Problems and Puzzles

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With the help of Mark Burton

The word "problem" is used generally to refer to a difficulty — "What is your problem?". In the mathematics classroom it has a particular usage referring to the set of exercises at the end of a chapter in the text. In North American classrooms, it has achieved the further particular meaning of "word problem" — the presentation of a mathematical situation in written language. The possible confusion which can be caused by these uses can be overcome by agreeing to use the term "exercise" to describe a mathematical question demanding the application of a known technique for the purpose of practice, and the phrase "word problem" as described above. What, then, is a problem?

When asked to give examples of currently faced problems, a class of adult students produced a list including:

how to vote in the forthcoming election
what to serve at a dinner party
how to pass the forthcoming examination

Dealing with such problems, which can range from the trivial to the life threatening, demands such procedures as:

collecting, classifying, analysing and using information
searching for relationships
making and testing hypotheses
discriminating between objective and subjective information (between needs and wants).

For example, the problem of how to vote could be dealt with by a careful sifting of party manifestos compared with whatever evidence is available of past records: it could be dealt with by a recognition of a subjective preference, preferring the look of candidate X to candidate Y; a value stand could be taken and compared to the value stand of each candidate; or some combination of all of these could be used.

Thinking about how such problems are overcome leads to the identification of a number of factors:

1. A problem is frequently a problem because it is ill-defined. Once the nature of the problem is identified, the method or methods for dealing with it are often clear.
2. Problems rarely have a single solution, indeed problems rarely have a final solution in that they are then "wrapped up" and concluded. More usually problems are open-search in the sense that the method chosen to deal with them is "best fit" rather than exact and relies upon an amalgam of objective and subjective information or even chance. What is then obtained is not a "solution" but a personal resolution of the problem, which the individual judges not as right or wrong but as adequate.
3. Each problem resolution opens up another field of problems.
4. Problems belong to people, they are real and involving to the individual. They present a challenge which the individual acknowledges.

Now consider a puzzle, perhaps a maze, or a crossword, or something like the following:

Use three continuous lines to join up four dots in the shape of a square.

Try it.

If the solution to that puzzle is obvious, try this one:

Replace the letters by numbers to make

$$\begin{array}{r} \text{H O C U S} \\ + \text{P O C U S} \\ \hline \text{P R E S T O} \\ \hline \end{array}$$

Or, perhaps, work out how many squares there are on an 8×8 chessboard.

Whether or not progress can be made with the chosen puzzle, notice some features of the activity

1. The puzzle was given to you — it was not 'your own'. In order to do it, you must not reject it but must make it your own. As a consequence, your attitudes to mathematical puzzles and the climate within which the puzzle is received are both relevant to acceptance

MOTIVATION — CREATIVE CURIOSITY

2. The puzzle has a solution which is known to the setter and unknown to the doer. Your problem then is to ease out that solution to the puzzle from somewhere

SEARCH BEHAVIOUR

3. The puzzle and its solution are in a known context and the rules for exploring it are established. Otherwise, you cannot even begin. Identifying the *type* of puzzle, or the type of appropriate procedure then becomes a first move. For example, if you were offered a computer puzzle and you knew nothing of computers you would reject the puzzle on the rational grounds that the type of puzzle was such that the context was unknown to you.

PARADIGM DEFINED

4. Puzzles often have a "trick" in them and once you have seen the nature of that trick you are well on the way to solving them. For example, the trick in the puzzle asking how many squares on a chessboard is that you usually think of a chessboard as being made up of unit squares. Now, if a hint is given by suggesting that there are squares of other sizes on a chessboard, can you go ahead and solve this puzzle?

TRICKS AND HINTS

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Examine some of these differences between problems and puzzles

Problems	*Puzzles*
Real and involving	Artificial and given
Open search resolutions	Single solutions
111-defined and often unconnected to known paradigms	Well-defined and related to a known paradigm.
Extend indefinitely, either in the range of possible methods of attack or in their development into new areas of investigation	Conclude with the satisfactory solution (answer)

There is one major difference which has not been discussed. Problems are usually serious and demanding on a cognitive level. The satisfactory resolution of a problem frequently provokes new learning or a new rearrangement of old learning in the problem solver. Dealing with problems involves creating a learning environment and the energy generated and consumed ensures that the learning is retained. This would suggest that becoming more efficient at coping with problems would give positive pay-off in terms of life style. Further, the link between problem solving and learning suggests that this experience can valuably be gained in the classroom. Puzzles, on the other hand, are diversionary and, as long as they do not create too much tension and frustration, they are "fun". There is not necessarily any new learning required in their solution — a shift in perception is frequently all that is necessary — and they do not have application or relevance to the world of the puzzler.

Puzzles and problems in the learning and teaching of mathematics

Many educationists who write in the fields of learning and teaching propose problem solving as the ultimate aim in mathematics. If current curricula in mathematics and most frequently observed classroom practice are considered, problem activities in the sense in which they have been discussed are not likely to feature. Much more consistent with general practice is the set of critieria listed under puzzles.

Artificial and given: much of the mathematics which pupils are required to perform falls into this category *from their point of view*. This not only results in poor motivation towards learning but also in a distorted experience of mathematics as being disconnected, unrelated to aspects of their lives which they value, irrelevant despite constant exhortations about utility.

Single solutions: the "right" answer phenomenon which itself is related to the "tick" phenomenon, that is, creating conditions where getting ticks (checks) is the major motivator.

Well-defined and related to a known paradigm: mathematics as presented in texts is broken down into small and discrete units which are taught and learned in context. Examinations are expected to replicate these conditions and the pupils learn to recognize the context in which they are being asked to work so that they can then apply the appropriate algorithms. Indeed, questions often use the format:

(i) prove the following theorem (by memorising?);
(ii) solve the related "problem" (by application).

Conclude with the satisfactory solution: this is part of the closed nature of the learning. Pupils' motivation is then directed to finishing and questions of elegance of solution, communication of experience, application of learning or extension of enquiring become irrelevant. Thus is generated, by the tricks and hints department, a view of mathematics as being somewhat arbitrary and full of trickery designed to catch out the unwary. This generates a style of learning which is dependent upon filling gaps in understanding by appealing for hints.

Are problems in the classroom desirable and possible?

Mathematicians view their subject as a searching/finding/proving/searching one. Each journey around the search/find/prove track results in a fragment of mathematical knowledge being established and a new set of tracks being revealed. So mathematics is both a body of established, recognised content and a process by which exploration and establishing takes place

There is mathematics to know
and mathematics to do

To be involved in mathematics, at whatever level, requires both content and process, otherwise the subject is being experienced in an unbalanced way

The very procedures which are part of the search behaviour required to pursue mathematics are those which are necessary to deal with problems in general. Earlier, four were listed:

- collecting, classifying, analysing and using information
- searching for relationships
- making and testing hypotheses
- discriminating between objective and subjective information.

To these could be added:

- using particular examples to give a "feel" for the problem (specialising)
- working systematically
- choosing a method by which to communicate with others the results of the search
- using verification techniques to "test" out the results
- deriving a statement about the perceived pattern of results (generalising)

These are some of the procedures which are necessary to deal with problems and which are specifically mathematical as well. If experience of them could be made available to pupils in the classroom it would appear to be most desirable from a mathematical and a general standpoint.

To make such experience possible requires a shift from a content dominated view of mathematics to a content/process view The purpose of a process lesson in mathematics is to emphasise for pupils the legitimate experience of problem solving procedures while simultaneously challenging them to concentrate upon the learning of new mathematical content. The mathematical content is thus subordinated to the mathematical processes. This inverts "normal" classroom procedure, which tends to legitimate facts and algorithms in mathematics and negate the meaningfulness of the experience of deriving mathematics. Attention can thus be focused on the problem solving process and its constituent procedures. Pupils can begin to develop problem solving awareness at the level of methodology, and also at the personal level as they confront their own feelings of competence and incompetence. They can use techniques which are helpful in overcoming the rigidities induced by negative feelings.

Once the shift from content to process has taken place, the fact that puzzles are enjoyable and that many of the problem solving procedures can be experienced through the medium of a puzzle can be used in the classroom if:

1. out of the puzzle arises a problem — out of context, open to definition and requiring more than just a perceptual shift;
2. there is open house on methods of tackling and on results;
3. pupils are expected to generate new questions where this is feasible;
4. pupils are asked to communicate their work to tell others what they have done;
5. the focus of attention of both teacher and pupils is on search behaviour.

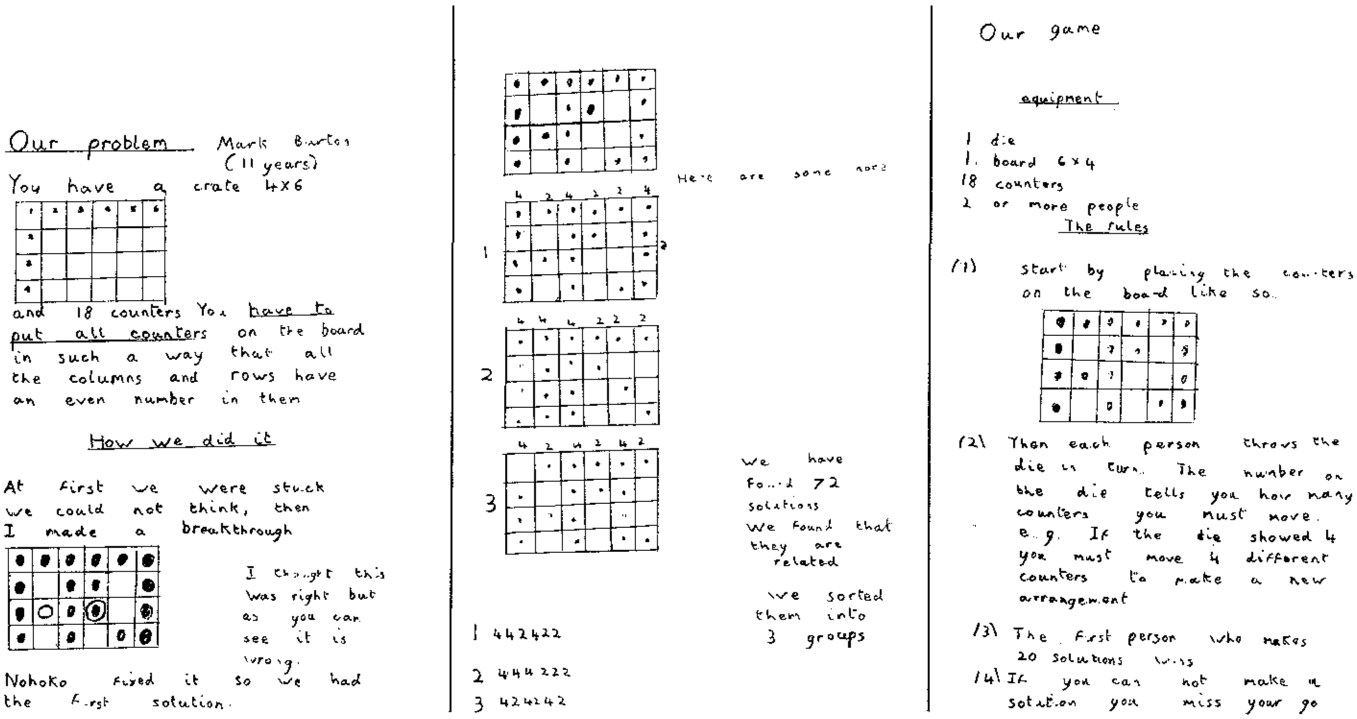
Example

A spatial puzzle which can meet the above criteria is;

> A milk crate holds 24 bottles in a rectangular array, four rows and six columns. Can you put 18 bottles of milk in the crate so that each row and each column of the crate has an even number of bottles in it?

This is a puzzle as long as the requirement is to find one arrangement of bottles satisfying the constraints. One pupil turned it into a problem by asking the question "Is that arrangement unique?" and, if not, "How many possible arrangements can be found?". Further, "Can these arrangements be grouped in any way to say that some are the same as others?" Finally, a game was invented using all the previous work.

The report below was the result of an extended piece of work done by two children, Mark and Nohoko, in a classroom offering time for the children to spend on problem solving as one aspect of their mathematical experience. The classroom is organized in such a way as to provide a problem corner, in which information relating to problem solving is displayed A problem a week is offered but the previous weeks' problems are not withdrawn so that, at any one time, groups of children can be working on extensions of previous problems, or choosing to work on the current week's problem. The most striking feature of the work of the children is the encouragement given to them to extend their work by asking questions which acknowledge change and interpretation. For example, in the work shown above, at one stage, the teacher had a discussion with the two children on symmetrical arrangements. She wished to exclude arrangements which could be obtained by a symmetry. At that stage, the two children did not wish to exclude such arrangements. They consequently sorted their arrangements labelling the group in which no symmetries were to be found as "Miss Osborne's types" (the teacher) and the



remaining group of arrangements resulting from a symmetry as their additional arrangements In the report as presented here, they themselves have excluded the symmetrical arrangements because of the logic of their own reasoning The teacher gave the children the space and time to arrive at that position themselves

Puzzles and problem

The thesis being advanced is that puzzle behaviour by teacher and pupils is not productive. However, the content of the puzzles does not have to be jettisoned. If the teacher is aware of the puzzle/problem distinction and its rationale, the emphasis will be changed from seeking a right answer to open search behaviour, from single solution to variation, comparison and evaluation of methods and resolutions, from meeting external requirements to becoming self-aware of one's own and consequently of others (different) thinking processes. Out of this environment springs motivation to learn mathematics. The key to making such a shift lies in the questions and expectations of teachers and pupils — the desire and interest to explore and understand. Both teaching and learning them become problem solving activities.

City Street Scenes II

LAWRENCE KUCHARZ

lights entering street night
empty dark summer sounds. walking dim silent man

summer night . dark empty
entering man. . . dim
walking silent lights. sounds street

dark walking man
empty sounds
summer. lights night-silent street
entering dim

silent man walking dim empty lights
dark . sounds night entering street summer

empty entering street-dark lights
dark . sounds night entering street summer

night-silent-dim, lights summer sounds
empty dark walking
street. entering light

dark empty summer night
man, entering silent street
dim lights. sounds walking

night, lights sounds
dark dim. entering
walking street man
empty summer . . silent

summer street entering night, sounds dark lights
empty dim walking man . silent

lights . . empty summer sounds. walking dim street
silent entering dark-night-man

walking dark night, . . street dim
entering silent man lights empty summer sounds

(11 of 48 word modules of City Street Scenes II)

original series (o) = dark empty summer. night man entering silent street dim lights sounds. walking .
retrograde form (r) = (o) in reverse order .
inversion form (i) = contour inversion or mirroring of (o) (contour created by arranging words in alphabetical order and using "word class number" as a contour determinant
(i) = complimentation (mod 12) of each word number of the series or (i) = (12-"word number" . .)
retrograde inversion (ri) = (i) in reverse order .
transposition . ((transposition (t) = adding (mod.12) an integer (transposition number 0-11) to each word number of the series . . or (t) = ("word number" + "transposition number")))

0	2	8	6	5	3	7	9	1	4	10	11	0	dark
10	0	6	4	3	1	5	7	11	2	8	9	1	dim
4	6	0	10	9	7	11	1	5	8	2	3	2	empty
6	8	2	0	11	9	1	3	7	10	4	5	3	entering
7	9	3	1	0	10	2	4	8	11	5	6	4	lights
9	11	5	3	2	0	4	6	10	1	7	8	5	man
5	7	1	11	10	8	0	2	6	9	3	4	6	night
3	5	11	9	8	6	10	0	4	7	1	2	7	silent
11	1	7	5	4	2	6	8	0	3	9	10	8	summer
8	10	4	2	1	11	3	5	9	0	6	7	9	street
2	4	10	8	7	5	9	11	3	6	0	1	10	sounds
1	3	9	7	6	4	8	10	2	5	11	0	11	walking

these series . . when translated back into words. . are projected into syntactic poetic lines . . . . these word modules. . . are then projected into a form of durations . . . . or TIME FORM . . . .

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