teachers in Canada, as we are unaware of any institution offering as part of their secondary school teacher education a course on "teaching analysis". In our informal survey, only 2 out of every 10 teachers explained the situation similarly to Miss Mauve, referring to inverse with respect to an operation. The most frequent response was that the case of exponent (-1) is not the only place where the same symbol is used for different purposes and the interpretation is context-dependent. Perhaps a survey of Spanish teachers would show a different trend and we invite our colleagues to share their results. Further, we do not claim that Miss Mauve was surprised about the mistake; we only reported her skillful care of the situation.

On horizon knowledge again

Foster (2011) writes: "the teacher's horizon knowledge could be regarded as mainstream mathematical knowledge [...] There is no reason to believe that the results referred to would be of more use to a mathematics teacher than to any other user of mathematics" (pp. 25-26). We maintain, based on our teaching and research experience, that the knowledge of mathematical connections and disciplinary structure exemplified in our article is not "mainstream". Furthermore, we see such knowledge as particularly useful for mathematics teachers, while it may also be helpful to other users of mathematics such as statisticians, engineers, carpenters, or bankers, should they decide to engage in instructional activities.

Figueiras *et al.* (2011) write: "they articulate all their reflection around the premise that mathematical teaching problems, and thus theoretical outcomes in the field of mathematics education, should be subordinated to the problem of teachers' learning of advanced mathematics" (p. 28). We were quite surprised to see such a statement. Indeed, we agree that "our professional task of teaching mathematics to primary and secondary students, as well as to future elementary and secondary school teachers, requires a much broader perspective on the nature of knowledge" (Figueiras *et al.,* 2011, p. 28). We reiterate that our article explored one particular aspect of teachers' subject matter knowledge and we made no suggestions regarding "theoretical outcomes in the field of mathematics education." Our article exemplified several teaching scenarios in which KMH is useful, presented a refined perspective on horizon knowledge, and argued that extended experiences of learning mathematics can be functional in teaching.

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Thinking about a mathematics for mathematics teachers

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Two comments from the November 2010 issue of *For the Learning of Mathematics* prompted this communication. One of them was made by Mason, who points out that:

> Many recent papers are so heavily theory-laden in the opening sections that by the time I get to the substance I have forgotten exactly which parts of which theory are actually being employed, and, indeed sometimes it is not even very easy to detect this. It seems to me that often only tiny fragments of theoretical frameworks are called upon. (Mason, 2010, p. 8)

In the other, Kajander and colleagues affirm that:

> Accepting the possibility that math for teaching is a particular mathematics sub-discipline, like math for engineers or carpenters or citizens, requires the dismissal or at least questioning of the premise that the math that teachers need will be a subset of mathematics that mathematicians value. (Kajander *et al.*, 2010, p. 56)

We agree with Mason that sometimes too much theory is included in mathematics education research papers. Nevertheless, we also believe that on many occasions, theory is lacking in the design of mathematics teacher education programmes. It is precisely the adoption of a theoretical framework that allows us to answer the question put forward by Kajander and colleagues:

> Can mathematics for teaching be viewed simply as the overlap of the two realms [the realm of academic mathematics and the realm of mathematics teaching in schools], or is it something that has yet to emerge, something that will encompass the full realities of both of its parent realms? (Kajander *et al.,* 2010, p. 56)

In this communication, we outline a proposal that responds to this question.

Developing a proposal of mathematics for teachers

In the design of our own teacher education programme, we have taken a situated perspective with respect to mathematics. This perspective leads us to think of mathematical knowledge as being what a teacher should know in order to be able to join the mathematics teacher community as a full member (Sánchez & García, 2008, 2009). For us, this implies the identification of intrinsic mathematical knowledge.

We characterize intrinsic mathematical knowledge as knowledge that considers mathematical elements from a dual point of view, integrating "operational" and "structural" aspects (Sfard, 1991), as well as the mathematical competence used in analysing these elements. This competence makes possible the "packing and unpacking" (Ball, Thames & Phelps, 2008) of the mathematical elements. People who understand and use mathematics in other contexts and situations do not necessarily have to share this competence, which adds specificity to mathematics teachers' knowledge and expands the idea of common content knowledge in Ball, Thames and Phelps's (2008) sense.

Now, in our case:

- adopting a theory,
- assuming the existence of intrinsic mathematical knowledge closely related to the development of the specific profession of a mathematics teacher, and
- considering that such knowledge must be transferred to the content of a mathematics teacher education programme

involves:

- translating the teacher's subdomain of knowledge (intrinsic mathematical knowledge) into a component of a teacher education programme, and
- identifying a few dimensions that allow the generation of spaces in which this component can be developed.

As mathematics teacher educators, going from a subdomain of mathematical knowledge to a component of a teacher education programme involves putting theoretical ideas into practice. To this end, we work with two dimensions that we believe allow us to develop such components (see Table 1).

- On the one hand, we consider activities of mathematical practice such as defining, justifying and modelling, among others (Rasmussen, Zandieh, King & Teppo, 2005). These activities underlie any mathematical content and are part of what we consider as "doing mathematics".
- On the other hand, we consider mathematical content to be organized into areas, taking into account those that have traditionally been considered part of mathematics. These areas include, among others, analysis, geometry, algebra, statistics and probability. The manner in which the content of these areas is considered will depend on the specialisation of the teachers (primary or secondary school teachers).

Activities of mathematical practice / Mathematical content areas	Defining	Justifying	Modelling	….
Analysis				
Geometry				
Algebra				
Statistics/ Probability				

Table 1. Dimensions related to what to teach in a course of mathematics for primary school teachers (from Sánchez & García, 2008, p. 286).

As can be seen in Table 1, the aspects that characterise intrinsic mathematical knowledge are situated at the intersections of both dimensions. Each of the different intersections gives rise to a different part of the program. For example, justifying in analysis, justifying in geometry, justifying in algebra and justifying in statistics/probability would be parts of the program. Research in mathematics and mathematics education allows us to identify the specific content to develop in each part. Below, we present one task that we use as a starting point for developing intrinsic mathematical knowledge: it is situated at the intersection of a mathematical content area (algebra) and an activity of mathematical practice (justifying).

Example: justifying in algebra

Suppose we want to consider the part of the programme "justifying in algebra". In relation to justifying we would include: the identification of premise, statement, or proposition, and the different roles they play (Balacheff, 1987; Hanna, 2000); and components such as the set of accepted statements, the modes of argumentation, and the modes of argument representation (Stylianides & Ball, 2008). With respect to algebraic elements, we would include polynomials, parts of polynomials (degree, coefficient …), operations with polynomials, polynomial long division, quotient of division, remainder of division, divisor of division, "evaluating" a polynomial, equation, factorization of polynomials, function, *etc*. Hence, we incorporate the competence of analysing the mathematical aspects included both in the activities of mathematical practice of justifying and in the mathematical content area of algebra.

One of the tasks that we use as a starting point is based on the Remainder Theorem (see Figure 1). It is designed so that students can build their knowledge through authentic activity.

From our point of view, this task allows students to analyse the different elements of a theorem carefully and in detail, going deep into its algebraic elements. In addition, it allows them to identify axioms, definitions and theorems, and distinguish between them. This leads them to notice that justifying is a characteristic of theorems, and not a characteristic of axioms or definitions. In the process of solving the task, students can: verbalize their ideas, recognize explicitly what they "see" and give reasons to support their comments; "unpack" characteristics and particularities that give form to a specific theorem and "pack" them only in a property that has been justified. This approach favours the clear characterization of the different parts of theorems and the role of premises, statements and propositions. We think that this type of task can contribute to the generation of specific mathematical knowlege that allows student teachers to put together different parts to form something whole, as well as to dismantle existing structures in order to identify their parts. To sum up, students are constructing and deconstructing mathematical elements. Of course, sometimes students "see" other things or provide other ideas that we had not taken into account. For us, the consideration of different points of view, which encourages student teachers to think flexibly, is a valuable skill that we try to foster.

Using the above ideas, and through several tasks, we are

TASK

a. Give the axiom, theorem, or definition justifying each step in the following proof, indicating the mathematical elements that intervened.

Proof	What you say
a. $p(x) = g(x)q(x) + r(x)$	
b. the degree of $r(x)$ is less than that of $g(x)$	
c. if $g(x)= (x\text{-}a)$ as the divisor	
d. degree of $r(x)$ as 0	
e. $r(x) = r$	
f. $p(x) = (x\text{-}a)q(x) + r$	
g. setting $x = a$ in the above equation	
h. $p(a) = (a\text{-}a)q(a) + r$	
i. $p(a) = r$	

b. Identify of premise/statement/proposition, and the different roles they play; state the property that is proved with that proof.

Figure 1. Task corresponding to justifying in algebra.

trying to develop the different parts of our course on specific mathematics for future mathematics teachers. Of course, this is only a first try. A better characterization of intrinsic mathematical knowledge and its transformation into the content of a teacher education programme is an important research agenda, which we believe must be addressed.

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Teacher discourse and the construction of school mathematics

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Mathematics educators and teachers often talk past each other. To outsiders, this might seem strange: people in ostensibly similar professions should see the world more or less the same way. Nonetheless, I have yet to work on a school-based research project where discordant moments do not arise. Having worked as both a teacher and a researcher, I find these moments curious and productive. To me, they do not reveal a fundamental irrationality of either teachers or researchers. Instead, they uncover underlying assumptions about the nature of mathematics and schooling that implicitly frame our discussions.

In this communication, I draw on my work studying secondary school mathematics teachers' conversations. These conversations are useful for understanding teachers' everyday thinking about their work. Specifically, in my work, I examine teachers' collegial dialogue to understand the locally constructed conceptions of teaching, mathematics, and student learning.

Language is a part of the larger meaning systems people use to make sense of the world. With this in mind, I share some teacher perspectives that are seen by researchers as problematic and explain why they make sense when viewed in the context of teachers' work. I selected these excerpts because they bothered people on my research team (myself included). I do not abdicate my judgment that the conceptions are ultimately problematic; I simply seek to understand them better so as to address them more effectively as a researcher, teacher educator and teacher advocate. Taking the anthropological imperative that people need to be understood in their own contexts, I arrive at an understanding of these apparently "irrational" perspectives that sees them as sensible in a certain light, namely, within a stance that equates schooling and learning

Paradoxes of schooling

The problematic teacher discourse that I focus on in this communication makes much more sense within a broader analysis of schooling. Schools are contradictory institutions, riddled with paradoxes. Teachers must manage the contradictions inherent in schooling and the bureaucratization of