Communications

Comments on Journals and Problems

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During the last few days, I have found myself, with no conscious plan, reading earlier issues of this journal, especially volume 4. Like Dick Tahta (4, 2) I have decided to take up the editor's invitations to comment.

My reading has caused me to reflect on a number of issues which I would not normally write about, such as the "house style" of this journal. I have always seen this journal to be very much the child of its editor, so this style, presumably, tells me something of his cast of mind. The main flavour that lingers in my mind is that of the social, cultural, historical and philosophical contexts of mathematics in educaton. Looking at an early editorial (2.3) I found this explicitly stated. David Wheeler also proposes a biology of mathematics and here I want to argue I do not believe that babies are born with the ability to think mathematically, but with the ability to learn how to learn. Starting with this virtual nothing ("I am therefore I think") the child performs a miracle of bootstrapping: learning how to perceive and more, learning language, and above all learning how to learn more efficiently. Only the roots of mathematics can be seen in this.

The contextual flavour of the journal lives in some reflective pieces, with what Dick Tahta calls the "authentic I" or voice (4, 2. I chuckled at the insertion of his closing plea under the longest bibliography in the issue). Thus, what holds Alan Bishop's piece (5, 1) together is a thread of intellectual autobiography which adds the flesh of his motives to the bones of his work. I am reminded (in miniature) of the past excitement I experienced in reading the intellectual autobiographies of Carnap, Popper and Russell, in the Library of Living Philosophers series.

When I first read Stephen Brown's reflections on a letter of acceptance (2, 1), I almost wrote a comment on a letter of rejection I had received. To respond to David Wheeler's (4, 2) discussion of the journal submission process, I can say that the rejection experience is an interesting one. I learn more from it than from acceptances, for I have to take stock, re-evaluate a creation, and come to terms with a personal rejection. However, this learning process does not usually inhibit me from resubmitting elsewhere. After all, one man's commonplace is another man's insight! I usually find revision suggestions result in an improved paper, but if too global I may begin again.

Ubiritan D'Ambrosio's paper (5, 1) contains some irritating assertions, for example that "the number one" is a different concept in oriental logic. But this irritant, like the oyster's grain of sand, started me on a chain of thought which connected in my mind Brouwer's "Two-one-ness," Parmenides' "Unity" and "the number one" for the first time. Once again, I find conflict more memorable than easy compliance or acceptability

Lately, I have found some very reflective and thought-provoking papers in three mathematics education journals: this one, *Mathematics Teaching,* and *Educational Studies in Mathematics. Mathematics Teaching* has a flavour of classroom honesty and enthusiasm, *Educational Studies* has a scholarly feel in its reflections (as well as a narrower, more technical tradition), and this journal has its own flavour and expertise. I am not putting the other journals in our field down (or am I?). They may inform me (when I read them), but lately they have not given me (to use a cliché sincerely) pause for thought.

Past and present editors of the three journals mentioned have figured in recent issues of this journal; either as contributors, or by invocation, in the case of Freudenthal. So has Jeremy Kilpatrick, editor of another prestigious journal in our field. (Perhaps just as you can have a painter's painter, you can have an editor's journal.) These editors have been contributing to David Wheeler's attempt to collect the central research problems of mathematics education, and thus to give the field definition and direction.

I have found the sixteen contributions to this joint endeavour fascinating and I am glad that David Wheeler persisted with it beyond his first attempt in (2, 1).

Reading the pieces, I am struck by how much they differ, both in focus and in content. Some authors seize the ball and insist on playing the game "their way" (or should I say that they reflect on the enterprise before churning out lists of research problems?) W. Brookes (4, 3) irritated me with his opening "it was an invitation to be artificial." But his sincerity and insights shine through. He seems to typify a peculiarly English stance, which combines personal charisma, erudition and heartfelt views with iconoclasm!

Jeremy Kilpatrick lists three surprisingly specific problems. Skills, learning hierarchies and transfer *are* important, and probably represent a very realistic choice of areas in which increased knowledge can be attained. Nevertheless, I found the choice disappointing, given (what I took to

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be) an opportunity to brainstorm. Doubtless I am being unfair in singling out Jeremy Kilpatrick, who reflected penetratingly on research in mathematics education in (2, 2). It is all a question of how the "Hilbert game" is conceived Are the experts choosing:

1) current research problems of interest,
2) research problems likely to be significant, or
3) research problems as growth points which will define the future of the field?

If the last of these alternatives is the self-imposed criterion, we are expecting success in excess of Hilbert's own (Howson: 4, 1)

I would like to go the other way, from the heights of portent to the pits of gut feeling. What problems in mathematics education cry out for research attention and clarification, irrespective of their tractability?

As a tyro, I can afford to be provocative by proposing problem areas such as the following:

1. The problems of dissemination
Effective instructional strategies for mathematics abound in theory (since Brownell, or earlier), but not in practice. How can progressive methods of teaching mathematics be disseminated?

What forms of pre-service training produce "good" mathematics teachers?

What forms of in-service training produce "good" mathematics teachers?

How can ordinary teachers be taught to overcome their prejudices against investigations, practical work, group work, pupil discussion, calculators, etc?

How wide is the gulf between aims and realisation?

2. The problems of the general aims of education
How can mathematics teaching make a real contribution to the education of an independent and critically thinking citizen?

Can any mathematics learning experiences transfer into logical thinking abilities (whatever they might be)?

How can ordinary students be taught to check their hasty and instinctive solution attempts (or keep them in check), and reflect on more problems?

How can we teach students to be confident, cooperative and sharing through mathematics lessons?

How can students be given real choices concerning the mathematics they learn (in a way which satisfies cognitive as well as affective aims)?

3 The problem of mathematical knowledge (as opposed to ritual)
Can we teach students to present plausible arguments and justifications in mathematics (instead of mere assertions or formal proofs, should we be so lucky)?

Can we separate the validity of mathematics from the authority of the teacher?

Can we teach students to *write* mathematics (and what would be the gain)?

4. The problems of social context
How can child centred methods be reconciled with the vocational preparation aspect of mathematics, and with the social control functions of the school?

How can progressive methods prevail in the face of urban decline and unemployment, the growth of the right wing in politics, and the erosion of civil liberties?

If the days of equal educational opportunities for all are passing, in developed countries, what should be the response of mathematics education(ists)?

5 Problems of criticising the sacred cows of mathematics education
Are the skills and processes of mathematical problem solving and investigations really useful? What palpable benefits arise from learning strategies such as "guess and check," "tabulate results," "conjecture relationships," "find related questions"?

What impact will the calculator have on mathematics teaching? Is it already just another fading novelty (used *ad hoc* like any other tool)?

Will the microcomputer benefit the mathematics curriculum, or will it die out, except for a few addicts? Will it cause conflicts (e.g. BASIC $X = X + 1$ for $f(x) = x + 1$) as Vergnaud (4, 2) hints?

What would be lost if we banished mathematics from the secondary school curriculum? If we have problem solving across in the curriculum, graphicacy in General Studies, spatial and pattern work in Art and Design, modelling in Physics, algorithms in Computer Studies, numeracy in Vocational and Business Studies, what more is needed? (Proof, for those that want it, can be taught later with Logic and Philosophy!)

6. Problems of the mathematics curriculum
How much of the mathematics curriculum can be justified (as Robitaille and Dirks ask: 2, 3)?

What do students of geometry *really* gain? (Whether neo-euclidean or transformational)

Is any real purpose served by the majority studying fractions, negative numbers, recurring decimals, vectors, matrices, topology, equations, or algebraic structure? Would it not be more useful and relevant to build a curriculum around current advertising claims, financial graphs, numerical business relationships, official statistical data, etc?

How many students meet and get excited by the big ideas of mathematics (infinity, paradox, fourth dimension, etc)?

How can the mathematics curriculum be unified, and what additional measures are required so that learners build an integrated body of mathematical knowledge?

7. Problems of mathematics education
Is an overall theoretical framework or theory for mathe-

matics learning possible? Would it be so detailed as to be incomprehensible? How should we set about trying to build such a theory? (I reject the naive baconian view attributed to Begle that all we need to do is to assemble sufficient observations and then scan them for regularities)

What defines a problem in mathematics education? Do these questions (including this one!) define problems in mathematics education?

What constitutes the answer to a problem in mathematics education?

What parameters delimit research in mathematics education?

What are the values characterising mathematics education and subscribed to by mathematics educationists What motives (pure and impure!) drive us? (Parallel questions concerning mathematics have been studied by C.S. Fisher in "Some Social Characteristics of Mathematicians and Their Work," *American Journal of Sociology* 78(5) 1094-1118).

It has been projected that if science publications are piled up as they are published, the top of the pile will be rising faster than the speed of light by 2000 A.D To what extent are we adding to this information pollution? Am I?

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A Teacher's Dilemma

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I remember my grandfather saying, "The man who never makes a mistake never makes anything." (Appropriately, this is a slight misquotation. The original, in a speech by Edward Phelps in 1899, was, "The man who makes no mistakes does not usually make anything.")

The first spring after I came to Lesotho I planted some peas. They failed; the harvest was just about enough for one helping I did not learn from my mistake but tried again the second spring, with an equally abysmal result. After that I learned to plant peas in the autumn: they do not grow in the winter but they stay alive and have time to produce a crop in spring, before the heat of summer

Last year the president of the Mathematical Association in the U.K. took up this theme and chose the title "The importance of mistakes" for his presidential address

Mistakes are indeed important and I make no apology for publicising some mistakes made by my pupils, boys and girls in their 8th to 12th years of formal education.

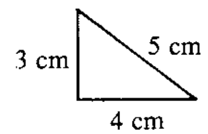
(1) $2\frac{1}{2} \times 4 = 8\frac{1}{2}$

(2) $63 \text{ mm}^2 = 6.3 \text{ cm}^2$

(3) $70 \text{ cm} = 7000 \text{ m}$

(4) $\frac{8435}{7} = 125$

(5)



area = 60 cm²

(6) (on Pythagoras' theorem)
$x = 10^2 = 100 - 8^2 = 64 = 36 = 6$

(7) $3p - (p + 5) = 3p - p + 5$

(8) (on a road going from altitude 1800 m to altitude 2500 m with an average gradient of 1 in 20)
length of road = 35 m

(9) $2^3 = 6$

(10) $\frac{0.204\not{0}}{10\not{0}} = 0.0204$

(11) radius = 2.8
$\Rightarrow$ diameter = 4.16

(12) 3.5 million = 3.5000000

(13) $\cos x = 0.5$
$\Rightarrow x = \frac{0.5}{\cos}$

(14) a wind is blowing at 12 km/h
in two minutes it blows 24 km/h

(15) $\left(\frac{1}{20} + \frac{1}{50}\right) \times \frac{1}{2} = \frac{1}{70} = \frac{1}{35}$

The trouble with most of the above statements is that they are not excusable When you think about what they mean you realise that something is wrong—or if you do not know what they mean, why write them in the first place?

(1) Thabiso knew the meaning of × 4, or he would not have got the 8 in his answer, but he did not bother to think of $2\frac{1}{2} \times 4$ as $2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2}$.

(2) Puleng was one of the top girls in the class; of course she could draw a square millimetre and a square centimetre if asked, and as soon as I queried the statement she realised that 63 mm² is less than 1 cm²—yet she made a similar mistake on several different occasions