The Didactical De Morgan: a Selection of Augustus De Morgan's Thoughts on Teaching and Learning Mathematics

ABRAHAM ARCAVI, MAXIM BRUCKHEIMER

> Less well known than Hamilton for any single achievement, less well known than Salmon for his thoroughness in science, familiar to a much larger circle of readers because of his wide range of interest and his skill in popularizing the science, was that eccentric but brilliant teacher, Augustus De Morgan. [1]

> In spite of the excellence and extent of his mathematical writings, it is probably as a logical reformer that De Morgan will be best known to future times. In this respect he stands alongside of his great contemporaries Hamilton and Boole, . .
>
> As a teacher of mathematics De Morgan was unrivalled. . . De Morgan's writings, however excellent, give little idea of the perspicuity and elegance of his *viva voce* expositions, which never failed to fix the attention of all who were worthy of hearing him. [2]

As a popularizer of science, as a logician, or as a mathematician, De Morgan has his place in the history of the 19th century mathematics. In this article, we would like to glance at a less well known aspect of his extraordinary creativity: his thoughts and insights on the teaching and learning of mathematics. It would take more serious historians than we pretend to be to present a considered view of his writings in this field, including any influence he may have had on his contemporaries and the following generations. (It would also require access to more extensive source materials than we have available.) The purpose of the present paper is to present a selection illustrating the sharpness of his ideas, his creative insights and his wit, for the enjoyment of the reader. Thus we present a small collection, which we believe still has meaning for anyone engaged in understanding the teaching and learning of mathematics, even though the extracts were written 150 years ago. The reader whose appetite has been whetted can find much more in the same vein in the references cited. We precede the selection by a short description of his life and work.

Life and work

Augustus De Morgan was born in Madras, India in 1806, where his father held a position in the army. When he was seven months old his family moved back to England. He attended private schools at which he acquired a mastery of Latin, Greek and Hebrew. Very early in his life he developed a strong interest in mathematics, although his talents were unnoticed.

At the age of 16 he entered Trinity College, Cambridge, where he came under the influence of such mathematicians as George Peacock. He graduated in 1827 as fourth wrangler.* He came a "mere" fourth apparently, because already then his interests interfered with the "drilling" necessary to "succeed" in the tripos. (De Morgan has left some severe criticism of the Cambridge system.**) Within a year, the University of London (what is now University College) was established, and De Morgan became the first professor of mathematics—at the age of 22—a post he held with a break of five years, for over 30 years.

His wife, Sophia Elizabeth De Morgan [3, pp. 98-99, 100-101] quotes the following description of his teaching at the University as given by one of his former students:

> As Professor of Pure Mathematics at University College, London, De Morgan regularly delivered four courses of lectures, each of three hours a week, and lasting throughout the academical year. He thus lectured two hours every day to his College classes, besides giving a course addressed to schoolmasters in the evening during a portion of the year. His courses embraced a systematic view of the whole field of Pure Mathematics, from the first book of Euclid and Elementary Arithmetic up to the Calculus of Variations. From two to three years were ordinarily spent by Mathematical students in attendance on his lectures. De Morgan was far from thinking the duties of his chair adequately performed by lecturing only. At the close of every lecture in each course he gave out a number of problems and examples illustrative of the subject which was then engaging the attention of the class. His students were expected to bring these to him worked out. He then looked them over, and returned them revised before the next lecture. Each example, if rightly done, was carefully marked with a tick, or if a mere inaccuracy occurred in the working it was crossed out, and the proper correction inserted. If, however, a mistake of *principle* was committed, the words 'show me' appeared on the exercise. The student so summoned was expected to present himself on the platform at the close of the lecture, when De Morgan would carefully go over the point with him privately, and endeavour to clear up whatever difficulty he experienced. The amount of labour thus involved

*A person placed in the first class of the mathematical tripos, which are any of the examinations for the B.A. degree with honors at Cambridge University

**See, for example, in the description of his teaching in the following and [3, p. 283]

For the Learning of Mathematics 9, 1 (February, 1989)
FLM Publishing Association, Montreal, Quebec, Canada

> was very considerable, as the number of students in attendance frequently exceeded one hundred The fundamental conceptions of each main department of Mathematics were dwelt upon and illustrated in such detail as to show that, in the judgement of the lecturer, a thorough comprehension and mental assimilation of great principles far outweighed in importance any mere analytical dexterity in the application of half-understood principles to particular cases. Thus, for instance, in Trigonometry, the wide generality of that subject, as the science of undulating or periodic magnitude, was brought out and insisted on from the very first. In like manner the Differential Calculus was approached through a rich conglomerate of elementary illustration, by which the notion of a differential coefficient was made thoroughly intelligible before any formal definition of its meaning had been given The amount of time spent on any one subject was regulated exclusively by the importance which De Morgan held it to possess in a systematic view of Mathematical science. The claims which University or College examinations might be supposed to have on the studies of his pupils were never allowed to influence his programme in the slightest degree He laboured to form sound scientific Mathematicians, and, if he succeeded in this, cared little whether his pupils could reproduce more or less of their knowledge on paper in a given time On one occasion, when I had expressed regret that a most distinguished student of his had been beaten, in the Cambridge Mathematical Tripos, by several men believed to be his inferiors. De Morgan quietly remarked that he "never thought—likely to do himself justice in THE GREAT WRITING RACE." All *cram* he held in the most sovereign contempt. I remember, during the last week of his course which preceded an annual College examination, his abruptly addressing his class as follows: "I notice that many of you have left off working my examples this week. I know perfectly well what you are doing; YOU ARE CRAMMING FOR THE EXAMINATION. But I will set you such a paper as shall make ALL YOUR CRAM of no use"

De Morgan was very active on two fronts: the popularization of science on the one hand and the raising of the level of scientific activity in England. He was a prolific writer both of treatises and articles on these issues His scientific endeavours found expression in his indefatigable participation and support of scientific organizations. In 1828 he was elected a Fellow of the Astronomical Society founded in 1820 In 1865, on the initiative of his son George*, he assisted at the birth of the London Mathematical Society, and became its first president. (Some extracts from his inaugural address are brought later.)

De Morgan was a man of integrity. He twice resigned his chair at University College on matters of principle (fully described in his wife's *Memoir*). He also resisted considerable pressure to become President of the Astronomical Society because "The President must be a man of brass—a micrometer-monger, a telescope-twiddler, a star-stringer, a planet-poker, and a nebula-nabber." [3, p. 154] De Morgan was never a practical astronomer, probably because of his deficient eyesight. But, typically, he was willing to work, and work hard, for the Society in a lesser capacity Thus, at the same time as refusing the Presidency, he proposes himself as secretary with the following self-description: "A De Morgan—well enough in his way, but cranky—for all miscellaneous work, from wax candles to Council minutes." [3, p. 154]

It was also a matter of principle with him not to accept "honours". Thus in reply to an offer of the honorary degree of LL.D of the University of Edinburgh, De Morgan writes: "I hope I shall give no offence by very respectfully declining the honour. I mean the diploma. The honour lies in the good opinion of the Senate, and that your communication gives me a right to say I have already earned My reason for declining the degree is my own pecular dislike of conventional titles, which are not what they seem to be" [3, p. 303]

The same principle extended even to the Royal Society Thus in a lengthy and mainly critical discussion of the Royal Society in the Introduction to his Budget of Paradoxes he writes: "Whether I could have been a Fellow, I cannot know; as the gentleman said who was asked if he could play the violin, I never tried Everyone who saw the three letters [F.R.S.] after my name would infer certain things as to my mode of thought which would not be true inference." [4, Vol 1, pp 25, 28]

Similarly De Morgan never "confessed with my lips" his Christian belief, "because in my time such confession has always been the way up in the world." [3, p 368]

A remarkable tribute to De Morgan's character is given by Mary Everest Boole, wife of George Boole: "Mr Boole's work was, as he gratefully acknowledged, made possible by the generous and self-forgetting aid freely given to him by many contemporary mathematicians; and in particular by Professor De Morgan, who seemed to take a pleasure in effacing himself to bring forward the man whom he might have been expected to feel a rival." [5]

Augustus De Morgan died in 1871 aged 65

The following extracts are quoted verbatim—only the headings designed to facilitate the reading, and one or two footnotes are ours

On education

"My view of the advantages of a liberal education is most assuredly not peculiar to myself. Let it be supposed that the former student had forgotten everything, that not a word of Latin is left, and not a proposition of Euclid What remains to him? If little or nothing, then his education has not deserved its name. But if, in spite of the loss of all that acquirement which he has had no daily need to recall, he be a man of trained mind, able to apply vigorously, to think justly, to doubt discreetly, and to decide wisely, he has been well educated, and the loss of the positive knowledge which I suppose him to have lost is comparatively a small matter. I do not underrate knowledge; I would educate for it, even if it gave no powers; but I am sure that if we take care of the habits, the acquirements will take care of themselves." [3, pp. 226-227]

*George, also a promising mathematician, died at the age of 26 in 1867

The reason for learning

"It is admitted by all that a finished or even a competent reasoner is not the work of nature alone; the experience of everyday makes it evident that education develops faculties which would otherwise never have manifested their existence. It is, therefore, as necessary to *learn to reason* before we can expect to be able to reason, as it is to learn to swim or fence, in order to attain either of those arts. Now, something must be reasoned upon, it matters not much what it is, provided that it can be reasoned upon with certainty. The properties of mind or matter, or the study of languages, mathematics, or natural history, may be chosen for this purpose Now, of all these, it is desirable to choose the one which admits of the reasoning being verified, that is, in which we can find out by other means, such as measurement and ocular demonstration of all sorts, whether the results are true or not. When the guiding property of the loadstone was first ascertained, and it was necessary to learn how to use this new discovery, and to find out how far it might be relied on, it would have been thought advisable to make many passages between ports that were well known before attempting a voyage of discovery So it is with our reasoning faculties: it is desirable that their powers should be exerted upon objects of such nature, that we can tell by other means whether the results which we obtain are true or false, and this before it is safe to trust entirely to reason. Now the mathematics are peculiarly well adapted for this purpose, . . When the conclusion is attained by reasoning, its truth or falsehood can be ascertained, in geometry by actual measurement, in algebra by common arithmetical calculation This gives confidence, and is absolutely necessary, if, as was said before, reason is not to be the instructor, but the pupil." [6, p 7-9]

Drill and practice

"Mathematics is becoming too much of a machinery; and this is more especially the case with reference to the elementary students They put the data of the problems into a mill and expect the result to come out ready ground at the other end. An operation which bears a close resemblance to that of putting in hemp seed at one end of a machine and taking out ruffled shirts ready for use at the other end. This mode is undoubtedly exceedingly effective in producing results, but it is certainly not so in teaching the mind and in exercising thought." [7]

"There is much truth in the assertion that new knowledge hooks on easily to a little of the old, thoroughly mastered. The day is coming when it will be found out that crammed erudition, got up for examinations, does not cast out any hooks for more" [4, Vol 1, pp. 278-279]

First steps in arithmetic

"It is a very common notion that this subject [arithmetic] is easy; that is, a child is called stupid who does not receive his first notions of number with facility. This, we are convinced, is a mistake Were it otherwise, savage nations would acquire a numeration and a power of using it, at least proportional to their actual wants, which is not the case Is the mind, by nature, nearer the use of its powers than the body? If not, let parents consider how man efforts are unsuccessfully made before a single articula sound is produced, and how imperfectly it is done aft all; and let them extend the same indulgence, and, if the will, the same admiration, to the rude essays of the thinl ing faculty, which they are so ready to bestow upon tho of the speaking power. Unfortunately the two cases a not equally interesting. The first attempts of the infant i arms to pronounce "papa" and "mamma", though much like one language as another, are received wi exultation as the promise of a future Demosthenes; b the subsequent discoveries of the little arithmetician, su as that six and four make thirteen, eight, seven, anythir but ten, far from giving visions of the Lucasian or Savili chairs*, are considered tiresome, and are frequent rewarded by charges of stupidity or inattention In t first case, the child is teaching himself by imitation, ar always succeeds; in the second, it is the parent wh instructs, and who does not always either succeed deserve to succeed. Irritated or wearied by his failur little manifestations of temper often take the place of t gentle tone with which the lesson commenced, by whic the child, whose perception of such a change is very acut is thoroughly cowed and discouraged, and left to belie that the fault was his own, when it really was that of h instructor." [8]

Extension and generalization

"We now come to a rule which presents more peculi difficulties in point of principle than any at which we ha yet arrived. If we could at once take the most general vie of numbers, and give the beginner the extended notio which he may afterwards attain, the mathematics wou present comparatively few impediments But the cons tution of our minds will not permit this It is by collecti facts and principles, one by one, and thus only, that arrive at what are called general notions; and we afte wards make comparisons of the facts which we ha acquired and discover analogies and resemblances whic while they bind together the fabric of our knowledg point out methods of increasing its extent and beauty the limited view which we first take of the operatio which we are performing, the names which we give a necessarily confined and partial; but when, after ad tional study and reflection, we recur to our form notions, we soon discover processes so resembling o another, and different rules so linked together, that feel it would destroy the symmetry of our language if were to call them by different names We are then induc to extend the meaning of our terms, so as to make tv rules into one. Also, suppose that when we have d covered and applied a rule and given the process which teaches a particular name, we find that this process is on a part of one more general, which applies to all cas contained under the first, and to others besides We ha only the alternative of inventing a new name, or of exten ing the meaning of the former one so as to merge t particular process in the more general one of which it is part." [6, p. 33-34]

On contextualized learning—or antideluvian logic

"The uncultivated reason proceeds by a process almost[1] entirely material. Though the necessary law of thought must determine the conclusion of the plough-boy as much as that of Aristotle himself, the plough-boy's conclusion will only be tolerably sure when the matter of it is such as comes within his usual cognizance. He knows that geese being all birds does not make all birds geese, but mainly because there are ducks, chickens, partidges, &c. A beginner in geometry[2], when asked what follows from "Every *A* is *B*", answers Every *B* is *A*, of course". That is, the necessary laws of thought, except in minds which have examined their tools, are not very sure to work correct conclusions except upon familiar matter. And above all, *relation* is a difficulty when the related terms are unusual names, even in the most[3] common cases

Footnotes

1 . . 2 .

3 Though I take the following only from a newspaper, yet I feel confident it really happened: there is the truth of nature about it. and the enormity of the case is not incredible to those who have taught beginners in reasoning. The scene is a ragged school. TEACHER. Now, boys, Shem, Ham, and Japheth were Noah's sons; who was the father of Shem, Ham, and Japheth? No answer. TEACHER. Boys, you know Mr Smith, the carpenter, opposite; has he any sons? BOYS. Oh! yes, Sir! there's Bill and Ben. TEACHER. And who is the father of Bill and Ben Smith? BOYS. Why, Mr Smith, to be sure. TEACHER. Well, then, once more, Shem. Ham. and Japheth were *Noah's* sons; who was the father of Shem, Ham, and Japheth? A long pause; at last a boy, indignant at what he thought the attempted trick, cried out, it *couldn't* have been Mr Smith! These boys had never converted the relation of father and son except under the material aid of a common surname: if Shem Arkwright, &c. had been described as the sons of Noah Arkwright, part of the difficulty, not all, would have been removed." [9]

On rules and reason

"We believe firmly, that to the mass of young students, general demonstrations afford no conviction whatever; and that the same may be said of almost every species of mathematical reasoning, when it is entirely new. We have before observed, that it is necessary to learn to reason; and in no case is the assertion more completely verified than in the study of algebra. It was probably the experience of the inutility of general demonstrations to the very young student that caused the abandonment of reasoning which prevailed so much in English works on elementary mathematics. Rules which the student could follow in practice supplied the place of arguments which he could not, and no pains appear to have been taken to adopt a middle course, by suiting the nature of proof to the student's capacity." [6, p. 184]

Euclidean logic

"We mathematicians may very easily improve our reasoning from the very beginning. For, though the Logic that Euclid used is very accurate, there has been no enquiry made with regard to it; and the consequence is that for two thousand years we have been proving, as we go through the elements of Geometry, that a thing is itself. That is to say, we have been proving, in the Elements of Geometry, by help of a syllogism, a thing which must be admitted before syllogism itself can be allowed to be valid. Thus, does Euclid not prove that, when there is but one *A* and but one*B*, if the *A* be the *B*, then the *B* is the *A*? He would not take such a thing as that without appearance of proof. "A thing is itself", that is the assertion, that is what Euclid would not take without proof! . . Is not this mode of proof in the third book of Euclid, being the way in which proposition 19 is deduced from proposition 18? Yet anybody who should use it out of geometry would be laughed at, though Euclid used it, and all those who have studied his Elements have been proving things in this manner for two thousand years." [7]

On the study of algebra

"In this chapter we shall give the student some advice as to the manner in which he should prosecute his studies in algebra .

1. The first thing to be attended to in reading any algebraic treatise, is the gaining a perfect understanding of the different processes there exhibited, and of their connexion with one another. This cannot be attained by a mere reading of the book, however great the attention which may be given. It is impossible, in a mathematical work, to fill up every process in the manner in which it must be filled up in the mind of the student before he can be said to have completely mastered it. Many results must be given, of which the details are suppressed, such are the additions, multiplications, extractions of the square root, etc., with which the investigations abound. These must not be taken on trust by the student, but must be worked by his own pen, which must never be out of his hand while engaged in any algebraic process. The method which we recommend is, to write the whole of the symbolical part of each investigation, filling up the parts to which we have alluded, adding only so much verbal elucidation as is absolutely necessary to explain the connexion of the different steps, which will generally be much less than what is given in the book. This may appear an alarming labor to one who has not tried it, nevertheless we are convinced that it is by far the shortest method of proceeding, since the deliberate consideration which the act of writing forces us to give, will prevent the confusion and difficulties which cannot fail to embarrass the beginner if he attempts, by mere perusal only, to understand new reasoning expressed in new language. If, while proceeding in this manner, any difficulty should occur, it should be written at full length, and it will often happen that the misconception which occasioned the embarrassment will not stand the trial to which it is thus brought. Should there be still any matter of doubt which is not removed by attentive reconsideration, the student should proceed, first making a note of the point which he is unable to perceive. To this he should recur in his subsequent progress, whenever he arrives at anything which appears to have any affinity, however remote, to the difficulty which stopped him, and thus he will frequently find himself in a condition to decipher what formerly appeared incomprehensible. In reasoning purely geometrical, there is less necessity for committing to writing the whole detail of the arguments, since the symbolical language is more quickly under-

stood, and the subject is in a great measure independent of the mechanism of operations; but, in the processes of algebra, there is no point on which so much depends, or on which it becomes an instructor more strongly to insist." [6, pp. 175-177]

Early misconceptions in geometry

". . we must observe, that this notion of a straight line will probably be, one which is parallel to the upper and under edges of the paper. Thus he has been told that he cannot write *straight* without lines, and so on This misconception must be corrected by drawing lines over the paper in all directions with the same ruler, and applying the term straight to them all. The learner must be made to understand, that the line which comes off the ruler is of the same form in whatever position the ruler may be held, and that a line which is straight in any one position is so in every other; that what he has been accustomed to call a straight line, means a straight line in the same direction as the top of the paper." [10, p. 37]

"The most serious difficulty in the definitions is, that of the word *angle*. because it contains a notion hitherto almost unconsidered. The best substitute is the word *opening*. Several intersecting straight lines may be drawn, making acute, obtuse, and right angles, care being taken that some of the longest lines shall contain very small angles, and some of the shortest, very obtuse angles" [10, p 38]

"He [the student] has been accustomed to the consideration of several things of the same kind, but rarely to that of the division of one of these objects into *equal* parts His *half* has, most probably, been merely a division into any two parts whatsoever, and he can accordingly, with perfect consistency, talk of the larger and the smaller half" [11]

On concepts and images

"To *imagine* is originally to *form an image* in the mind. But it has been transformed into a synonym of to *conceive,* to form a *concept* The distance from here to the sun is a *concept*. I have no image of it But of six feet I have both image and concept when I shut my eyes. Now many persons, when they cannot *image,* speak as if they could not *conceive,* and use the ambiguous word *imagine*. We cannot, they say, *imagine* infinite space I grant they can't *image* it, but I am sure by their modes of denial that they have a *conception* of it. Locke and others affirm that we arrive at the notion of infinity by finding out that when, say, we add number to number, we *find* the succession incapable of termination, and so fashion interminability in our minds. I say the process is precisely the reverse. If it were not for our *conception* of infinity we should not know the interminability.

Who ever tried up to 10,000,000,000,000,000? It is certainly not experience. If anyone were to affirm that 10^{16} is only a symbol, and that any one who should try would find himself brought up by the nature of things, Locke has no answer, unless, as would probably be the case, he should ask permission to bring on the conception of infinity.

I therefore affirm the concept *infinite* as a subject reality of my consciousness of space and time, as rea[l] my consciousness of either, because inseparable from consciousness of either. When, therefore, I think o[f] finite space—say a cubic foot—if I compare it with totality of space, I say *infinitely small,* if I compare *totum* with *it* I say *infinitely great*." [3, p. 313]

A clinical interview

"I tried an experiment yesterday with my daughter* 8 1/3 years old as to the ideas of necessity, and there wa[s] dialogue as follows:

Q If you let a stone go, what will happen?
A. It will fall, to be sure
Q Always?
A. Always
Q. How do you know?
A I'm sure of it
Q. How are you sure of it? Would it be true at the No[rth] Pole, where nobody has been?
A Oh yes, people have been to the North Pole, else h[ow] could they know about the people who live there, a[nd] their kissing with their noses?
Q That's only *near* the North Pole. Nobody has e[ver] been *at* the Pole.
A. Well, but there's the same ground there and the sa[me] air Hotter or colder can't make the air heavier so as [to] make it keep up the stones Besides, I've read in t[he] *Evenings at Home* that there is something in the grou[nd] which draws the stones I am quite sure they would fa[ll]. Now, is there anything else you want to be a little mo[re] convinced of?
Q How many do 7 and 3 make?
A. Why, 10, to be sure.
Q At the North Pole as well as here?
A Yes, of course.
Q Which are you most sure of, that the pebbles fall to t[he] ground at the North Pole or that 7 and 3 make 10?
A I am quite as sure of both
Q. Can you imagine a pebble falling upwards?
A. No, it's impossible. Perhaps the birds might take the[m] up in their beaks, but even then they wouldn't go up themselves They would be held up.
Q. Well, but can't you think of their falling up?
A Oh yes, I can fancy three thousand of them going up [if] you like, and talking to each other too, but it's an impo[s]sible thing, I know
A. Can you imagine 7 and 3 making 12 at the Pole?
A. (Decided hesitation.) No, I don't think I can No, [it] can't be; there aren't enough

Here her mother came into the room As long as th[e] questions were challenges from me it was all defiance an[d] certainty, but the moment Mrs. De M appeared she ra[n] up to her and said, "What do you think papa has bee[n] saying? He says the stones at the North Pole don't fall t[o] the ground Now isn't it *very* likely they fall just as they d[o] here and everywhere?" But she did not mention the 7 an[d]

*This daughter, Alice, was his eldest child She died in 1853 at the age o[f] 15 The "experiment" took place in 1846

3 = 12 question, nor appeal to her mother about it." [3, p 197-198]

Postscript

As we wrote in the introduction, we are not able to give a detailed picture of De Morgan's influence on mathematics education We have the impression, however, that it was considerable, both directly as a result of his talks, articles and books, and indirectly by his own example to his students, who later became figures of influence. Thus in the notice about De Morgan in the Encyclopaedia Britannica [2] quoted in the introduction we read: "Many of his pupils have distinguished themselves, and, through Mr. Todhunter and Mr. Routh, he has had an important influence on the modern Cambridge school"

While reading on De Morgan and searching for sources, we found a few examples which explicitly acknowledge a debt to him—and to close this anthology we bring examples of these

On the fly leaf of Part II of the Arithmetic by Sonnenschein and Nesbitt [12] there are extracts of reviews of Part I

> Mr Sonnenschein is a pupil, and a thoroughly taught pupil, of Mr. De Morgan's, and it is scarcely necessary to say more in order to convince all who know Mr. De Morgan's works that there is nothing like half-digested work in this arithmetic . . Brevity and lucidity in the exposition of principle are its main characteristics as a scientific book; and great care in the explanation of simple practical rules for shortening or verifying calculations is its main characteristic in reference to the art of computation It gives a clear proof of all the rules—insisting on the exact meaning of the various operations and their interpretation—and contains a remarkably good chapter on the general properties of numbers, so far as they can be explained to beginners who have only mastered the arithmetic of integers It is hardly possible to speak too well of this little book, which we have examined very carefully. —*Spectator*

> Forty years have elapsed since the appearance of Prof De Morgan's "Elements of Arithmetic", at a time when perhaps few teachers, as they submitted the rules of the science to their pupils, cared to establish them upon reason for demonstration The effect of this work was that a rational arithmetic began to be taught generally, and the mere committing of rules to memory took its due subordinate position in the course of instruction . . . The book before us is avowedly drawn up in agreement with the principles of Mr De Morgan's work, and the aim of the authors is to lead the student "to the discovery of the several rules by some path such as an original discoverer might have travelled" . —*Nature*

In the preface to the sixth edition of his Arithmetic [13], Brook-Smith writes as follows:

> In the following pages I have endeavoured to reason out in a clear and accurate manner the leading propositions of the science of Arithmetic, and to illustrate and apply those propositions in practice . . . I have to express my many and great obligations to the College Lectures on Arithmetic of Professor Kelland, and of the late Professor De Morgan ."

Finally we quote again from the notice in the Encyclopaedia Britannica [2]:

> In 1830 appeared the first edition of his well-known *Elements of Arithmetic* which has been widely used in schools. and has done much to raise the character of elementary training. It is distinguished by a simple yet thoroughly philosophical treatment of the idea of number and magnitude, as well as by the introduction of new abbreviated processes of computation to which De Morgan always attributed much practical importance. Second and third editons were called for in 1832 and 1835, and more than 20,000 copies have been sold; the book is still in use, a sixth edition having been issued in 1876.

References

[1] Smith, D.E. *History of Mathematics* Vol I Dover Publications Inc, New York. 1958. p. 462

[2] *Encyclopaedia Britannica* Vol. VII. The New Werner Twentieth Century Edition The Werner Co., Akron, Ohio, 1905, pp. 57-59 The entry on De Morgan was written by William Stanley Jevons, F.R.S. W.S. Jevons (1835-1882) was a leading English economist educated at University College. London

[3] De Morgan, S.E. *Memoir of Augustus De Morgan* Longmans. Green and Co., London, 1882

[4] De Morgan, A. *A Budget of Paradoxes* Vol. 1 & 2 (Second Edition edited by David Eugene Smith) The Open Court Publishing Co, Chicago—London, 1915

[5] Boole. M.E. *Collected Works* Vol. II, edited by E M Cobham, The C W Daniel Company. London. 1931, p. 432

[6] De Morgan. A *On the Study and Difficulties of Mathematics* The Open Court Publishing Co. Chicago, 1898 (New Edition)

[7] De Morgan, A Speech of Professor De Morgan. President. At the First Meeting of The London Mathematical Society. January 16th. 1865. *Proceedings of The London Mathematical Society* 1865. pp 1-9

[8] De Morgan. A. On Teaching Arithmetic. *The Quarterly Journal of Education* Vol. V, 1833, pp. 1-16

[9] De Morgan, A. *On the Syllogism and other Logical Writings* (Edited, with an Introduction by Peter Heath) Routledge & Kegan Paul. London, 1966, pp. 211-212

[10] De Morgan, A On the Method of Teaching the Elements of Geometry Part I *The Quarterly Journal of Education* Vol. VI. 1833

[11] De Morgan, A On the Method of Teaching Fractional Arithmetic *The Quarterly Journal of Education* Vol. V. 1833, p. 210

[12] Sonnenschein, A. and Nesbitt H.A *The Science and Art of Arithmetic for the Use of Schools* Parts II and III Whittaker and Co. 1870

[13] Brook-Smith, J. *Arithmetic in Theory and Practice* Macmillan. London. 1898