Mathematics Learning in Narrative Classroom Cultures: Studies of Argumentation in Primary Mathematics Education [1]

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> Cognition, like any other object of academic study, is brought into being by us, as soon as we start to define its nature and scope [. .] cognition is already, before we have got very far, clearly a cultural and discursive matter (Edwards, 1997, p 28)

The central research purpose of this article is to examine the relationship between the participation of students in classroom interaction and individual content-related learning. The empirical focus is on mathematics teaching and learning situations in typical elementary classes, while the theoretical orientation is drawn from ethnomethodology (Garfinkel, 1967), symbolic interactionism (Blumer, 1969) and cultural psychology (Bruner, 1996)

Within all of these perspectives, classroom situations are defined as *processes of interaction*: students and teachers contribute to according to their sense and purpose of these events Students act in ways that seem *sensible* and *tenable* to them. In order to do this, they *interpret* their classroom situations: they reflect, set up and review hypotheses, and make rational decisions - common features are accomplished that temporarily enable them to participate In such a classroom situation, students develop their content-related understanding not only to be mutually regarded as responsible and capable, but also in order to participate in the joint creation of the interactions Thus, in such a situation, their minds are challenged, which they employ and develop simultaneously.

This approach emphasizes the cognitive aspect of content-related learning processes from a sociological perspective. It is therefore related to the typical interests of psychology For this reason, it seems necessary to point out the principal difference between the theoretical approaches of cognitive psychology and an interactional theory of learning.

Cognitive psychology deals with the way of thinking of individuals that enables them to handle *their own lives* According to this, the unit of analysis is the *individual.* An interactional approach deals with the rational contributions of individuals which enable them to accomplish a *commonly shared life*: accordingly, the system is the *situation of interaction* (Cobb and Bauersfeld, 1995; Edwards, 1997; Howard, 1994; Krummheuer, 1992; Laucken, 1995)

In the following section, the elements of an interactional theory concerning content-related learning in classroom situations are discussed in greater detail First, I clarify the concept of the 'social constitution of learning' and the role played by the concept of 'culture' Second, with regard to the empirical material of interaction processes in primary school education, I outline the narrative character of the reconstructed classroom culture. Here, I contend that the classroom culture is characterized by means of processes of common storytelling, seen as a feature of rational acting in primary classroom interaction, and is based on reflexive argumentation. Finally, the empirical results presented in these sections will be brought together to create elements of a theory of interaction of content-related learning.

The social constitution of learning

The entire phenomenon of human learning is not encompassed if one confines one's studies to the interior, mental processes of the learner Instead, learning is a social process that takes place in the interaction between human beings:

> where human beings are concerned, learning [ ] is an interactive process in which people learn from each other (Bruner, 1996, p 22)

Because learning is socially constituted, social interaction is a necessary component for learning to take place This insight led Bruner to formulate a form of a psychology which constitutionally considers social elements for cognitive processes, a *cultural* psychology (1990, p 35) He also explained that each individual's development must be expressible in the particular symbolic system of a given culture: members of a culture not only have access to the general resources of their language but also employ specific culturally-accomplished ways to interpret the psychological characteristics of individuals - which he terms "folk psychology" (p 35). Regarding the teaching and learning process in a classroom situation, Bruner speaks of "folk pedagogy" (1996, p. 46)

These concepts include the implicit, basic assumptions of a culture about members' psychological functioning

> All cultures have as one of their powerful constitutive instruments a folk psychology, a set of more or less connected, more or less normative descriptions about how human beings 'tick,' what our own and other minds are like, what one can expect situated action to be like, what are possible modes of life, how one commits oneself to them, and so on We learn our culture's folk psychology early, learn it as we learn to use the very language we acquire and to conduct the interpersonal transactions required in communal life (1990, p. 35)

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Here, Bruner suggests that folk psychological insights are acquired at an early age and that they are already linked to the learning of one's own native language. Folk psychology is thought to be acquired through narrative interaction: in other words, students' insights are learned from stories. He designates this way of "learning from stories" as an independent mode of thinking, and contrasts it with a logical scientific way of thinking, regarding the former as a learning mechanism through which a child develops fundamental views and perspectives of the inhabited world (1986, p. 11; 1996, p. 39).

Learning one's native language has a paradigmatic status for the learning process of human beings in general. In one of his earlier studies, Bruner (1983) examined the initial processes of learning one's native language, and introduced the term 'format', defined as:

> [a] standardized, initially microcosmic interaction pattern between an adult and an infant that contains demarcated roles that eventually become reversible (p. 120)

In such structures of interaction, ones that are characterized by an increasing autonomy on the part of the learner, the learning process takes place in an indirect way (Bauersfeld, 1995; Krummheuer, 1995). The main focus of the learner and all other participants is to maintain themselves and the interaction through *participation*, which can be described by means of processes of sharing and taking part in the interaction.

In a learning situation, students are sharing partners and are co-creators of learning. At the same time, as those taking part, they try to orient their way of thinking toward the stated ideas of their partners (teacher and peers) in the interaction. In any social learning situation, both aspects are in effect at the same time. At the beginning of a learning process, the predominance of one of these two aspects of participation is not clearly evident (see Markowitz, 1986, p. 9).

Successful development of a learner in the sense of increasing autonomy is noted in terms of a gradual shift that takes place from the learner as an observer, seeking a powerful and sensible interpretation, to a more and more influential way of participation. This final state is also understood as the sharing of meanings and values of a culture, which are effective during the entire learning process through the actions and remarks of all members (cf. Bauersfeld, 1995, p. 179).

Bruner's work on the concept of 'format' within the frame of early child language acquisition merges into his later works on cultural psychology. The term *format*, however, hardly appears again.

At some point, it seems as if he described a similar phenomenon with the term 'scaffolding' (e.g. Bruner and Haste, 1987 and Bruner, 1996, p. 21). Additionally, in his later works, he often employs the concept of 'plot', by means of which he articulates learning through participation in formats in a more general and more advanced way as learning through participation in processes of story-telling, stating:

> When we enter human life, it is as if we walk on stage into a play whose enactment is already in progress - a play whose somewhat open plot determines what parts we may play and toward what denouements we may be heading. Others on stage already have a sense of what the play is about, enough of a sense to make negotiation with a newcomer possible (1990, p. 34)

In summary, Bruner configures a theoretical frame of learning and teaching in which the participation of the learner in the process of interaction is the constitutive social condition for learning. From this perspective, the *structure* of the interaction is relatively stable, while the process of *role-taking* is continually changing towards increasing autonomy for the learner.

Therefore, learning is not simply the appropriation of culture: rather, it occurs during the co-creation of interaction. It is often stated, especially with regard to primary education, that basic cultural contents such as reading or arithmetic are taught and acquired. From the perspective of cultural psychology, this seems lacking. Children do not only learn the subject matters of culture: instead, through their contributions in reading or calculating, they also create 'a' or 'the' culture. If we integrate these two aspects, we arrive at what could be described as 'classroom culture'. *It is a culture of subject matter and a culture of learning.*

It appears plausible that learning at an early age can be increased through the specific intervention of a more knowledgeable adult (e.g. a teacher): it is possibly even a pre-requisite. Regarding the learning of toddlers or teenagers, it is striking that the interaction among persons of the same age evoke the social processes for learning. In the late state of independent learning, adults are able to reconstruct mentally the corresponding processes of interaction with an "ideal 'alter ego'" (Miller, 1986, p. 339), that is, with a virtual interlocutor. In the first place, the learning conditions are interactive, heteronomously and reciprocally structured. Later on, one can relocate them in the individual in the sense of autonomous learning (p. 338).

This ontogenetic design of learning is based on Piaget's model of individual development (see e.g. Piaget, 1948, 1972, 1974; Youniss, 1982, 1984, 1994; Krummheuer, 1992). The individual's learning trajectory is characterized by an inverse proportional relationship between the developmental tendency from egocentrism to decentrism and the degree of dependence on social participation, which proceeds from subordination via homology to autonomy. Thus, the more the thinking is decentralized (socialized), the more the learning can occur autonomously.

According to this model of development for primary school age, children usually act in learning stages which are interlocked with heteronomous and reciprocal structures of interaction. In the related image of schooling, the social basis of learning is teacher-guided 'whole-class discussion' and 'group work' among peers.

The narrativity of classroom culture

It is well-known that in primary education children like to tell and listen to stories. The argument presented here goes beyond such empirical evidence: I claim that children learn

by participating in the telling of these stories. In addition, they learn the content of different school subjects. When the classroom culture of primary schools is characterized by narrativity, the social constitution of classroom learning can be described by models of participation in story-telling situations. This is also relevant for mathematics classes, and analysis of processes of interaction concerning this subject matter can demonstrate the importance of this thesis in general.

I begin by presenting the characteristics of such narrations in observed mathematics classes, aiming to clarify which aspects of narrativity are most relevant to mathematics classroom interaction. This interaction is of an argumentative nature and argumentation dissolves in a narrative presentation. This means that:

> *the narrative classroom culture of primary education is based on rationality, and the social constitution of classroom learning is participation in the interactional accomplishment of argumentative, narratively-structured sequences of actions*

This thesis does not imply that 'stories' in classroom situations are told endlessly and that beyond educational goals in native language classes, classroom education intends to teach children the telling of stories. Rather, the negotiated theme in a classroom interaction emerges more frequently in such a way that one can reconstruct aspects of a narrative process. Thus, the concept of 'narrative' is used here to describe a specific phenomenon of everyday classroom conversation. It is not meant in the sense of narrative found in literary studies.

Bruner (1990, p. 50) identifies four characteristics of narrative accomplishments:

1. sequentiality;
2. a factual 'indifference' between the real and the imaginary;
3. a unique way of managing departures from the canonical;
4. a dramatic quality

Here, the first and third points are of special interest. The supposed narrativity of classroom culture can be seen in the patterned sequentiality of classroom interaction, and the specificity of an event (such as the elaborated solution process for a new mathematical task) is presented in relation to the canonical management of such events or problems.

Classroom processes display some specificities in which they differ from usual narrations.

- Frequently, students and teacher complement each other in the role of the story-teller. Thus, there are no definite roles of the 'listener' and the 'teller'. Usually, several persons are engaged in generating a story.
- Not only are stories about the past told, but also stories emerge about something new. Usually, one associates with the concept of 'narration' the image that something that is already over comes to the fore. One can describe this as the *presentation of facts* (Kallmeyer and Schütze, 1977, p. 159). In our observed mathematics classrooms, we also see a 'narratively characterized interaction', during which the children accomplish their single steps of calculation and at the same time express what they do. In such cases, they 'tell' or 'narrate' how they came to their solution, or better put how one can come to a solution. This could be called the *constitution of facts*. In this constitution, one can identify typical elements of a narration, which is the ordered presentation of a concrete case, in which a problematical situation is managed (see e.g. Bruner, 1986, p. 16; 1990, p. 47; Krummheuer, 1997).

In the following, evidence is provided for this theoretical approach by reconstructing several mathematical classroom episodes in detail.

Introductory example – the narrative plot

In this section, I illustrate the narrative character of classroom culture with empirical data from a second-grade mathematics class. The students are supposed to present their results to word problems in front of the class after having previously solved them in pairs. In a certain sense, this resembles a typical situation of narrating: one reports what one did and experienced (a presentation of facts). But it is not only the single reporting child who generates a story. Rather, in interaction with the teacher, a new story is accomplished, one which tells how one *should* solve problems of the given kind (a constitution of facts).

One requirement which results in the presentation of the children is to reorganise all that happened in the work of the pair in a comprehensive order according to the solution of the problem. Two second graders, Sibylle and Stefano, for instance, are coping with this narrative problem of 'sequentiality' in the following way. (On the worksheet, a drawing showing five plants in a flower box accompanies the text.)

Sibylle: [*Reads*] The gardener puts the plants into the flower-box for Mrs. Müller. He charges her four marks for one plant.

Stefano: Four times five [.] marks equals twenty marks.

They read aloud the text of the task (Sibylle) and then proceed to the result (Stefano). An additional example including a small supplement by the teacher may point more clearly to a typical characteristic of a narrative presentation.

Tamara: [*Reads*] Twenty-eight students want to go on a boat-trip during their class outing. One boat can carry four students at a time.

Barbara: Twenty-eight divided by four equals seven.

Teacher: What does the seven stand for?

[3 sec]

Teacher: They need [ ]

Barbara: Seven boats

Teacher: Yes [ ] see, very good [ ] [*nodding*] Okay [*Tamara and Barbara walk back to their seats.*]

Here again, one can recognize the sequentiality as described above: read text - calculate - give result. The questioning of the teacher reveals a principal problem of listeners On the one hand, students have to understand the details of the presentation, and, on the other, they have simultaneously to infer the 'sense' of the whole story that is implicit because it is co-delivered

This issue can be differentiated more precisely in an episode from the same lesson in which two children present a wrong answer

Lisa: [*Reads*] Elke has thirty-five marbles. Four friends of hers are playing with her. Every child gets the same amount of marbles.

Marcel: Thirty-five [ ] divided by four equals five.

[2 sec: Marcel and Lisa are on their way back to their seats]

Teacher: So what did you figure out now?

[2 sec]

Marcel: Actually, she figured it out [*pointing at Lisa*]

Lisa: You

Teacher: I don't know, four friends are playing with her Every child gets the same amount of marbles

Marcel: Five marbles.

Teacher: Five [ ]

Marcel: Five divided [ ] from those five [ ] so the four children get five marbles

Again, one can recognize in this episode the sequentiality of a narrative presentation as mentioned above. However, this time the delivered story is told incorrectly: both the approach to the problem and the result of the calculation are wrong. The teacher asks for an interpretation of the result ("So what did you figure out now?"). She does not ask about the process of the children's prior co-operative group work ('How did you solve it?'). This one can understand in a lesson: in front of the class, it is not appropriate to investigate the genesis of the solution of these two children It seems more appropriate to stress the missing credibility or verisimilitude of the whole story.

As Bruner (1990) puts it:

> *The function of the story is to find an intentional state that mitigates or at least makes comprehensible a deviation from a canonical cultural pattern.* It is this achievement that gives a story verisimilitude. (pp. 49-50; *italics in original*)

Together, the participants of such a narratively-structured process of interaction must fulfill the demand and expectation of concluding both its concatenation and the continuity of the whole from the details of a given story. Frequently, from the perspective of a theory of narrativity, one uses the concept of 'plot' for this relationship Bruner (1986) defines it as:

> how and in what order the reader becomes aware of what happened. (p. 19)

He refers to the 'reader', because the context of this quotation refers to written stories One can easily substitute for the reader a listener in relation to a verbally-presented story or a participant with respect to a process of interaction in which a story emerges Important in this quotation is that a plot characterizes the sequence of action in its totality: it describes something that is already fixed (such as that of a movie) But an unfolding plot connotes something fragile, not yet entirely executed, still changeable. Both aspects are essential and the tension between these two dimensions of this concept is crucial for its adaptation for classroom interaction and its function for learning

Elements of such a plot in the previous class discussion become clearer in the following example with Stefanie. (On the problem sheet one can see a container with twelve bottles)

Stefanie: [*Reads*] Susi carries that to the kindergarten. She can bear two bottles at one time Calculation ( ) Two times six equals twelve Answer She must go six times

First, Stefanie reports the text of the task, then she presents her calculation explicitly labelled as such, and finally she formulates a sentence as her answer, entitled as such. This is the rough draft of the plot of the working process on a word problem. With respect to the whole-class report, the plot consists of the following parts:

- reading the text;
- possibly rephrasing and emphasizing the relevant parts of the text;
- developing a solution approach which is restricted to those parts;
- conducting the calculations;
- formulating sentences as an answer which contains the results of the calculation.

In the episodes presented, the plot is only partially realized

The children not only follow the single steps of the presented solution to the mathematics, they also comprehend these steps as parts of a more global scheme or plot for solving word problems. This scheme is 'wrapped' in the story of pair-work reports. In this lesson, it is neither explained nor commented upon as a recommended working scheme – nor is it argumentatively brought into prominence by contrasting it, for example, with different (possibly inappropriate or wrong) solution procedures. Thus, this scheme is not only a plan for the presentation of an already-solved problem, but also the plot for future solution activity for such tasks

The academic task structure (ATS)

The material chosen from this project contains whole-class discussion and group work as well. Here, I will present episodes from the group-work data (generally two or three children working together), in order to reach a deeper clarification of the narrative character of the processes of interactive mathematical problem-solving in elementary classes. Their successful co-operation demands two different achievements. They have to clarify (a) what shall be done at what time and (b) who shall do it at what time. Both points have to do with making decisions about the right moment. However:

- the first deals with the insight within a sequence of solution steps and their chronological order;
- the second involves the sequence of interactional moves and the chronological change of speakers

The structure of actions and interactions with regard to the first issue can be called the 'academic task structure' (Erickson, 1982) and is based on an understanding of the situation of the problem as shared amongst the students. It is not identical with logical considerations about a sequence of solution steps according to the subject matter of mathematics (see Vollmer and Krummheuer, 1997). The second issue can be described analogously as the 'social participation structure'. Both structures are mutually dependent on one another.

> The academic task structure (ATS) can be thought of as a patterned set of constraints provided by the logic of sequencing in the subject-matter content of the lesson. The social participation structure (SPS) can be thought of as a patterned set of constraints on the allocation of interactional rights and obligations of various members of the interacting group (Erickson, 1982, p. 154)

A class episode might help to identify typical features of the ATS.

In the 'shirt task' described below, three third-grade boys Daniel, Slawa and Stanislaw are confronted with the presentation of numbers on the back of T-shirts which represent the first parts of a number sequence. Their task is to determine the fifth element of this sequence.

{3 – 8 – 15 – 24 – ?}

For this problem, Slawa quickly gives a solution.

What number?
Here, members of a sports club sit together. The numbers on their shirts create a sequence of numbers. Which number should the boy to the very right have on the front of his T-shirt?

Once again: What number?
And which number should be on his shirt?

Figure 1

Slawa:	[*Pointing at the picture*] Here's five, here's seven . here's [ ] nine . He gets an eleven –
Daniel:	Why eleven?
Stanislaw:	Why?
Slawa:	Well, eleven. Look [*precariously whispering*] how much plus three, look at this number five –
Daniel:	Well, from three to eight is five
Slawa:	[*Directed to Daniel and still pointing at the picture*] Here's seven already, seven –
Daniel:	Seven –
Slawa:	Nine [ ] eleven
Stanislaw:	[Inaudibly] Well, yeah
Slawa:	Eleven plus twenty-four. Add it here. Then you get [*figures for about 2 sec*] thirty-five

From a mathematical stance, one can see in Slawa's solution the thematization of the general concept of the sequence of differences and the first four numbers of a specific sequence of differences {5 – 7 – 9 – 11}. The boy cannot name them, nor does he define them explicitly. In a sense, he is not talking *about* them, but *through* them. His two classmates cannot follow him. Slawa is obliged to explain. Generally, he reacts in the way just described, by saying the four elements of the sequence of differences

Slawa:	This is five. Here [*points at paper*] then seven, then comes nine
Daniel:	Five [*mumbles inaudibly*] from eight to fifteen is seven.
Slawa:	Always add two to it. . . Thus, here comes eleven, Daniel [*points at the number sequence*] here.

Daniel: Seven yes from, yes.

Slawa: Eleven comes to that number.

Daniel: From fifteen to twenty-four is nine

Slawa: So here you get thirty-five - [*inarticulate*] thirty-five.

Stanislaw: Whoop -

Daniel: Yes, nine.

In three separate turns, Daniel agrees to the numbers five, seven and nine as the differences between the given elements of the initial number sequence. Both he and Stanislaw apparently do not conceptualize these numbers as elements of a different number sequence, one which emerges from finding differences. Even Slawa's meta-comment about the rule for this sequence of differences ("Always add two to it") does not help.

Slawa's finding of the solution, his presentation and his justifications are narratively oriented. In order to understand his solution, one must firstly recognize the phenomenon of a sequence of differences and, secondly, perceive the defining characterization $x_{n+1} = x_n+2$ while repeating the numbers five, seven, nine, eleven aloud. Those who cannot infer this argument from the saying of the numbers alone cannot make sense of the story.

In summary, the four following conclusions can be illustrated by my interpretation of this episode:

1. Mathematical concepts which are necessary for understanding the ATS are not introduced explicitly: rather, in a narrative way, they are pointed at implicitly. Not all students are able to recognize the ATS by this means of presentation. The plausibility of this solution process might be inscrutable to them.

2. To accomplish the different steps of the ATS, the boys need a certain mathematical competence, such as addition and subtraction of whole numbers.

3. Only a few or no meta-comments about the functionality of the ATS or certain steps of the ATS itself are given, which characterizes narratively-organized interaction.

4. The presentation of the solution steps proceeds mainly by verbalization: the boys do not use alternative presentations such as visualization or embodiments. This is an additional characteristic for narratively-organized processes of interaction in mathematical group work.

These four aspects describe features of the ATS and its narrative generation in group work. In general, these are:

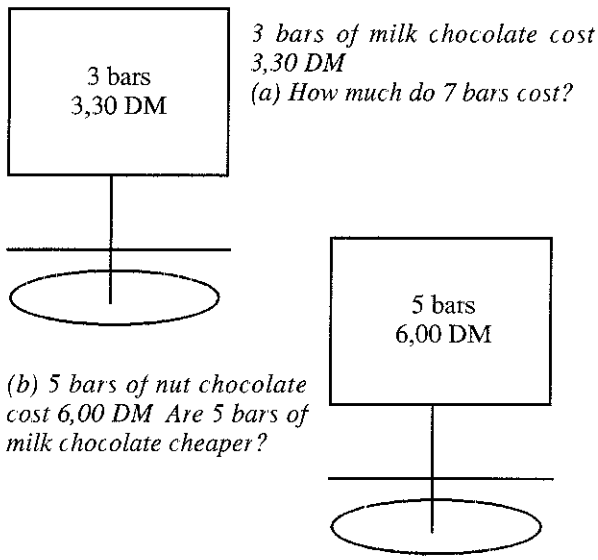
> (a) the logic of subject matter sequencing; (b) the information content of the various sequential steps; (c) the 'meta-content' cues toward steps and strategies for completing the task; and (d) the physical materials through which tasks and task components are manifested and with which tasks are accomplished. (Erickson, 1982, p. 154)

In the following, these four aspects of the academic task structure (ATS) are illustrated by presenting two more examples from our research project. My main emphasis is on the 'logic of subject matter sequencing', which with regard to the current widespread interest in argumentation will be called the 'inner logic' of the ATS. The three additional aspects are discussed subsequently.

The inner logic of the ATS

The chocolate task

Two third-graders, Linda and Esther, worked on the following problem [2]:



3 bars of milk chocolate cost 3,30 DM
(a) How much do 7 bars cost?

(b) 5 bars of nut chocolate cost 6,00 DM. Are 5 bars of milk chocolate cheaper?

Esther: [*Writes their names on working-sheet and reads task*] Three bars of milk chocolate cost three marks thirty. How much do seven bars cost?

Linda: [*Looking at teacher*] Can we write down our calculations right here?

Esther: Hey, look here, five bars cost six marks [*points at the picture at the right margin, which belongs to part (b) of the problem, after this, she points again to part (a)*] nine thirty, are then for si... nine thirty

Linda: Three ... [*not reconstructable, both murmur incomprehensibly*]

Esther: ... So, they are six bars of chocolate.

Linda: Nine ... [*turns away and murmurs something to herself – 4 sec*]

[Both think 8 sec]

Esther: Just three bars cost [*points at left margin of sheet*]

Linda: Twelve sixty.

Esther presents a first approach for a solution: 9,30 DM is supposed to be the price for six bars of chocolate. We can assume that she takes 6 marks as the price for five bars, as it is indicated on the right-hand picture, and she takes 3,30 DM from the left one, apparently ignoring the information that this is the price for 3 bars.

Thereafter, both children think silently for a while. Linda begins to reject Esther's first attempt at a solution, an interpretation that finds further support below. Contrarily, Esther's proposal of 12,60 DM is understood as a somehow-consistent continuation of the former approach: the mentioned amount of 9,30 DM was interpreted here as the sum of 6 DM for five bars and 3 30 DM for one bar. Thus adding another 3,30 DM to 9,30 DM leads to the result of 12,60 DM which stands for the price for seven bars of chocolate.

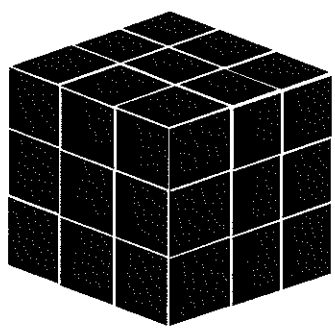
If we take into account the difficulties one has as an observer attempting to understand this part of the transcript, the apparent ease of the exchanges between the two girls is striking. They do not need any further clarification and comment. It seems as if their actions, or more precisely the specificity of the sequence of their actions, speak for themselves. Obviously, for the two girls, the rationality of this accomplished procedure is 'self-explanatory' - any explicit explanation is unnecessary. For the girls, the logic of subject-matter sequencing appears to be simultaneously expressed in the presentation of the sequence.

Shortly after, the girls recognize that they have to separate the two parts of the problem and that you have to calculate solutions separately. They develop an ATS in which they first of all determine the price for one bar of chocolate and then work out the price for seven. In order to avert criticism of the formulation of the problem, we must take into account the fact that the two girls find the right solution relatively quickly. In a certain sense, one assumes that mistakes might very often be produced in these social encounters and that the children learn from them nevertheless.

Again, Linda and Esther work together on the following task.

The cube task

Imagine you have a big cube made of light coloured wood. You could paint it black entirely. Then you would saw it into pieces as shown in the picture. Question: how many small cubes would have three black sides?



Linda: [*Pointing at the picture*] So three pieces, hold on [*counting the cubes of the edges*] one, two, three, four, five, six, seven, eight, nine. [ ] Three times nine.

Esther: Three, six, nine.

Linda: [*After she has been calculating for 3 sec*] Twenty-seven.

Esther: [*whispering her calculation*] Twenty-seven. [*Writes 27 in the upper right corner of the sheet, both of them happily looking at the teacher*] Let's do another one.

Linda: Could we do one more?

Teacher: And what have you done?

Linda: [*Circling over the paper sheet*] Em. We have counted one of them and multiplied it by three. [*Not reconstructable further*]

Esther: And then we have multiplied it by three.

After reading the text of the task, Linda comments "So three pieces, hold on". She seems to count the small cubes of the edges of the two-dimensional picture of the big cube and ends up with the number nine. After this, she says three times nine. Both students get to the result of twenty-seven. They need some time to do this during which they say no word.

Obviously, the two girls develop a consensus about what they are doing and that this is sensible. A detailed exchange seems unnecessary, for, as the common adage suggests, what they are working at 'goes without saying'. Presumably, they produce a kind of argumentation, one which is already reflected by or in the sequentiality of their solution steps (its inner logic). As observer, one is not quite sure what the consensus is about.

At least three interpretations are possible. First, one can assume that they try to count the number of the small cubes generated through the sawing. One can think of three layers of nine small cubes each. The product 3x9 represents the total number of these small cubes. Second, the students identify three sides in the picture of the big cube, each containing nine little areas. The product 3x9 represents the total of these areas shown in the picture. Third, Linda states that each visible edge of the big cube consists of three small 'cubes'. Additionally, she finds out by counting that there are nine of these edges in the picture. Based on that, they figure out the product 3 x 9.

Furthermore, the on-going interaction, which is not presented here, does not help to reduce these options (see Krummheuer, 1997, p. 77). They then signal their belief that they have finished the task to the teacher. The teacher asks for their solution procedure and as answer she gets "Em we have counted one of them and multiplied it by three".

The expression "we have counted one of them" in their answer is highly indexical. It could refer to the areas in the sense of the visible sides of the small cubes. It could also

refer to the edges or the sides of the big cube Nevertheless, the inner logic of their way of proceeding seems to be clear to the two girls in this way of telling - a narrative kind: they tell what they did The logic of their doing remains implicit One can only infer it by analyzing the sequencing of their actions (counting 9, stating the product 3x9, and figuring out the result 27). This is typical for a narratively-emerging ATS The rationality of the whole solution procedure has to be inferred from the plot of the story being created

Additional points concerning the ATS

Of Erickson's four features characterizing the ATS, in my analyses of the three episodes above I have so far stressed (a) the logic of subject matter sequencing, which will be pursued in the following section in more detail Here, though, I want briefly to elaborate on the other three: (b) the information content of the various sequential steps; (c) the 'meta-content' cues toward steps and strategies for completing the task; (d) the physical materials through which tasks and tasks components are manifested and with which tasks are accomplished

(b) The children needed specific mathematical competence in order to conduct the sequential steps in the tasks

- *Word problems*:

  1 basic ideas about the concept of multiplication;

  2 multiplicative number-facts

- *The shirt task*:

  3 competence in calculating differences

- *The chocolate task*:

  4 simple multiplication and division with money

- *The cube task*:

  5 (probably) knowledge about the calculation of volumes;

  7 counting;

  8 spatial imagery

  Except for point 8, the children seemed to have an appropriately-developed competence in order to carry out the different calculations ordered in an interactively-emerging sequence of steps

(c) In these episodes, one can scarcely find any meta-content cues toward steps and strategies for completing the task Presumably, it is difficult for the children to come up with such commentaries while struggling for a solution at the same time

(d) In the same way, it is apparent that the students do not make refererence 'voluntarily' to additional physical materials, such as the usual physical embodiments, drawings or notations of provisional results The interaction during the common solution process is exclusively based on the spoken word

In summary, the interaction in such classroom processes is characterized by the fact that students:

- do not talk explicitly about the rationality of their task-solution process: rather, they and anybody else can only infer the inner logic of the ATS from the specific sequentiality of the accomplished solution-steps;
- do talk explicitly about the accomplishment of these do different solution-steps (thus, their competence is visible/audible in the moment);
- do not provide any meta-cues about the nature, efficiency and/or reasonableness of their ATS as a whole or even parts of it;
- do not produce any written notation for illustrating special solution steps or marking of specific interim results, etc

Reflexive argumentation in narrative classroom interaction

Substantial argumentation

In such solution processes, these four issues demonstrate that on the surface of their communication the participants only talk about calculation steps and not much else This widely-known phenomenon of students' mere verbalization of the steps in calculation is considered within narratively-structured processes of interaction which contain an argumentative aspect under the surface Typically, these processes are characterized by (a) their specific sequentiality of the actions and their dominant verbal presentation, and (b) the missing explication of the plot of the story and the implicit assumption that the participants and other listeners are able to follow this plot by themselves

With regard to the four components of the ATS in a narratively-structured realization of a problem-solving process, components 2 and 4 are explicitly dealt with, whereas components 1 and 3 are implicitly taken as effective Thus, this kind of classroom interaction fosters a rationality of task-oriented acting that reveals argumentation about the 'rightness' of a solution to the students to the extent that they themselves are able to infer the expectation for argumentation from the plot of the evolving story.

If we use the concept of argumentation in the field of mathematics, we might tend to bind it closely to that of proof The analysis of argumentation in a classroom, then, could misleadingly be understood as a treatise on proof We should therefore notice that both the concept of argument and that of argumentation do not need to be connected exclusively with formal logic, either as we know it from mathematical proofs or as the subject matter of logic There are more human activities and efforts which are still argumentative in intent, but not in a strictly logical sense As

Toulmin (1969, p. 40) points out, if these formal logical conclusions were to be the only legitimate form of argumentation, then the domain of rational communication would be considerably restricted and argumentation would be irrelevant as a possible way of undertaking it.

A logically-correct deduction, for example, contains in its conclusion nothing that is not already a potential part of the premises. It explicates aspects of the meaning of the premises by means of deduction. These kinds of arguments are termed *analytic* (Toulmin, 1969, p. 113). In contrast, *substantial* arguments (p. 113) expand the meaning of such propositions in so far as they successfully relate a specific case to them by actualization, modification and/or application. Thus, substantial arguments are informative in the sense that the meaning of the premises increases or changes by the application of a new case to it, whereas analytic ones are tautological, that is a latent aspect of the premises is elaborated visibly (p. 125). Usually, these kinds of substantial arguments do not have the logical stringency of formal deduction, which is not to be taken as a weakness, but rather as a sign that fields of problems exist which are not accessible to formal logic.

Toulmin strongly recommends that substantial arguments should not be subordinated or related to analytic ones in the sense that the latter is the ideal type of arguing and that one can always identify the logical gulf in substantial arguments (cf. p. 234) in comparison with analytic ones. Substantial argumentation has a right to exist in itself. With substantial argumentation, a statement or decision is gradually supported (see also Perelman and Olbrechts-Tyteca, 1969, p. 1 and Krummheuer, 1995, p. 236 and 1997, p. 29).

This distinction helps to clarify the conceptual framework chosen for the analysis of argumentation in mathematics classroom situations in primary education. It is substantial argument that is seen here as more adequate for the reconstruction. With regard to folk psychological assumptions of learning, the impact of argumentation can be reconstructed in a more appropriate manner when the analysis is based on the concept of substantial argument (for further clarification, see Krummheuer, 1995, p. 236).

Ethnomethodology and rationality

The previous discussion suggests that the rationality of everyday activities, such as those in the observed classes, should not be measured by the standards of a scientific model of rationality which in mathematics is oriented toward deductive logic. Specifically, ethnomethodology emphasizes that the participants in the affairs of everyday life constitute their model of rationality interactively, which one might call the 'informal logic' (Klein, 1980) or 'inner logic' (see above) emerging in these situations. One of the aims of ethnomethodology is to explore such interactive constitutions of rational acting (Garfinkel, 1972). Thus, from the stance of ethnomethodology, the rationality of everyday actions is interactively generated while acting in a social setting; it is not an invariant part of such actions.

The participants of everyday encounters are continuously concerned with showing and clarifying the rationality of their actions for themselves and for others as well (see Lehmann, 1988, p. 167). The participants use so-called 'accounting practices', which are techniques and methods that help to demonstrate the rationality of the action while acting. In the process of accomplishing an action, the participant is already trying to make his actions accountable. This concept of 'account' is key in Garfinkel's work (see e.g. Attewell, 1974, p. 183).

The concept has an application that is broader than that of argumentation. Rational processing envelops all types of experience. The unifying character of all accounts is that the denoted action is to be made understandable, and that the proposed claims are intersubjectively acceptable (see also Voigt, 1995). In order to do so, one endows it with the "status of an intersubjective object" (Leiter, 1980, p. 162). With regard to the concept of substantial argument, this transfer changes the collectively accepted basis as well as the collectively shared means for this transfer through the interactive establishment of such a form of argumentation.

The reflexivity of interactional activities

For the ethnomethodological approach, it is an essential awareness that the accomplishment of an action and the demonstration of its accountability are not separate activities:

> the activities whereby members produce and manage settings of organized everyday affairs are identical with members' procedures for making those settings 'account-able'. (Garfinkel, 1967, p. 1)

Thus, in the process of social interaction, participants make their actions understandable and accountable as well, a fact which is usually termed 'reflexivity' (Mehan and Wood, 1975, p. 137; Lehmann, 1988, p. 174; Voigt, 1995 and Yackel, 1995). This notion of reflexivity does not necessarily mean, as 'account-able' suggests, that the rationality of one's actions is explicated entirely or sufficiently while accomplishing these acts. Rather, in the episodes presented here, reflexivity constantly emerges from the specific sequentiality of children's solution steps which reflexively expresses the accountability of both the single calculation steps and their entire solution approach.

Summary: arguing and learning in a narrative classroom culture

Parts of the classroom culture in primary mathematics education can be reconstructed as narratively structured. The following describes how the content-related learning of mathematics is situated in such kinds of interaction.

The basic assumption is that the students do learn mathematics content during these interaction processes. From a cultural psychological perspective, it is essential that the participants try to realize social conditions of (mathematics) learning according to their folk psychology. They create the classroom processes in this way, because they assume that learning is made feasible as a result and that the students carry along the cognitive supply (equipment) so that such narrative processes can work as social constitutents for content-related learning.

In order to reconstruct this folk psychological assumption, I use the perspective of interactionism (cf. Bauersfeld *et al.*, 1985), which contains as a basic assumption that participants

make sense of a situation by means of their individual interpretations. The related inner-psychic process is termed the 'definition of the situation' The learning-theoretical consideration presented in the following applies to this inner-psychic disposition and examines the social conditions of the possibility of affecting (influencing) these processes which comprise the definition of the situation.

A second basic assumption of this theoretical approach is that the definition of the situation evolves from co-operative, interactive processes, for example through the resolution of contradictions. At the level of interaction, the processes of 'negotiation of meaning' about different interpretations of the situation result in ideas which are taken as shared In a successful instance, a 'working consensus' is accomplished.

Principally, it might happen that such working consensus interpretations of the situation are created, which are both taken as shared and go systematically beyond the definition of the situation for at least some of the participants. Frequently, in mutually-complementing interactional moves, a tightly woven sequence of utterances emerges which:

(a) could not be created solely by single participants, because they could not have produced the contributions of the other participants based on their idiosyncratic definitions of the situation;

(b) leads to the negotiation of meaning, a process which has to be ascribed to the 'dynamics' of the interaction process.

Such interactive accomplishments can be transient and the participants might judge them as less important However, when these common constitutions of meaning appear with a certain regularity and credibility in the sense of a format or a plot, one can assume that these social events become more influential for the content-related internal processings of the participants. The moment-to-moment interactional moves, as well as those expected for the next moment, can be anticipated by such individuals, if they are able to comprehend *the rationality of the whole plot*.

The rationality of the whole process expresses itself through the conduct of an ATS which is already known or which is newly created. The ATS relates to a sequence of working steps seen by the proceeding persons as appropriate for leading to a plausible result In the presented episodes, this happens in the form of a narrative. According to the sequence of actions of an ATS, the working steps are told to the extent that the participants are able to proceed with these steps Typically, in a narratively-structured ATS, the inner logic of the whole endeavour is not expressed explicitly. It is left to the participants to infer the inner logic of the plot for themselves.

On this base, I have elaborated a model of participation in narratively-structured classroom processes. Thus, the children are participating in an ATS-governed interaction and they carry out those steps of the interaction that they realize Through several repetitions of such sequences, patterned according to a specific plot, a student might then be able to conduct all of the steps of the ATS by him-/herself. This gain in autonomy might be an indicator of successful learning. But it can also evoke an illusion of learning, if children do not recognize the similarity of the argumentation presented in the plot of each problem-solving situation, and if they do not construct on this basis a convincing argument for the correctness of all solutions. (For more details see e.g. the concept of *topos* in Krummheuer, 1995.)

In such classroom situations, the properties of content-related learning might not occur consistently or in a complete way The solution process was the core event In order to increase the likelihood of successful learning, the teacher could repeatedly initiate stories of a similarly-structured plot But possibly only the solution of a certain class of problem (defined by its ATS) becomes routinized. Practically the same problem with changed numbers could be solved autonomously as long as the student is able to conduct the single solution steps. This is the case of the illusion of an increasing autonomy. The essential learning increment, which plays out at the level of the plot, does not necessarily occur: *learning happens indirectly*.

This kind of narrative classroom culture is characterized by stability in everyday primary-school teaching and learning situations and there are many students who daily successfully proceed in their content-related learning development by participating in this classroom culture.

With regard to those students who do not proceed successfully, one issue needs to be addressed: the fact that writing and application of other illustrative tools are missing. The observed interaction processes are solely based on oral exchange. Bruner (1996) speaks with regard to this point of the necessity of an externalization tenet:

> Externalizing, in a word, rescues cognitive activity from implicitness, making it more public, negotiable, and 'solidary'. At the same time, it makes it more accessible to subsequent reflection and meta-cognition (p. 24)

The starting point of my argument is that in the project episodes the preference for verbal productions can be observed Generally, the quick evaporation and the situational uniqueness of verbal accomplishments impedes reflection on such interactive procedures - at least for some, the so-called 'weak' students. Complementing such reflections with a written presentation of the result (especially of the work process) seems helpful. Bruner (1996, p. 22) refers to French psychologist Ignace Meyerson's concept of *œuvre* It does not mean a somehow standardized scientific presentation: rather, it means that the children find a productive form of written presentation of their thoughts by themselves. *Œuvres*, produced in such a way, facilitate easier listening and if necessary, possible repetition.

'Creative and productive writing', in such a sense, is not only a category appropriate to native language classes, but provides in general a platform for reflection on classroom-related processes of symbolization It is not a question of whether the children should write something down that is correct in the sense of the subject matter, but rather that the children are able to find a means for presenting their thoughts which lasts over a longer span of time

Such classroom cultures provide all participants with well-founded possibilities for negotiating meaning productively and to produce shared meaning. The specific problem

might be to identify forms of externalization which enable all students (not only the teacher or researcher) to pursue a specific solution process. Regarding arithmetic, we can refer here to standardized iconic ways of presentation. However, they need to be assessed and enhanced for this special use of providing reflection for the students

Notes

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[2] The decimal place separator employed in Germany is the comma rather than the dot. "DM" is the abbreviation for "Deutsche Mark" (German Mark).

References

Attewell, P. (1974) 'Ethnomethodology since Garfinkel', *Theory and Society* **1**(2), 179-210.

Bauersfeld, H. (1995) '"Language games" in the mathematics classroom: their function and their effects', in Cobb, P. and Bauersfeld, H. (eds), *The Emergence of Mathematical Meaning. Interaction in Classroom Cultures*. Hillsdale, NJ, Lawrence Erlbaum, pp. 271-291.

Bauersfeld, H. *et al* (1985) 'Interactional theory of learning and teaching mathematics and related microethnographical studies', in Steiner, H. G. and Vermandel, A. (eds), *Foundations and Methodology of the Discipline of Mathematics Education (Didactics of Mathematics)*, Antwerp, University of Antwerp, pp. 174-188.

Blumer, H. (1969) *Symbolic Interactionism*, Englewood Cliffs, NJ, Prentice-Hall.

Bruner, J. (1983) *Child's Talk: Learning to Use Language*, Oxford, Oxford University Press.

Bruner, J. (1986) *Actual Minds, Possible Worlds*. Cambridge, MA. Harvard University Press.

Bruner, J. (1990) *Acts of Meaning*, Cambridge, MA, Harvard University Press.

Bruner, J. (1996) *The Culture of Education*, Cambridge, MA, Harvard University Press.

Bruner, J. and Haste, H. (1987) *Making Sense: the Child's Construction of the World*, London, Methuen.

Cobb, P. and Bauersfeld, H. (1995) 'Introduction: the coordination of psychological and sociological perspectives in mathematics education', in Cobb, P. and Bauersfeld, H. (eds), *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures*, Hillsdale, NJ, Lawrence Erlbaum, pp. 1-16.

Edwards, D. (1997) *Discourse and Cognition*, London, Sage.

Erickson, F. (1982) 'Classroom discourse as improvisation: relationships between academic task structure and social participation structure in lessons', in Wilkinson, L. C. (ed.), *Communicating in the Classroom*. New York, NY, Academic Press, pp. 153-181.

Garfinkel, H. (1967) *Studies in Ethnomethodology*, Englewood Cliffs, NJ, Prentice-Hall.

Garfinkel, H. (1972) 'Remarks on ethnomethodology', in Gumperz, J. J. and Hymes, D. (eds), *Directions in Sociolinguistics*, New York, NY, Holt, Rinehart and Winston, pp. 301-324.

Howard, J. A. (1994) 'A social cognition conception of social structure', *Social Psychology Quarterly* **57**(3), 210-227.

Kallmeyer, W. and Schütze, F. (1977) 'Zur Konstitution von Kommunikationsschemata der Sachverhaltsdarstellung', in Wegner, D. (ed.), *Gesprächsanalysen*, Hamburg, Helmut Buske, pp. 159-274.

Klein, W. (1980) 'Argumentation und Argument', *Zeitschrift für Literaturwissenschaft und Linguistik* **38/39**, 9-57.

Krummheuer, G. (1992) *Lernen mit "Format": Elemente einer interaktionistischen Lerntheorie. Diskutiert an Beispielen mathematischen Unterrichts*, Weinheim, Deutscher Studien Verlag.

Krummheuer, G. (1995) 'The ethnography of argumentation', in Cobb, P. and Bauersfeld, H. (eds), *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures*, Hillsdale, NJ, Lawrence Erlbaum, pp. 229-269.

Krummheuer, G. (1997) *Narrativität und Lernen: Mikrosoziologische Studien zur Sozialen Konstitution Schulischen Lernens*, Weinheim, Deutscher Studien Verlag.

Laucken, U. (1995) 'Sozialkonstruktivistische Sozialpsychologie: Grundlagen, Einordnung und Abgrenzung, offene Fragen', *Handlung Kultur Interpretation* **7**, 173-206.

Lehmann, B. E. (1988) *Rationalität im Alltag? Zur Konstitution Sinnhaften Handelns in der Perspektive Interpretativer Soziologie*, Münster, Waxmann.

Leiter, K. (1980) *A Primer on Ethnomethodology*, Oxford, Oxford University Press.

Markowitz, J. (1986) *Verhalten im Systemkontext: zum Begriff des Sozialen Epigramms. Diskutiert am Beispiel des Schulunterrichts*, Frankfurt am Main, Suhrkamp.

Mehan, H. and Wood, H. (1975) *The Reality of Ethnomethodology*, New York, NY, Wiley.

Miller, M. (1986) *Kollektive Lernprozesse*, Frankfurt am Main. Suhrkamp.

Perelman, C. and Olbrechts-Tyteca, L. (1969) *The New Rhetoric: a Treatise on Argumentation*, Notre Dame, IN, University of Notre Dame Press.

Piaget, J. (1948) *Psychologie der Intelligenz*, Zürich, Rascher.

Piaget, J. (1972) *Denken und Sprechen des Kindes*, Düsseldorf, Schwann.

Piaget, J. (1974) *Der Aufbau der Wirklichkeit beim Kinde*, Stuttgart, Klett.

Toulmin, S. E. (1969) *The Uses of Argument*, Cambridge, Cambridge University Press.

Voigt, J. (1995) 'Thematic patterns of interaction and sociomathematical norms', in Cobb, P. and Bauersfeld, H. (eds), *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures*, Hillsdale, NJ, Lawrence Erlbaum, pp. 163-201.

Vollmer, N. and Krummheuer, G. (1997) 'Anfangen - Machen - Helfen. Zur Beziehung zwischen Arbeitsteilung und Aufgabenverständnis während einer Partnerarbeit im Mathematikunterricht', *Journal für Mathematikdidaktik* **18** (2/3), 217-244.

Yackel, E. (1995) 'Children's talk in inquiry mathematics classrooms', in Cobb, P. and Bauersfeld, H. (eds), *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures*, Hillsdale, NJ, Lawrence Erlbaum, pp. 131-162.

Youniss, J. (1982) 'Die Entwicklung und Funktion von Freunschaftsbeziehungen', in Edelstein, W. and Keller, M. (eds), *Perspektivität und Interpretation*, Frankfurt am Main, Suhrkamp, pp. 78-109.

Youniss, J. (1984) 'Moral, kommunikative Beziehung und die Entwicklung von Reziprozität', in Edelstein, W. and Habermas, J. (eds), *Soziale Interaktion und Soziales Verstehen*, Frankfurt am Main, Suhrkamp, pp. 34-60.

Youniss, J. (1994) *Soziale Konstruktion und Psychische Entwicklung*, Frankfurt am Main, Suhrkamp.