part of the mathematical enterprise and teachers can help decide what is to be considered significant at any one time [ibid., p. 22]

It is, indeed, unsettling to suggest that reason cannot lead us to the unique right answers to questions about the nature of Pythagoreanism, the origins of deductive proof, the purpose of the arithmetisation of analysis . . or whatever. Well, we shall just have to be as brave as we require our students to be when we prise them away from their treasured beliefs in the unique rightness of solutions to mathematical problems

4. Consider the words of the title

LIMIT — ULTIMATE / BOUNDARY / OBSTACLE

RATIONALITY — REASON / REASONABLENESS / RATIONALISATION

The alternatives seem to run from "high" to "low". This is particularly obvious in the second case. "Reason" cries out for a capital letter: for some it is the greatest of the mental powers, the characteristic that makes a human being human. "Reasonableness", on the other hand, is moderate and modest, a characteristic of the ordinary man, whether in the street or on the Clapham omnibus. "Rationalisation" is a low form of reason, a misprision of reason's power to grasp phenomena and make them comprehensible.

Rationality reminds us of the sober virtue of getting things "in proportion". Is it a coincidence that intelligence tests are full of questions of the form, "A is to B as C is to ?"? On the other hand, being rational may be no more than exhibiting common sense. It is this latter connection that supplies the essential social and consensual flavour. Rationality is an endowment of all human beings in the sense that everyone has the possibility of learning to be rational just as everyone is born able to acquire a spoken language, but the particular form of rationality (i e. common sense) that a person acquires is determined by social and cultural factors as is the particular language that the person learns to speak

5. David Bloor, in a speculative article contrasting Hamilton's and Peacock's views on the essence of algebra, talks of Hamilton's involvement with Idealism, which he learned mainly from Coleridge and Carlyle

> Carlyle .. goes on to explain precisely how Idealism has a practical bearing ... By making matter dependent on mind, rather than something in its own right, Idealism removes the threat of a rival conception of Reality. [Bloor, 1981, p. 208]

In Carlyle's view, all conclusions of the Understanding have only a relative truth: "the Understanding is but one of our mental faculties There is a *higher* faculty which transcends the Understanding and gives us contact with non-relative and non-dependent Absolutes." [ibid., p 209] This higher faculty is, of course, Reason which, in Carlyle's words, should

> "conquer . . all provinces of human thought, and everywhere reduce its vassal, Understanding, into fealty, the right and only useful relation for it "

This elevation of Reason to the level of the sacred (echoes of "which passeth all understanding"?) has powerful social and political implications, but I will not follow that track here. Bloor suggests that in relating algebra to our intuition of pure time, Hamilton was attempting to raise algebra to the level of the holy too.

> The essence of algebra was given a direct association with the Reason, with what was prior to and determined the form of experience. At the same time it was thereby put in close proximity to our insights into moral truths and their divine origin In a word, Hamilton was irradiating algebra with spirit. [ibid , p. 216]

In the controversy between British mathematicians about the nature of algebra, Hamilton took neither the side of Frend, for whom algebra was universal arithmetic, nor the side of Peacock, for whom algebra was a symbolic system with arbitrary rules, but implied that "its essence was derived from the laws and constitution of the mind itself—and the most exalted part of the mind at that." [p. 217]

It may be arguable whether this last propostion necessarily belongs to Idealism or not, but the whole story (which I have not been able to offer here) suggests that attempts to give Reason an autonomous role, a position above all conflict, safe from refutation, only succeeds in embedding it the more firmly in a local, contingent, metaphysics.

6. In "Reflections on gender and science", Evelyn Fox Keller says:

> I argue that we cannot properly understand the development of modern science without attending to the role played by metaphors of gender in the formation of the particular set of values, aims, and goals embodied in the scientific enterprise. [Keller, 1985, p. 43]

At around the time of the foundation of the Royal Society, intellectual history could be described schematically in terms of two competing philosophies: hermetic and mechanical: "two visions of a "new science" that often competed even within the minds of individual thinkers." [p. 44]

> In the hermetic tradition, material nature was suffused with spirit; its understanding accordingly required the joint and integrated effort of heart, hand, and mind. By contrast, the mechanical philosopher sought to divorce matter from spirit, and hand and mind from heart [ibid , p 44]

The founding of the Royal Society in 1662 marked the victory of the mechanical philosophers and the defeat of the alchemists, stigmatised as anti-rationalists. The Baconian programme was adopted, and with it, the sexual metaphors in which it was expressed

> A recurrent token of this is their Baconian use of "masculine" as an epithet for privileged, productive knowledge As Thomas Sprat (1667) explained in his defense of the Royal Society, "the *Wit* that is founded on the *Arts* of men's hands is masculine and durable." In true Baconian idiom, Joseph Glanvill adds that the function of science is to discover "the ways of

captivating Nature, and making her *subserve* our *purposes*." (Easlea 1980, p. 214) [ibid., p. 54]

The last quotation suggests a clear association between scientific rationality and the act of rape. I am not sure one could wish that the hermetic alternative had entirely won, but the metaphors give an appalling indication of the social price that had to be paid for the establishment of modern science and certainly supply a motive for considering whether any of its damaging side-effects may be ameliorated. Three hundred and more years later, are we any wiser in our day?

7. The achievements of scientific rationality may seem so substantial that we choose to forget its tendency to tip over into irrationality. The process is more apparent in the human sciences where the danger of pushing rationality too far and forcing it to tip over is only too obvious. Or should be.

Pedagogy provides an illuminating example. It is a reasonable pedagogical principle to break up what is to be learned into manageable pieces; but this principle becomes an absurdity when *everything* presented to be learned is broken into separate pieces, each as small as possible, so that the totality cannot be perceived. It is a reasonable pedagogical principle to guide students in such a way that they do not fall into egregious error; but this principle tips over into foolishness when it becomes an attempt to prevent students from making any mistakes, denying them access to an important source of feedback. It seems to me a legitimate matter for rage and the gnashing of teeth when *teachers* (ha!) and *educators* (ha! ha!) close their minds to the irrationality of their actions. In my more pessimistic moments I fear that the educational system will *always* manage to pervert *any* rational principle in short order by pushing it further than it will stretch.

Of course, for many people, including a lot of teachers and educators, pedagogy has a dubious existence. They don't believe teaching is an activity one need be, or can be, scientific about. But teaching is not a transparent process for transporting something from place A to place B; it is not a catalyst, facilitating learning without influencing it. Consider how one may introduce students to, say, the solution of simple linear equations in algebra. The metaphor of the balance may suggest certain operations on an equation while making others, algebraically just as important, seem implausible. It is well known that the "think of a number" approach and the "unravelling" technique it suggests work admirably for equations with a single appearance of the unknown but fail to give a lead to the solution of, say $5x = 3x + 6$. On the other hand, the Dienes method of representing both sides of a linear equation with suitable pieces of wood gets around the particular limitation of the "think of a number" approach while introducing another obstacle: that of regarding two manifestly different amounts of wood as representing two equivalent algebraic expressions.

All pedagogical devices cast their imprint on the matter they are designed to teach. And in case one would be so naive as to suppose that this difficulty might be avoided by suppressing pedagogical devices altogether, let us remember that when we teach anything to someone who does not yet know it, we cannot proceed without offering the person at least an implicit model of what is to be learned.

The need for pedagogy comes from another source too. There is an inevitable tension between engaging with mathematics in order to use it and engaging with it in order to teach it. The teacher and the mathematician do not have the same professional insights into mathematics; what is illuminating for one is not necessarily so for the other. The Hindu-arabic notation, when it reached Europe, played hell with the teaching of arithmetic, causing teachers to substitute "ciphering" for the counting and manipulation of beads and other objects. [Smith, 1900] Giving the number system a solid foundation in set theory was a liberation for mathematics and an aberration in the classroom. The HP 28C is a remarkable mathematical aid, but it is not the calculator that educators would like to have been able to design to sort out some of the difficulties for the learner of college mathematics. Indeed, what is best for mathematics and the mathematician is not always best for teachers and would-be mathematicians.

8. In coming to the end of this slide show, it seems appropriate to ask whether rationality is an instrument of human liberation or of human enslavement. To the extent that rationality is institutionalised and embedded in a specific culture, it has the power to be both. As Jules Henry puts it:

> Thus, the dialectic of man's effort to understand the universe has always decreed that he should be alternately pulled forward by what has made him *homo inquisitor* and held back by the fear that if he knew too much he would destroy himself, i.e. his culture. So it is that though language has been an instrument with which man might cleave open the universe and peer within, it has also been an iron matrix that bound his brain to ancient modes of thought. And thus it is that though man has poured what he knows into his culture patterns, they have also frozen round him and held him fast. [Henry, 1960, closing passage]

Henry, as always, stresses the negative side of the evolutionary dialectic. However difficult it may be to bring about certain shifts, nevertheless new knowledge *can* be constructed, language *does* gradually change, and cultural patterns *are* transformable. Past achievements are indeed a potential obstacle to future achievements. But that poses the challenge: to *break* the grip of past knowledge, *fight* the hegemony of language, and *evade* the restrictions of one's culture. One can't always win, but one won't always lose. These constraints are all *inside* us, in the mental schemata we have formed out of the experience of living in our world. As Bartlett reminds us, we have the power to "turn round upon our own schemata". [Bartlett, 1932, p. 301] That is what human consciousness is for.

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