MATHEMATICS AND THE MIND GYM: HOW SUBJECT TEACHING DEVELOPS A LEARNING MENTALITY

GUY CLAXTON [1][2]

Be reasonable – see it *my* way, implies the Ashleigh Brilliant pot-shot (see Figure 1, opposite) [3], pointing to one of the defining characteristics of a 'culture' or a 'tribe'. From inside a culture, a whole lot of things look absolutely 'reasonable'. There is a largely unspoken, dominant consensus about what is to be taken as important or trivial, creative or mundane, funny or silly, problematic or 'obvious'. And this is usually balanced by deviant sub-cultures: groups of insiders that seek to contest some of these assumptions. Mathematics educators, like any professional group, constitute such a partly consensual, partly dissenting tribe.

I, inevitably, belong to many tribes, and, partly consciously, largely unconsciously, subscribe to their partial perspective on the world. I have the *deformation professionelle*, the distorting glasses, of the cognitive scientist, of the white middle-aged male, of the Buddhist meditator. But I do not belong to the mathematics education tribe, and the tentative views that follow are liable, to an unpredictable extent, to sound trite, ill-informed or ill-conceived to its insiders. Whether I am seeing the world of school mathematics freshly, or wrongly, or most likely a bit of both, remains to be seen.

What I see, as I look at mathematics education from the outside, is something like the House of Lords (the 'upper house' of the UK parliament): an institution long certain of its own central importance to public life, but under threat, with foundations and justifications that are shakier than they have been thought to be. For both mathematics education and 'The Lords', the world has changed, privilege is no longer taken for granted, and the question is what depth of reform is required.

Mathematics has long assumed, as of right, a central place in the curriculum, but its warrant is under scrutiny. Both social demands and psychological assumptions have changed; the core purpose of education is deeply contested, powerful new models of teaching and learning are being proposed, and mathematics is in need of a new rationale. In this article I want to reflect on how the world around mathematics has changed, and what implications and possibilities arise.

Three challenges for mathematics education

From my vantage point outside the tribe, I see three main challenges.

First, it is not clear that much of mathematics is as directly useful as it has claimed to be. Many people, at all levels of society, lead happy and successful lives without any mathematical knowledge beyond checking their change, adjusting the quantities in recipes, and so on. There is evidence that street mathematics works best with local heuristics, rather than the powerful but cumbersome procedures that are taught in school:

> We suggest that educators should question the practice of treating mathematical systems as formal subjects from the outset and should instead seek ways of introducing these systems in contexts which allow them to be sustained by human daily sense. (Carraher *et al.*, 1985, p. 28)

Cheap, smart machines shoulder most of the burden of calculation. A professor of mathematics education confesses that the only time in her life when she has actually been required to add fractions was in computing the 'credit' she could claim for her contributions to a collection of multiply-authored research papers. Matrices and partial differentials are needed in some careers, but that in itself is no more justification for including them in a general syllabus than making everyone learn advanced knitting skills because a few people are going to be glad of them later. (I have heard a medical lecturer claim recently that any motivated, literate eighteen-year-old could acquire the practical knowledge needed to be a general practitioner in six months. The rest of what they are required to study is history and ritual. The same kind of challenge applies to mathematics.) Anyway, a good deal of what is learned to pass examinations – 75% in some cases – is forgotten within a year (Conway, Cohen and Stanhope, 1991).

Second, it is not clear that mathematics is the new Latin – in the sense of providing any kind of effective, generic 'training of the mind'. Of course, Latin never was, despite the rear-guard rhetoric of its adherents, and there is no evidence that I am aware of that students of mathematics show any enhancement of their spontaneous, real-life powers of deduction, logical argument and so on.

Perkins (1985) has shown, in fact, that *no* academic study beyond primary level improves informal reasoning. There are ways of teaching subjects that *do* encourage disembedding and transfer of useful generic skills, but normal 'good teaching' does not reliably achieve it (Halpern, 1998). The fact that algebra deals in *x*s and *y*s, rather than kings and queens, or acids and bases, does *not* mean that it constitutes a Platonic domain that is content-free and universally applicable. On the contrary, mathematical domains are, at least to students, highly specific, if arcane, worlds akin to, but (except to a few) much less interesting than, those of Lewis Carroll or, more recently, Joanna Rowling's Harry Potter. (Recall Hughes's young respondent who could happily add "2 wuggles and 3 wuggles" without having a clue what a wuggle was, but refused to add "2 thousands and 3 thousands" because "we haven't done thousands yet" (Hughes, 1983).)

The *third* challenge suggests that mathematics has achieved its high status in the curriculum not through its utility, either specific or generic, but because it forms a closed,

Figure 1: "One of the defining characteristics of a 'culture' or a 'tribe'" (see the beginning of the article by Guy Claxton, opposite).

elegant, timeless domain, capable of being neatly turned into an apparently logical succession of curriculum packages that require clear expository teaching, and that can be assessed objectively, unambiguously and rigorously. Mathematics sits at the centre of a national curriculum, says this challenge, only because it is the prototypical 'subject'. And the epistemology on which this judgement rests has been under serious attack for several decades. If 'knowledge' is not, after all, to be best construed as packages of reliable, eternal, communicable propositions and operations – then where does that leave mathematics?

This third challenge to the warrant of mathematics education is perhaps the most serious. Mathematics is vulnerable because its status depends on an epistemological, cultural, scientific and educational worldview that is increasingly seen to be archaic. Let me say a word about each of these facets of the changing *zeitgeist*.

In the relativistic, post-modern, just-in-time rather than just-in-case world, the epistemological emphasis has shifted from knowledge to knowing; from the application of generic knowledge and procedures to particular problems by intelligent, well-informed individuals, to the generation of problem-specific knowledge *in situ*, often by specially formed teams of people with different, complementary domains of expertise. The fast pace of business, for example, routinely requires such localised, targeted knowledge generation. Much of the 'knowledge' that is needed is not pre-existing, and no one person possesses more than a few pieces of the collective jigsaw puzzle.

The value of such situated knowledge is judged in terms of its use – does it get the job done – rather than, as in a traditional university, for example, in terms of dispassionate canons of 'truth'. There are multiple paths to a solution, no 'right answer', and a considerable degree of uncertainty at all stages of the problem-solving process (Gibbons *et al.* 1994). In this creative, situated world, the boundaries between the traditional 'disciplines' become increasingly permeable, and even irrelevant. Mathematics becomes a tool, usually enshrined in a computer programme, that people need to know how and when to use, but not how to derive or justify. The programme designer may need (access to) that understanding, but the home or business user absolutely does not.

Culturally, young people are now well aware that traditional, long, intellectual apprenticeships are not always necessary to be an effective 'knowledge worker'. Though statistically a degree still leads to a higher income, there are many well-sung young entrepreneurial heroes and heroines who have succeeded without. Lewis (2001) in *Next: the future just happened* recounts many stories of fourteen-year-olds making a killing on the stockmarket, Finnish adolescents who invent texting, and the fifteen-year-old who set up a highly successful – that is, lucrative – legal aid website based on nothing more than his many hours spent watching *Judge Judy* on 'reality' TV (Lewis, 2001, p. 99). Professional gate-keepers have been swept aside by the Internet, and knowledge is radically democratised.

There is also a general appreciation of the fact that education does not – cannot – give you the knowledge you are going to need for life, and that leisure, work and learning interpenetrate throughout life in much more variable and complicated ways than the traditional model of 'school' admits. Young adults take their time to 'mill and churn', as a recent Organisation for Economic Cooperation and Development report (OECD, 2000) put it, to the despair of their elders, but not necessarily to their own longer-term detriment. The foundation of abstract knowledge – such as mathematics – that pre-arms you to deal with all contingencies seems to be a thing of the past.

Scientifically – that is to say, in terms of current neuro- and cognitive science – the idea that minds are more-or-less fixed-sized, individual containers of 'knowledge' is also giving way to much more multiple, flexible and social notions.

First, the supremacy of rational, scholarly forms of knowing – the Enlightenment ideal of the pure, abstract, 'formal operational' intellect – is being challenged by a plethora of other kinds of 'intelligence': musical intelligence, interpersonal intelligence, emotional intelligence, practical intelligence, creative intelligence – even spiritual intelligence. Whether all of these ideas will turn out to be robust is doubtful, but it is safe to say that the exclusive association of 'intelligence' with linguistic and mathematical forms of reasoning has been shattered beyond repair. There is a growing realisation that engaging effectively with uncertainty and complexity requires personal characteristics such as perseverance and tolerance for confusion, as well as non-analytical forms of cognition such as intuition, imagination and reflection. The boundary between intellect and personality rapidly dissolves as these more sophisticated approaches to 'mind' develop (Chiu, Hong and Dweck, 1994).

The *second* scientific shift has discredited the idea that minds can be distinguished in terms of the size of some kind of fixed endowment of general-purpose 'ability' (Howe, 1997; Richardson, 1999). How smart people are turns out to be much more situation- and task-dependent than this old 'ability' model would have us believe. Measured IQ is uncorrelated with real-life smart. And 'intelligence' is much more malleable too. Piaget's (1959) definition of intelligence as "knowing what to do when you don't know what to do" is being revived and much quoted because it promises a more expandable mind, associating intelligence with learning – with the developing resources with which one *comes* to know – rather than with knowing itself. (As one savvy seven-year-old told me recently, "Intelligence isn't knowing lots; it's what you *do* to know lots".) Intelligent learning reflects a complex interplay of acquired "habits of mind" (Costa and Kallick, 2000).

Finally, science shows that 'mind' expands beyond the skin of the individual. If we look at how people actually do clever things, rather than at how the Cartesian model *says* they do them, we find that 'the organ of intelligence' has to include all kinds of human and non-human adjuncts – filing systems, databases, the internet, teams and contacts.

As I write this paper, my own mental processes are in continual interaction with the record on the screen, the reference books I go to find, the piles of paper I search through, the *Encyclopedia Britannica* installed on my hard drive – as well as with my memories of talks I attended, and my sense of what makes a good sentence. How I think is inextricably moulded by the external tools and resources I habitually make use of, and by the communities and individuals with which I continually interact (Clark, 2003; Hutchins, 1995; Salomon, 1993). In the real world, it makes no more sense to sit someone down at an isolated desk, deprived of all their resources, and require them to show how smart they are, than it would to isolate David Beckham from the rest of Real Madrid, take away his ball, and say "now show me what a good footballer you are". In the real world, most mathematical thinking is socially and technologically distributed in just this way. 'Student + textbook + notepad + Biro' just does not begin to do justice to the situation.

All of these epistemological, cultural and scientific considerations serve to undermine the industrial age model of education from which the structure of 'school' still derives. Modelled explicitly on the 'assembly line', the schools of the late nineteenth century sought to process students in 'batches' (year groups); to pass them through a standardised process in which a number of specialists (particularly at secondary school) added their own knowledge contribution to the developing person; to subject them to 'quality control' every so often; and to grade, sort and label them according to how well they did. There were plenty of low-skill jobs for the 'rejects', who were expected, at least, to have mastered loyalty and compliance, while the 'top quality' products became managers and professionals.

But now this model is self-evidently in tatters: wasteful, divisive and anachronistic (Gilbert, 2004; Senge *et al.*, 2000). It no longer corresponds to the predominant view of knowledge and knowing; to the nature and demands of society; nor to what we know about real minds and brains. The mines have closed and robots rule the assembly lines. The web gives millions of students instant access to knowledge that is more comprehensive, attractive and up-to-date than their strapped-for-cash teachers can hope to offer. The world is fluid and turbulent, and to flourish rather than flounder young people have to acquire the skill and the courage to flow with it.

From the knowledge factory to the mental gymnasium

What can and should education do in the face of these challenges? I have argued elsewhere (Claxton, 1999, 2002) that there are a range of pedagogical attitudes and techniques that systematically cultivate students' generic capacity to be effective real-life learners. The way teachers comment on students' learning, what they themselves model in the face of uncertainty, the ways they orchestrate learning contexts and activities, and the kinds of conversations *about* learning which they encourage: all of these can contribute to the cumulative building of young people's so-called 'learning power'.

However, these previous discussions have not addressed the place of specific subject content in this process. Obviously, to develop one's 'learning muscles' and 'learning stamina', one has to have interesting things to learn *about* (just as, for example, you can only develop the general skill of welding by working on particular jobs and materials). But the content you are working with inevitably affords, invites or foregrounds some kinds of learning, and neglects others. How can we look at the content curriculum, and especially mathematics, in this light?

Perhaps to start with, we need to replace the assembly-line model of education and the 'fixed pot' model of the mind with new 'root metaphors' for what minds are, and how schools affect them. I propose the following, which better captures the new epistemology and sociology: minds are like *bodies*; and schools are like *gymnasia* for developing fitter minds. Let me try to unpack this metaphor, to show how it leads to new possibilities, and new ideas about the role of specific subjects, including mathematics.

1. Bodies come in all shapes and sizes, but they can all get fitter. And so can minds. There may be some hypothetical limits to how strong and fast and supple I can get, but that does not mean I cannot get fitter, nor that there is not enormous potential for me to do so. One's mentality can be seen primarily as expandable, rather than merely fillable.

2. Being relatively fit is a good thing to be. It provides the physical platform that enables people to aspire to and achieve a whole variety of more specific goals. And so does having a fit mind. The moral imperative behind the 'mental gymnasium' model of education asserts that there are a set of learning-positive dispositions and capabilities which it is useful for all young people to have; that they can be cultivated; and that it is one of the core purposes of education to do so. The corollary of this is that not all forms of education are mind-expanding, and most, in fact, have not been.

3. Fitness trainers have a model, based in sound physiology, of what 'fitness' consists of: suppleness, skeletal muscle strength, speed, stamina, cardiovascular recovery rate, body-mass index, and so on. Likewise we now have the beginnings of a good model of what 'mental fitness'– what I have elsewhere called 'learning power' – comprises (Claxton 2002). There are:

- *emotional components* like the ability to tolerate frustration and confusion
- *attentional components* like the ability to manage distractions and sustain concentration
- *cognitive components* like curiosity, imagination and 'critical thinking'
- *reflective components* like the ability to assess one's own level of development and become one's own 'coach' and
- *social components* like the ability to work in a team, resist group-think, and know how to put yourself in someone else's shoes.

4. Getting all-round fit requires a variety of different forms of exercise that cumulatively stretch and cultivate the different aspects of fitness. Keeping on working only on the same muscle group day after day, even if the activities vary a little, is not developing general fitness. Likewise, education for mental fitness has to vary the *kind* of learning that students are involved in. Learning in the same way, day after day, even if you are learning different things, will not build all-round 'learning power'. (You'll end up with massive biceps and a weak chest, so to speak.)

5. Activity is not the same as exercise. Activities can be very interesting and engaging, but unless they 'stretch' the system, regularly pushing it near its limits, they are not

developing general fitness. Going to the gym without getting hot and sweaty is a waste of time, however long you spend there. Students who are busy and engaged are not necessarily developing their mental fitness. From the mind-gym point of view, 'bright' students who find it easy to gain high marks are likewise wasting their time. (David Beckham gains no fitness benefit from taking part in his son's kindergarten's egg-and-spoon race.) So what counts as exercise is entirely relative to the existing level of a person's fitness. For me, a brisk walk to the top of my lane is exercise, while a stroll round the back garden is not. To a recovering stroke patient, a few steps may be challenging enough.

6. Gyms have equipment, the use of which invites desirable kinds of exercise. There are many kinds of exercise you cannot do on your own in a bare room. You have to have something to exercise *on*. Likewise, you have to have interesting, challenging things to think and learn about in the 'mind gym'. That is what 'topics' and 'subject-matter' are. 'Mathematics' is a set of apparatus, in one corner of the mental gymnasium, which affords and invites certain kinds of 'mental work-out'. *Prioritising the development of a learning mentality does not mean that 'content' is neglected.* It may mean, though, that content is selected and used on the basis of the kind of exercise it invites and affords, and not (exclusively) on the basis of 'intrinsic worthwhileness' or 'teachability'. (You do not go to the gym in order to study the equipment: to learn how it is made, how it was developed, or the names of its parts. You go, primarily, to use it.)

7. Different kinds of exercise contribute to the cultivation of all-round fitness. Each piece of equipment in the fitness centre has its own specific job to do, and there is no room for different-looking bits of kit that essentially duplicate each other's roles. Likewise, the curriculum as a whole has to provide well-rounded mind training, and each subject and topic has to justify its inclusion partly on the grounds that it affords a kind of mental exercise that is complementary to all the others. If, looked at this way, it turns out that mathematics, in whole or part, duplicates the kind of work-out that is provided by 'science', say, then its place is vulnerable.

8. Especially when a piece of apparatus is unfamiliar, it helps to have some expert advice about how to use it, for how long, and with what degree of difficulty. "Do two lots of twelve presses with 10kg to start with, and see how you go." If you make it too easy you do not stretch the system; if you make it too hard you will get dispirited, and even injured. Teachers function as 'learning coaches' in the same way; their experience and their knowledge of their specialised learning apparatus, as well as of their students, enables them to make educated suggestions about appropriate levels of difficulty, to get them going. However, they might also want to encourage students to take over this coaching function for themselves, becoming better at setting their own mental fitness targets and designing their own 'training programmes'.

9. Equipment is designed so that the level of difficulty can be varied to suit a wide range of starting levels. (*NB*, 'starting level' is not the same thing as 'ability'. The level you start at says nothing at all about how far you can go.) Topics and activities in the 'mind gym' should likewise be selected so that they afford a range of different levels of engagement. Learning coaches might encourage students to think about, and take responsibility for, moderating their own level of 'difficulty', learning how to raise or lower the level of difficulty of an activity for themselves so that it affords an appropriate level of challenge. Topics need to be selected and 'framed' so that they allow for that.

10. Getting fitter requires sustained commitment. It is of no long-term use doing 'binge exercising' – renewing that gym membership and going five nights a week for the first fortnight of January, only to have totally lapsed again by the beginning of February. Gentle persistence and slowing pushing your limits of stamina and strength are what works. Likewise we must assume that 'building learning power' takes time. The learning world, like the physical fitness and weight loss worlds, is full of exaggerated claims and 'miracle cures', which are not to be trusted. Simple hints and tips (mnemonics, mind maps, bottles of water and so on) may get you started but they do not do the sustained work of developing mental fitness. There is a good deal of vacuous sloganeering and commercial branding in the 'learning to learn' world that ought to be treated with suspicion.

11. The key 'performance indicators' of developing physical fitness are increases in your 'personal bests'. You run four kilometres in less time. You can lift a 20 kg weight 15 times, when a month ago you could only do 10. Likewise, developing learning power is best assessed *ipsatively* – comparing yourself with your own earlier performance – rather than against group norms or fixed criteria. Seeing how your learning power is expanding is more motivating than being told that you have moved up or down the class league-table (Harlen and Deakin-Crick, 2003). However, *provided the main focus is on improving your personal best*, we might argue that a bit of 'healthy competition' is often useful – as it is in a group of athletes in training. A learning coach might need to know when and how to harness this competitiveness positively, and when to focus back on individual progress.

12. Coaches support their athletes in setting and achieving their own goals. The athletes have ultimate responsibility for whether they exercise or not. The deal is: they lend some of this self-determination to the coaches – trusting them to boss them about and set them challenging and sometimes uncomfortable targets and activities – in the belief that by doing so they will be helped to achieve their own goals. The coaches may remind them – forcibly sometimes – of what they said those goals would be, but they avoid taking too much responsibility. They cannot force the athletes to put in the work, and it is not their job to do so. Likewise, learning coaches can remind their students of the value and purpose of stretching their learning power, and make the exercises as attractive as possible. But they cannot make them, and, ultimately, will not try.

13. It helps to sustain motivation, and boost pride in progress, if you belong to a 'team' all of whom, at their own levels, are trying to get fitter. Team members support each other, and celebrate each other's (maybe very different) achievements. Smart learning coaches, like the physical coaches, might be aware of and seek to foster this team spirit and camaraderie. They could notice and acknowledge both commitment and progress, and encourage their students to do the same with each other.

14. Physical coaches help you set realistic targets – maybe moderating your own enthusiasm or impatience – helps you monitor progress, and suggests activities and programmes that will help you narrow the gap between current and target perfor- mance. So do learning coaches. Their feedback is useful and precise, not global and vague. And they help you focus on one aspect of your developing fitness at a time, and not be overwhelmed by how much there is to do. Recent work on 'formative assessment' and 'assessment for learning' has identified many practical ways for teachers to do this (Black and Wiliam, 1998; Clarke, 1998).

15. Finally, effective coaches tend to walk their own talk. They model their own skill and commitment in raising their own level of fitness. They are not afraid to be red in the face and out of breath, alongside their students. Analogously, learning power coaches would see the importance of modelling and demonstrating learning-positive habits of mind. They are happy to say "I don't know" or "That's a good question; how could we find out?" The classic example from mathematics education is given by Schoenfeld (1992, 1996), who regularly invites his higher education students to throw tricky mathematics problems at him, and models not only skill in mathematical thinking, but also enjoyment in the process of trying to figure things out in public. In the 'assembly line' model, every teacher had to be an expert in their own area and any ignorance was to be denied or covered up. In the 'mind gym' model, it is part of each teacher's responsibility to be an expert in *not* knowing, and to model confidence, openness and equanimity – pleasure even – in the face of uncertainty.

The implications for mathematics education

So much for the mind gym metaphor. Even with such a brief presentation, it is clear that the role of the 'learning coach' is in many ways distinct from the role of the traditional 'good teacher', and also more precise than that of the 'liberal facilitator'. But the implications that I have hinted at so far have been designed to be food for thought for any curriculum area. The interesting question to come back to is: are there any particular implications for each curriculum subject, such as mathematics?

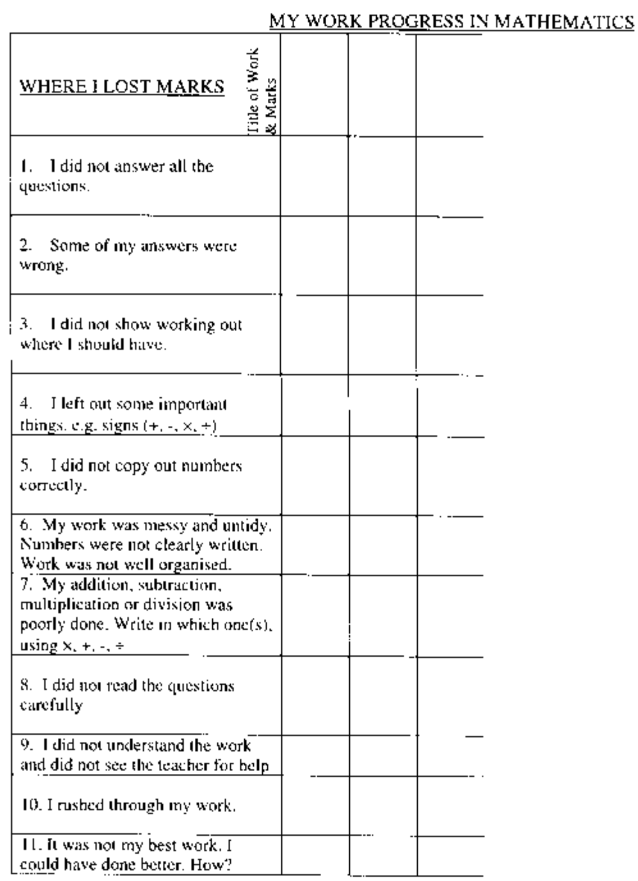
If the metaphor holds up, then each subject can be seen through a new lens, complementary to the others that I mentioned earlier. We can argue about what is intrinsically worth knowing – mathematics as a remarkable cultural achievement that, we believe, everyone should get a feel for, just as we might argue for Beethoven and Shakespeare. We can argue about which bits of mathematics will be of real, practical utility to the digital natives of the mid-twenty-first century. But we can now also ask: what kinds of mental, emotional and social exercise are afforded by each topic, and how can it be presented so that, by engaging with it, useful generic skills and attitudes are likely to be strengthened?

And we must not be allowed to answer this with wishful thinking or dogmatic assertion. If 'adding fractions' is to defend its place in the school curriculum, for students who will never need to add fractions in real life, then it can potentially do so in terms of the general-purpose exercise which adding fractions affords – and which is not equally well or better afforded by a different topic that has the added benefit of also being valuable in its own right. And it must be shown how this specific form of exercise translates into a genuine contribution to generic fitness, and complements those provided by other subjects and topics.

This is not the place to try to make a detailed mapping of the affordances of different mathematical topics onto the different aspects of 'mental fitness'. But here are a few possibilities:

- Obviously, mathematical problem-solving can be used as a site to practise and compare learning alone with learning collaboratively, in different kinds of groups and in different kinds of setting. Students can be invited to reflect on which kind of grouping works best for different kinds of task, and on individual differences.

- Obviously, mathematical investigations afford many opportunities for planning and assembling resources, and for reflecting on progress made, obstacles met, and strategies deployed. Students can practise anticipating their likelihood of success, how long a task will take, and who or what is likely to help them most effectively.



MY WORK PROGRESS IN MATHEMATICS

WHERE I LOST MARKS	Title of Work & Marks		
1. I did not answer all the questions.			
2. Some of my answers were wrong.			
3. I did not show working out where I should have.			
4. I left out some important things, e.g. signs (+, -, ×, ÷)			
5. I did not copy out numbers correctly.			
6. My work was messy and untidy. Numbers were not clearly written. Work was not well organised.			
7. My addition, subtraction, multiplication or division was poorly done. Write in which one(s), using ×, +, -, ÷			
8. I did not read the questions carefully			
9. I did not understand the work and did not see the teacher for help			
10. I rushed through my work.			
11. It was not my best work. I could have done better. How?			

Figure 2: What I did wrong checklist. (Image reproduced with permission from Baird and Northfield (eds), 1992, p. 259.) [4]

- Obviously, many kinds of mathematics problems afford opportunities to practise error-detection and fault-tracing, and for devising helpful aids to self-evaluation. Many students have enjoyed the collaborative process of reflecting on the common ways in which they 'go wrong' or 'lose marks', and of devising practical tools for spotting and circumventing these recurrent pitfalls (see for example Figure 2).

- Obviously, mathematics is a prime site for developing strategies for self-help, and strengthening the disposition to persist in the face of uncertainty or frustration. (One strategy I have seen some teachers use to help to tackle this is by initiating public discussion in their classrooms of the 'thirty second rule', which some students have unconsciously absorbed: "If I can't do it in thirty seconds – or two minutes, or whatever – then I can't do it at all, so it's not worth trying beyond that point").

All of these suggestions are pretty self-evident beginnings of much more intricate lines of thought. It took years to develop exercise machines that most reliably and attractively contributed to the development of generic fitness; it will take a while to select and craft the mathematical exercise machines that deserve a place in the mental gymnasium. The mind gym perspective is unusual to many people; the questions that it raises are often unfamiliar, and some of the possible implications unwelcome. It may be that a good many venerable topics – highly 'teachable' in the old dispensation – will find it hard to keep their place.

And affection and familiarity should not be allowed to save them. I have learned, as a writer, that I am not the best editor of my own work. Slowly, painfully, I have come to accept that sometimes my fine prose and great ideas have to go, and that someone else can see, from the outside, where the cuts needs to be, in order that the overall work be better fit-for-purpose. I cannot bear, in the famous phrase, to 'murder my darlings', and I have to trust a stranger with a red pen to do the dark work of verbal infanticide for me. It may be, when we look at mathematics education from the point of view of its place as one set of apparatus, in one corner of the mind gym, that people outside the tribe – people who do not share the history, the war stories, the sentiments and the 'common sense' assumptions – might be better placed to make those difficult judgements. But then, what do I know?

Notes

[1] An earlier version of some of these ideas was presented to the Annual Conference of the New Zealand Council for Educational Research, Wellington, July 2003, and I thank NZCER Director, Robyn Baker, for the invitation, and her colleagues Jane Gilbert and Karen Vaughan for their thought-provoking inputs to the discussion.

[2] There is a commercial organisation called *The Mind Gym* that offers a wide range of mental 'work-outs' to businesses worldwide, though not currently to schools. That organisation shares many, though not all, of the assumptions of the present argument.

[3] Image of *Pot-Shots #5230*, copyright Ashleigh Brilliant, reproduced with permission. For more information about these cards go to: http://www.ashleighbrilliant.com.

[4] For more information about the PEEL project, from which this example is taken, contact ian.mitchell@education.monash.edu.au.

References

Baird, J. and Northfield, J. (eds) (1992) *Learning from the PEEL experience*, Melbourne, Australia, Monash University Printing Services.

Black, P. and Wiliam, D. (1998) 'Inside the black box', *Phi Delta Kappan* **80**(2), 139-144.

Carraher, T., Carraher, D. and Schliemann, A. (1985) 'Mathematics in the streets and in schools', *British Journal of Developmental Psychology* **3**, 21-29.

Chiu, C., Hong, Y. and Dweck, C. (1994) 'Toward an integrative model of personality and intelligence: a general framework and some preliminary steps', in Sternberg, R. and Ruzgis, P. (eds), *Personality and intelligence*, Cambridge, UK, Cambridge University Press, pp. 104-134.

Clark, A. (2003) *Natural-born cyborgs: minds, technologies and the future of human intelligence*, Oxford, UK, Oxford University Press.

Clarke, S. (1998) *Targeting assessment in the primary classroom,* London, UK, Hodder and Stoughton.

Claxton, G. (1999) *Wise up: learning to live the learning life,* London, UK and New York, NY, Bloomsbury.

Claxton, G. (2002) *Building learning power: how to help young people become better learners,* Bristol, UK, TLO Ltd.

Conway, M., Cohen, G. and Stanhope, N. (1991) 'On the very long-term retention of knowledge acquired through formal education: twelve years of cognitive psychology', *Journal of Experimental Psychology: General* **120**(4), 395-409.

Costa, A. and Kallick, B. (2000) *Habits of mind: a developmental series*, Alexandria, VA, Association for Supervision and Curriculum Development.

Halpern, D. (1998), 'Teaching critical thinking for transfer across domains: dispositions, skills, structure training and metacognitive monitoring', *American Psychologist* **53**(4) 449-455.

Gibbons, M., Limoges, C., Nowotny, H., Schwartzmann, S., Scott, P. and Trow, M. (1994) *The new production of knowledge: the dynamics of science and research in contemporary societies*, London, UK, Sage.

Gilbert, J. (2004, in press) 'Catching the knowledge wave?: 'knowledge society' developments and the future of public education in New Zealand', in Codd, J. and Sullivan, K. (eds), *Education policy directions in Aotearoa/New Zealand: looking for a third way?*, Palmerston North, NZ, Dunmore Press.

Harlen, W. and Deakin-Crick, R. (2002) 'A systematic review of the impact of summative assessment and tests on students' motivation for learning (EPPI-Centre review, version 1.1)', in *Research Evidence in Education Library*, London, UK, EPPI-Centre, Social Science Research Unit, Institute of Education.

Howe, M. (1997) *IQ in question*, London, UK, Sage.

Hughes, M. (1983) 'What is difficult about learning arithmetic?', in Donaldson, M., Grieve, R. and Pratt, C. (eds), *Early childhood development and education: readings in psychology*, Oxford, UK, Blackwell, pp. 204-221.

Hutchins, E. (1995) *Cognition in the wild*, Cambridge, MA, MIT Press.

Lewis, M. (2001) *Next: the future just happened*, New York, NY, Norton.

Organisation for Economic Cooperation and Development (2000) *From initial education to working life: making transitions work*, Paris, France, OECD Publications.

Perkins, D. (1985) 'Postprimary education has little impact on informal reasoning', *Journal of Educational Psychology* **77**(5), 562-71.

Piaget, J. (1959) *The language and thought of the child*, London, UK, Routledge and Kegan Paul.

Richardson, K. (1999) *The making of intelligence*, London, UK, Weidenfeld and Nicholson.

Salomon, G. (ed.) (1993) *Distributed cognitions: psychological and educational considerations*, Cambridge, UK, Cambridge University Press.

Senge, P., Cambron-McCabe, N., Lucas, T., Smith, B., Dutton, J. and Kleiner, A. (2000) *Schools that learn: a fifth discipline fieldbook for educators, parents, and everyone who cares about education*, New York, UK, Doubleday.

Schoenfeld, A. (1992) 'Learning to think mathematically: problem solving, metacognition, and sense-making in mathematics', in Grouws, D. (ed.), *Handbook for research on mathematics teaching and learning,* New York, NY, MacMillan, pp. 334-370.

Schoenfeld, A. (1996) 'In fostering communities of inquiry, must it matter that the teacher knows the "answer"?', *For the Learning of Mathematics* **16**(3), 11-16.