Learning from Mathematics Practice in Out-of-School Situations*

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Research in the last fifteen years has indicated a burgeoning interest in examining the mathematics practice in distinct cultures [e.g. Brenner, 1985; Lancy, 1983; Saxe, 1991] and everyday situations within cultures [e.g. Carraher, Carraher & Schliemann, 1985; de la Rocha, 1985; Harris, 1987; Lave, 1988]. Research on mathematics practice in distinct cultures has tended to look at the mathematics practice of a whole culture, whereas research on mathematics practice in everyday situations within cultures has focused on one situation or work context (e.g. grocery shopping, carpentry) within a culture. Some of this research [e.g. Carraher, Carraher & Schliemann, 1985] has contrasted mathematics practice in school with mathematics practice in everyday situations and noted the gap between these two.

Knowledge gained in out-of-school situations often develops out of activities which: (a) occur in a familiar setting, (b) are dilemma driven, (c) are goal directed, (d) use the learner's own natural language, and (e) often occur in an apprenticeship situation allowing for observation of the skill and thinking involved in expert performance [Lester, 1989]. Knowledge acquired in school all too often grows out of a transmission paradigm of instruction and is largely devoid of meaning (lack of context, relevance, specific goal). Resnick [1989] has argued that schools place too much emphasis on the transmission of syntax (procedures) rather than on the teaching of semantics (meaning) and this "discourages children from bringing their intuitions to bear on school learning tasks" [p. 166].

Students need in-school mathematical experiences to build on and formalize their mathematical knowledge gained in out-of-school situations. An important part of mathematical experience in school is the guidance and structure that can be provided by a teacher to help students make connections among mathematical ideas. Some researchers [e.g. Lave, Smith & Butler, 1989; Schoenfeld, 1989] have suggested that teachers should establish master-apprentice relationships with their students to help initiate them into the mathematics community (persons who are learners and doers of mathematics). Working with others toward common goals, being actively involved in doing mathematics, and discussing and refining mathematical ideas are all ways of being a part of the mathematics community. At present, however, many students become isolated from the mathematics community rather than becoming a part of it. A key reason for this isolation is the wide gap that exists between mathematics practice in school and in out-of-school situations.

My interest lies in working to close the gap between doing mathematics in school and doing mathematics in out-of-school situations. This article discusses suggestions for the learning and teaching of mathematics stemming from a study examining the mathematics practice of carpet layers. In this study [Masingila, 1992], I examined the mathematics concepts and processes used by a group of carpet layers, as well as the process of becoming an expert carpet layer via apprenticeship. I then compared: (a) the concept of measurement and the process of measuring as presented in six seventh- and eighth-grade textbooks with their occurrence in the carpet laying context, and (b) the school-based knowledge of general mathematics students with the experience-based knowledge of floor covering workers (carpet layers) in solving floor covering problems.

The following list of conclusions briefly summarizes my findings from this research:

1. Carpet layers engage in doing mathematics. That is, mathematics concepts are present in floor covering work and estimators and installers use mathematical processes as they solve problems they encounter in doing their gobs. Job situations are problematic because of the numerous constraints inherent in floor covering work. For example: (a) floor covering materials come in specified sizes (e.g. most carpet is 12′ wide, most tile is 1′ × 1′), (b) carpet in a room (and often throughout a building) must have the nap (the dense, fuzzy surface on carpet formed by fibers from the underlying material) running in the same direction, (c) consideration of seam placement is very important because of traffic patterns and the type of carpet being installed, and (d) tile must be laid to be lengthwise and widthwise symmetrical about the center of the room.
2. Apprenticeship serves as a good method of learning and teaching in the floor covering context.
3. Textbooks often do not provide constraint-filled situations to engage students in problem solving.
4. Students often have a narrow concept of area and a limited range of problem-solving skills and strategies, perhaps because they have not been exposed to constraint-filled problems that engage them in problem solving.

Suggestions from research

In reflecting on this study and other research examining mathematics practice in everyday situations, I have formed some suggestions for the following areas of mathematics

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education: (a) the school mathematics curriculum, (b) the methods used to teach school mathematics, and (c) research in mathematics education.

For the school mathematics curriculum

In comparing the school-based knowledge of general mathematics students with the experienced-based knowledge of the carpet layers I found that some of the difficulty the students had in solving the floor covering problems was related to the students' lack of exposure to rich, constraint-filled problems.

The main suggestion for the school mathematics curriculum stemming from research on mathematics practice in everyday situations is that the curriculum should include a wide variety of rich problems that: (a) build upon the mathematical understanding students have from their everyday experiences, and (b) engage students in doing mathematics in ways that are similar to doing mathematics in out-of-school situations At present, students are not encouraged to and maybe even discouraged from making connections between how they do mathematics in school and how they do mathematics out of school.

Students often do not see school mathematics as connected to the real world, and as a result they often do not evaluate solutions to school exercises to see if they make sense The following exercise, taken from the Third National Assessment of Educational Progress [Carpenter, Lindquist, Matthews & Silver, 1983], illustrates the fact that students often do not try to make sense of school mathematics:

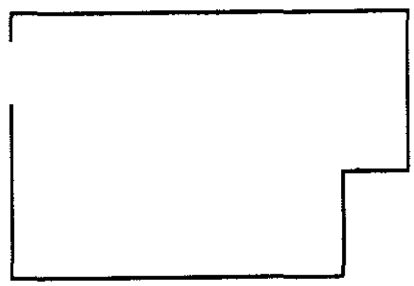
> An army bus holds 36 soldiers If 1128 soldiers are being bussed to their training site, how many buses are needed? [p 656]

Of the students who worked this exercise only 23% gave the correct answer of 32 buses, while 229% indicated that "31 remainder 12" was the number of buses needed. These students would never give this answer if they were thinking of this exercise in its real-world context. Instead, these students saw this exercise in the context of a school situation and thus they did not evaluate the sense of their answer.

Often students do try to bring their out-of-school experiences to bear upon work in school when the work engages them in problem solving. For example, several of the general mathematics students I worked with used personal experiences to give them insights into solving the problems. However, the present curriculum is frequently so divorced from students' out-of-school experiences that it is very difficult for students to connect the out-of-school world and the in-school world. Thus, doing mathematics in school often has little meaning for students.

Besides having problems that build upon the students' everyday mathematical experiences, the school mathematics curriculum should engage students in doing mathematics by providing them with rich, constraint-filled problems. Schoenfeld [1987] has suggested that what is needed in mathematics knowledge outside the school context" [p. 214] Consider the following exercise taken from a pre-algebra book: "Find the cost of carpeting a floor that is 15 feet by 12 feet at $24.95 per square yard" [Shulte & Peterson, 1986, p. 527]. Although this problem uses an everyday context, the absence of actual constraints makes the problem artificial; the textbook example is simply a computational exercise placed in the context of an everyday situation. The effort required by a student to find an answer to this exercise does not reflect the way mathematical knowledge is used in carpet laying Situations in a carpet laying context involve using measurement concepts in problem-solving situations compounded by real-life constraints.

Instead of stating the exercise as it is above, this example could be made into a problem solving situation by changing it to the following:



1 Find the measurements of this room The diagram is drawn in a scale of 1/8 inch = 1 foot.

2 Find the most cost-efficient way to carpet this room given the following constraints;

- carpet pieces are 12 feet wide;
- the nap of different carpet pieces must all run in the same direction;
- seams should be placed out of normal traffic patterns.

Not only does the above problem have real-world constraints, but it also engages the students in the processes of measuring and problem solving. The effort required by a student to find a solution to this problem does reflect the way mathematical knowledge is used in carpet laying. Problems of this type engage students in mathematics practice and encourage mathematical understanding of the concepts involved

Besides the suggestion that the school mathematics curriculum should include rich problems that build upon the out-of-school mathematical understandings of the students and engage them in mathematics practice, this study has some suggestions for the teaching of school mathematics.

For teaching school mathematics

The above discussion concerning changes in the school mathematics curriculum is all for naught if far more important changes do not occur in the chain of events that affects the way students learn mathematics in school The research into the mathematics practice in carpet laying and the way the general mathematics students approached the floor covering problems suggests three key ideas for teaching school mathematics.

The first is that teachers should build upon the mathematical knowledge that students' bring to school from their out-of-school situations As discussed above, students try to make connections between the in-school and out-of-school worlds, but the mathematics practice in school often discourages or prevents such connections.

The second suggestion arises out of the way in which floor-covering helpers learn about installation work and from the obvious lack of understanding of the concept of area that the general mathematics students demonstrated: Teachers should introduce mathematical ideas through situations that engage students in problem solving.

The third suggestion comes directly from the master-apprentice relationship that enables floor covering workers to become expert estimators and installers: Teachers should establish master-apprentice relationships with their students to guide students in doing mathematics and help initiate them into the mathematics community. Note that while the apprenticeship model used in work situations is a viable model for those contexts, the model needs to be adapted for use in the classroom. This adaptation will be discussed later in the article.

Building on students' out-of-school knowledge. All students bring to school mathematical knowledge from everyday situations they have experienced. This knowledge is often hidden and unused by the students in school as they learn to use the mathematical procedures that the teacher demonstrates and evaluates.

Just as the mathematics practice of everyday activity is ignored by teachers in school, mathematics practice in schools is likewise devalued by students because of their lack of use of it in out-of-school situations: "There exists no legitimate field of practice other than the classroom itself" [Lave, Smith & Butler, 1989, p. 74]. As D'Ambrosio [1985] has noted, people in out-of-school situations often use very different mathematical procedures and thinking processes than those taught in school.

However, if teachers engage their students in conversation, listen to them, and encourage and observe their informal methods of solving problems, much can be learned about the out-of-school situations their students have experienced. Lester [1989] offered several suggestions for helping students make connections between the in-school and out-of-school world. One suggestion was to have students create their own problems. Another suggestion was to encourage "students to solve problems in more than one way and to share their approaches with each other. By sharing approaches, students will learn about methods used by their peers and also that it is acceptable to use the informal methods they have developed on their own" [p. 34]. A third suggestion involved engaging students in mathematics projects that encourage the use of mathematical methods and reasoning.

By building upon the mathematical knowledge students bring to school from their everyday experiences, teachers can encourage students to: (a) make connections between these two worlds in a manner that will help formalize the students' informal mathematical knowledge, and (b) learn mathematics in a more meaningful, relevant way. "Mathematics teaching can be more effective and will yield more equal opportunities, provided it starts from and feeds on the cultural knowledge or cognitive background" of the students [Pinxten, 1989, p. 28].

Teaching via problem solving. Although each of the general mathematics students I engaged in solving some floor covering problems had been taught about area, none of them were able to use their knowledge of area to solve the following problems:

> Suppose you need a piece of carpet 12 feet by 9 feet. How many square yards should you order from the carpet supplier?

The students were able to change feet into yards. But when they were asked how many square yards were needed, they either found the perimeter or squared each dimension. When I asked what it meant to find square yards, none of the students provided an answer. When I finally identified the task as finding the area of the carpet piece, the students typically replied, "Oh, length times width."

To these students, area is a formula dependent upon the geometric shape of the object. Their understanding of the concept of area is narrow and strongly tied to algorithms for finding area. Because the students have not experienced finding area in a real-life way (at least not in school), they do not have an understanding of area that can be applied to concrete situations. On the other hand, the carpet layers work with area in concrete ways every day and are able to apply their knowledge of area to a wide variety of floor covering situations.

If the general mathematics students had been involved in solving problems where they encountered dilemmas that required them to find areas, their understanding of the concept of area would have been constructed out of their mathematics practice. This is the goal of teaching mathematics via problem solving. In teaching via problem solving, "problems are valued not only as a purpose for learning mathematics, but also as a primary means of doing so. The teaching of a mathematical topic begins with a problem situation that embodies key aspects of the topic, and mathematical techniques are developed as reasonable responses to reasonable problems" [Schroeder & Lester, 1989, p. 33].

This approach embodies three key recommendations of the *Curriculum and evaluation standards for school mathematics* (National Council of Teachers of Mathematics [NCTM, 1989]: (a) mathematics instruction should be carried out in an inquiry-oriented, problem-solving atmosphere, (b) mathematics concepts and skills should be learned in the context of solving problems, and (c) the development of higher-level thinking processes should be fostered through problem-solving experiences.

Teaching via problem solving deviates from the traditional instructional approach of the teacher presenting information (facts) and then assigning exercises in which students practice and apply this information. Using a teaching via problem solving approach means that the mathematical information arises out of the problem-solving activity, along with an understanding of the mathematical concepts and processes involved.

Teaching via problem solving is also consistent with the way in which apprentice floor-covering workers learn about estimating and installing. The apprentices are engaged in problem solving every day and gain mathematical knowledge (although they may not be aware of this) through solving these problems. An important aspect of this problem solving is that it occurs in the context of a master-apprentice relationship.

Using the apprenticeship model in the classroom. The floor covering helpers I observed became expert carpet layers by observing the installation process, questioning the installer, participating in the installation process, learning from their mistakes, and coming to know what the installer knows. The installers contribute to this process by maintaining control of the installation process, giving the helpers the opportunity to develop a feel for installation work, and determining the progress of the helper.

A number of researchers have discussed apprenticeship and its application to the classroom [Collins, Brown & Newman, 1987; Lave, 1977, 1988; Lave, Smith & Butler, 1989; Rogoff, 1990; Schoenfeld, 1987, 1989] and have found the apprenticeship model to be a viable one for teaching and learning. However, the apprenticeship model that could be used in a classroom is different in two important ways from the apprenticeship model used in work situations, and in particular in the carpet laying context.

The first difference involved the master-apprentice relationship: In the workplace, a master and apprentice are working one-on-one; in the classroom, a teacher and possibly 30 or more students are working together. In the workplace, the apprentice is guided and directed by the master as he or she participates in the work activity; in the classroom, the students are guided by the teacher, but more importantly are guided and challenged by other students as they work cooperatively in doing mathematics, Thus, applying the apprenticeship model to the classroom implies a heavy reliance on cooperative learning: The teacher creates a classroom environment where students (apprentices) work with other students (apprentices) and in this way the teacher helps initiate the students into the mathematics community.

A second difference between the use of the apprenticeship model in the workplace and in the mathematics classroom is that apprentices in the workplace are constructing situation-specific knowledge; in the mathematics classroom students are constructing mathematics content and processes that are more general, and hopefully can be applied to a variety of situations. The end goal of my suggestion that the school mathematics curriculum should contain rich, constraint-filled problems (e.g. problems from a carpet laying context) is not that students acquire the knowledge necessary to become expert carpet layers. Rather, problems of this type are vehicles for engaging students in doing mathematics and aiding them in developing the mathematical reasoning and problem-solving abilities used by expert problem solvers.

There are three key reasons for using the apprenticeship model in the mathematics classroom: (a) an apprenticeship model enables mathematical knowledge to be developed within a context, (b) cognitive development can occur as students work cooperatively with their teacher, and (c) a mathematics culture is developed within the classroom and students are initiated into this mathematics community

Using the apprenticeship model, mathematical knowledge is developed within a context and is framed by that context: A "general advantage of learning-as-apprenticeship is that it assumes that knowing, thinking, and indeed, problem-solving activity, are generated in practice, in situations whose specific characteristics are part of the practice as it unfolds" [Lave, Smith & Butler, 1989, p 64]. The apprenticeship model "implies continuity between ways of learning and thinking in school and nonschool settings: Learning is learning, mathematical thinking is mathematical thinking, in and out of school" [Scribner & Stevens, 1989, p. 1].

Vygotsky's concept of the zone of proximal development included the proposal that cognitive processes occur first on the social plane. The individual plane is then formed as these shared processes are internalized and transformed. Thus, the zone of proximal development is a dynamic region of sensitivity to learning the skills of culture, and children develop in this region through participation in problem-solving activities with more members of the culture [Rogoff, 1990; Vygotsky, 1978].

Not only do children develop mathematical knowledge by working closely with more experienced members of the culture, but they themselves become part of the mathematics community Using an apprenticeship model in the classroom provides a way for initiation into the mathematics community. Schoenfeld [1987] has noted that whereas schools traditionally provide "training in ready-made, prepacked mathematical procedures.....apprenticeship does more than provide training: it provides an initiation into a culture" [p. 204].

Thus, using the apprenticeship model in the mathematics classroom allows students to do mathematics in a natural way and be active learners in a "community of people who support, challenge, and guide novices as they increasingly participate in skilled, valued sociocultural activity" [Rogoff, 1990, p 39].

Research in mathematics education

The examination of mathematics practice in carpet laying has several suggestions for future research in mathematics education First, there is a need for further research on mathematics practice in everyday situations to add to the growing body of knowledge Everyday situations other than the ones already researched could be examined to broaden the knowledge of mathematics practice in everyday situations.

To extend this study, other measurement situations could be examined (e.g. wallpapering work, bricklaying work) to see the mathematical concepts and processes used in these contexts. In addition, problems from one everyday measurement situation (e.g. floor-covering work) could be given to workers in a different measurement context (wallpapering work) to see if the individuals transfer problem-solving strategies and measurement concepts from a familiar situation to an unfamiliar one. There is also a need for research on children's mathematics practice in out-of-school situations Research in both of these areas can enable the mathematics community to develop and teach school mathematics in a way that builds upon the students' out-of-school mathematical knowledge. Case studies of classroom teaching (at different levels) that use the apprenticeship model are also needed to help teachers at every level (elementary through university) develop and implement this model in their own classrooms.

Summary

In this article I have offered suggestions related to: (a) the school mathematics curriculum, (b) how school mathematics is taught, and (c) research in mathematics education. Overriding all of these suggestions is that in order for students to learn mathematics meaningfully, the gap between doing mathematics in school and doing mathematics in out-of-school situations must be reduced

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Behind the mountains there also dwell people. Be modest. You have never invented or discovered anything that others have not invented or discovered before you. And even if you have, consider it as a gift from above which it is your duty to share with others.

Robert Schumann
