INTERSECTING VIEWS ON LEARNING MATHEMATICS: TDS AND TDI

ELENA POLOTSKAIA

The *Race to Twenty* (course à vingt) is a game in which players take turns to take one or two chips from a line of twenty chips. The player who takes the twentieth wins the game. Brousseau (2004) proposes a didactical scenario involving this game to enable mathematical learning for students. How can we understand the didactical mechanisms involved from the perspective of two different theories? I propose to examine the Theory of Didactical Situations (Théorie des situation didactiques, TDS, Brousseau, 2004) and the Theory of Developmental Instruction [1] (TDI, Davydov, 2008) through the example of the Race to Twenty.

Distinct origins and goals

In a few words, the Theory of Developmental Instruction has its roots in the work of Vygotsky on the socio-historical nature of teaching and learning as socio-cultural phenomena. In his book, Davydov integrates and synthesises the theoretical work of several researchers over 40 years [2]. He uses the concept of 'activity' [3] (деятельность, *deiatelnost*, which describes any human activity with a form and structure developed by a society in the course of its history) and the concept of 'learning activity' and even 'conscious learning' (which according to him implies 'theoretical thinking' on the part of the student). According to the theory, this type of activity must be developed in students to ensure effective learning and, above all, intellectual and personal development. TDI pays great attention to the analysis of the cognitive and historical genesis (epistemological analysis) of mathematical concepts in order to design a specific curriculum for the development of conscious learning activity in learners.

Since the 1970s, Brousseau's work has given rise to the Theory of Didactical Situations, which has become a core research area in didactique des mathématiques in the francophone world and elsewhere. Through the study of mathematics classrooms, Brousseau seeks to model teaching by separating it into situations that allow the acquisition of knowledge. The concept of the 'didactical situation' and various essential components related to the organisation of learning, found in TDS, have gained prominence in didactical research. A didactical situation is generally understood as a set of relationships (related to learning) established explicitly or implicitly between students and their environment (*e.g.,* educational system, teacher's behaviour, the task, *etc.*) in the learning process. The observation and analysis of teaching practices from this perspective have given rise to several important didactical concepts: a-didactical situation, milieu, didactical contract, didactical time, didactical memory, didactical engineering, *etc*.

Radford suggests that "even the simplest connection [between theories] requires *dialogue*" (2008, p. 318), and dialogue is unimaginable without a common language. The first step is to ensure that we understand the meaning attributed to the central concepts of each theory. For example, Davydov talks about knowledge *transfer* and Brousseau mentions knowledge *construction*. Are these conflicting understandings of the learning phenomenon? I do not intend to answer this question fully here, but more modestly to compare a few concepts using the context of the Race to Twenty. I would like to see to what extent one can jointly interpret the concepts proposed by the two theories without running into major obstacles. This exercise will allow us to see how these two theories shed light on certain phenomena in different ways, and to identify points (of divergence?) to explore.

Some basic concepts

It can be seen that TDI and TDS are very distinct in terms of their origins and their objectives. TDS is a 'didactical' theory born in the search for answers to the unsatisfactory processes of teaching mathematics in the classroom (which is a specific goal), whereas TDI is a 'psychological' theory whose goal is the personal development of the child as a future citizen (a very broad goal). This difference does not prevent the two theories from intersecting at several points. Let us take a closer look at how some of the concepts used by these two theories compare to highlight important nuances.

Thinking and abstraction

Thinking and abstraction are at the heart of mathematics and mathematics teaching. An activity such as the Race to Twenty supports both, because it engages players in mathematical thinking about how the game works. But what can be said about this thinking?

TDI distinguishes between empirical and theoretical thinking. To explain empirical thinking, Davydov writes:

> [empirical thinking is] interpreting the general merely as that which is identical and similar in some group of objects, [...] interpreting the essential merely as the distinguishing attribute of a class of objects. (2008, p. 76)

Thus, according to Davydov, empirical thinking is a generalisation from observable attributes of the objects under study. This generalisation serves to distinguish one class of objects from another. Empirical thinking does not seek to understand the internal and external relationships between

Authorised translation of Polotskaia, E. (2020) Regards croisés sur l'apprentissage des mathématiques : TSD et TID
For the Learning of Mathematics **40**, 1

attributes, as these relationships are not directly observable in a static object that is not developing or interacting with other objects. This is the role of theoretical thinking, which seeks to understand the object rather than describe it. Theoretical thinking tries to explain the origin and the purpose of the object, why it exists as it does, and how it relates to and interacts with the world. According to Davydov, theoretical thinking is the process of making visible what is invisible, the objectification of the implicit relations of the object and its structure. It is theoretical thinking that primarily supports the psychological development of children and enables the growth of their cognitive abilities. This distinction between theoretical and empirical thinking seems similar to the one proposed by Piaget (1978) when he distinguishes 'empirical' abstraction from 'reflective' abstraction.

In the case of the Race to Twenty, TDI invites us to distinguish the recognition of facts (*e.g.,* "whoever wins takes the 20, the 17, the 14, *etc*.") from the recognition of the mathematical relationship between these numbers (*e.g.,* that these numbers are equivalent modulo 3). The first is the result of empirical thinking. The second is the result of theoretical thinking.

The process of abstraction evoked by Brousseau's TDS reflects a similar idea to theoretical thinking. Discussing Dienes' ideas, Brousseau accepts that :

> L'abstraction consiste alors à identifier, en tant qu'objet de connaissance la « structure commune » à divers jeux isomorphes. La structure est ici l'ensemble des propriétés qui, indépendamment des particularités de chaque exemple, les régissent tous. (2004, p. 192) [a]

This definition of abstraction appeals to the idea of structure: for example, to the set of rules that govern a game such as the Race to Twenty. However, it is not clear which properties are observable and which are not, and whether the set includes internal and external relationships. TDS places great emphasis on what is considered to be the *raison d'être* of the game: in the Race to Twenty, the rules of the game entirely determine the knowledge potentially developed in playing. From this point of view, then, TDI makes some interesting distinctions compared to TDS. It invites us to revisit what we believe students should be getting out of a situation by recognising the part played by empirical thinking.

In the light of TDI, the idea of empirical thinking leads us to clearly distinguish between the rules of the game, which are explicit and immediately available, and the internal mathematical laws (divisibility/comparability) which are implicit and 'to be understood'. Indeed, one can always win at the Race to Twenty using empirical knowledge of the 'winning' numbers, which are based on the rules of this particular version of the game. One can empirically work out the sequence of numbers and thus simply learn to produce the series 2, 5, 8, *etc*. in order to win. But this is obviously not the intended knowledge.

What is this knowledge? In his book, Brousseau talks about the Race to Twenty as a way of approaching the concept of division as repeated subtraction (with a remainder), and for the introduction of the culture of proof in mathematics, insofar as the game is played by asking the students to formulate and justify a winning strategy. From the point of view of TDI, on the other hand, in addition to the proof, it is above all the mathematical relationship between the 'winning' numbers that would be the reason for the game's use in a classroom: we can indeed observe how these numbers are equivalent in modular arithmetic, a principle that can be generalised to other numbers or rules. The two notions are obviously related, but TDI, talking in terms of theoretical thinking, calls for a broader reflection on the relationships involved. We will see later how this, in conjunction with the particular orientations of each of the theories (discussed in the previous section), relates to a certain vision of child development.

Concepts (or notions)

In TDS, the idea of a structure and rules governing a game (or problem) implies the existence of a process of assimilating this structure or rules, and thereby developing mathematical *notions/concepts* [4] in the students who play the game (or solve the problem). Davydov proposes to look at the concept from two perspectives: as a form of mental reflection and as a form of human activity. He explains:

> Here a concept is a form of thinking activity that reproduces an idealized object together with its system of links. [...] The concept is simultaneously both a form of reflection of the material object and a means for mentally reproducing or constructing it. (2008, pp. 90–91)

According to Davydov, the concept as a form of mental reflection is an instrument that the thinker uses to understand and operate on the object. The concept as a form of human activity, on the other hand, allows Davydov to establish a genetic relationship between (scientific) knowledge and its development within a historical process. Once developed, knowledge becomes part of culture, begins to determine human activities and becomes their form. Davydov argues that humans, and specifically children, have the ability to grasp the form of an activity and transform it into a form of mental reflection. For example, children appropriate social and moral concepts by participating in social and cultural activities. Their actions in these activities are transformed into thought actions. Concretely, this is recognised in the part of the Race to Twenty that is linked to proof. Within this game, students are called upon to validate and justify their strategies. Participating in this part of the scenario can form the concept of mathematical proof as an element of mathematical culture, even if the concept of proof is not formally reflected upon.

In a similar framework, Brousseau uses the terms *notion* and *theory*. In TDS, a notion represents a key, a mental instrument for addressing (understanding and solving) a set of situations sharing important characteristics. In validation and institutionalisation situations, students discuss their (partial or even incorrect) theories in order to arrive at a common institutionalised understanding. Brousseau explains the emergence of a notion for the learner as follows:

> Mais pour que ces théories aient un sens pour celui qui les utilise, il « faut » qu'elles aient préalablement fonctionné comme solution à un problème posé à chaque

élève dans des conditions qui lui permettent, soit de trouver lui-meme cette solution, ou plus exactement de la construire (éventuellement progressivement), soit de l'emprunter toute faite, de lui-meme, entre plusieurs qu'il pouvait envisager sans qu'une intention didactique ou une pression culturelle l'y contraigne en se substituant à son jugement. (2004, p. 218) [b]

We can see that what interests Brousseau most is particular knowledge. In the case of the Race to Twenty, Brousseau talks about strategies to win the game, and it is division (by repeated subtraction) that is targeted. TDS explicitly states that knowledge can emerge in the learner through interactions with situations with this notion as a solution. However, Brousseau recognises that the mere participation in the game (action situation), although it occupies the important place in TDS, is not sufficient to provoke a generalisation and the formation of the concept. Formulation-validation-institutionalisation situations, managed by the teacher, are necessary. The individual student's action in learning is treated differently in TDI.

The distinction probably stems from the fact that Brousseau very clearly suggests a detachment, for him, between individual action and social activity, insisting on the absence of 'cultural pressure' (more precisely, pressure from the teacher) in the action situation. From the point of view of TDI, this separation is not possible, as it is the teacher who is responsible for guiding the student towards theoretical thinking, so that the student does not remain at a superficial level in relation to the object of study. On the other hand, the concrete actions of the student are not necessarily targeted by this orientation. In general, individuals' actions are *always* part of some form of activity (or situation) that puts pressure on them. However, this pressure may direct them towards various goals. For example, the teacher may suggest a systematic study of the Race to Twenty rather than proposing to play several games in the hope that the students will discover something for themselves. Nevertheless, it is possible to think that the different moments identified by TDS (*e.g.,* institutionalisation) are there to provide this orientation and this 'cultural pressure' so valued by TDI.

We note then that TDI does not necessarily advocate that each student, or the group, should 'figure things out for themselves'. To clarify this difference, we will discuss the nuances in the way the two theories recognise the situation and the structured activity as the main sources of knowledge development in the learner.

Knowledge and its development

According to TDS, the process of developing particular knowledge (read *concept*) in students consists of several consecutive stages: action situation, communication situation, validation situation and institutionalisation situation. In the action situation, the learner has to construct a mental tool to solve one or more concrete problems. In the Race to Twenty, the game is played until some students discover what they have to do in order to win. Then, in the communication situation, the learner has to recognise this tool as valuable enough to formulate it explicitly and communicate it to others; the learner has to explain how to win. The validation situation allows the learner to explicitly and logically confirm the validity of the conception; the learner explains and validate the strategy mathematically. Finally, in the institutionalisation situation, the learner should modify their conception to fit a culturally established form, as well as relate it to other existing notions (*e.g.,* the notion of division with whole numbers more generally). Working within the framework of TDS consists of establishing a development of knowledge in the form of a tool for solving a concrete problem before studying the concept itself as an object of knowledge. There appears to be a progression from the particular to the more general.

The way the learning process is organised in TDI differs formally from the description given by TDS. Davydov suggests that a child can grasp the general idea directly from culturally and historically structured human activity. For this reason, TDI suggests that the acquisition of a concept must begin with the grasp of its most general form. Thus, from the perspective of TDI one can view the Race to Twenty as an activity whose purpose is to immerse the student in the mathematical culture of proof. The learning of a concept such as division should, according to this theory, be carried out in a different way. It should be situated in a continuum rooted in the concept of number, itself thought of more generally through the notion of magnitude (indeterminate quantity) and the relations between magnitudes. The notion of magnitude is then to be worked on with the idea of comparisons (additive and multiplicative) between physical objects (and their measurable properties, including length, area, volume or weight). Thus, 'operations' and relationships are already addressed without numbers. For example, a law of commutativity can be grasped and formulated from the manipulation of objects of various lengths, and then, at some point, applied to numbers or algebraic expressions. The Race to Twenty would then not be an opportunity for 'discovery', but rather for the enrichment of a certain form of thinking.

> The children first discover the initial general relation in some field of study. Then they use it to construct a contentful generalization, and then they use that to determine the content of the "cell" of the subject, converting it into a means for deriving other, more particular relations—i.e. into a concept. (Davydov, 2008, p. 122)

However, a closer look at TDI reveals that the 'general-particular' difference regarding the order of development of a concept is rather superficial from the point of view of didactical scenarios. Both theories advocate the development of new knowledge starting with concrete activities in particular contexts. The difference mainly concerns the explicit choice of the most generalisable form of the concept (and thus the context of the activity to be used at the beginning) that TDI advocates. It is from this particular context (an activity with a particular form) that the theoretical study of the concept takes place. This theoretical study is always based on a practical activity (such as comparing objects, or playing a game) and seems to join several aspects of the formulation-validation-institutionalisation of TDS but without having the particular language that allows TDS to cut them up so well and to describe the didactical role of each type of situation. Proponents of TDI will nonetheless stress that a situation like the Race to Twenty does not allow the student to directly grasp the mathematical relation at the origin of the aimed concept; the repeated subtraction does not represent the true character of the relation of comparability (by modulo) in spite of the link between the two. It should also be noted that it does not engage the student in realising the limits of his or her knowledge on this subject from the outset. Let us discuss this in a little more detail.

An important element from the point of view of knowledge and its development that distinguishes the two theories is the emphasis on 'ruptures'. TDS pays special attention to the phenomenon of socio-cognitive conflict and learning obstacles that may arise when encountering a new concept. According to Brousseau, students may 'naturally' form and use conceptions that may conflict with the desired learning and thus represent an obstacle. TDS states that learning situations (including devolution and peer validation) should be specifically designed to minimise obstacles, while recognising that some cognitive conflicts are inevitable.

With regard to the idea of a progression from the general to the particular, TDI approaches this differently. In the context of his studies, Davydov carried out such a teaching project, which today is known as the El'konin-Davydov curriculum (Schmittau, 2011). This curriculum is constructed, according to TDI, to allow for work on genetically more general concepts first. The other, more specific, derived knowledge can then be developed later as an enrichment of the basic general thinking without causing ruptures in meaning. For example, the concept of number is presented from the outset from the point of view of properties that are common to all types of numbers and quantities (order, equality, *etc*.). They are first constructed without numbers, in the context of the magnitudes of physical objects. The properties of natural numbers will therefore be 'derived' from the more general properties of quantities.

According to this logic, as I said above, the Race to Twenty would be offered to students at a time when students have already mastered the concept of multiplicative relationships on quantities in its general and abstract form. Students would also have a broad understanding of the base 10 positional system, seen as one of several base systems. Thus, the game could be an opportunity to look, for example, at how the winning numbers of the Race to Twenty are written in base 3 (202, 122, 112, *etc*.), thus noticing a regularity that could be another interesting mathematical exploration.

In a curriculum where multiplicative relationships have not been worked on beforehand on the basis of the notion of measurement, students introduced to the Race to Twenty could be naturally inclined to adopt an additive view of division (repeated subtraction), and subsequently develop cognitive ruptures (when this model no longer works, for example in the case of multiplicative comparisons). In short, it is in the general organisation of the curriculum that the question of cognitive conflicts or logical ruptures is essentially addressed by TDI. This theory does not therefore propose didactical tools to remedy a rupture, if such a situation occurs in the learning process.

TDS obviously recognises the value of reflection about the order in which knowledge is taught. Speaking of decimals, Brousseau states:

> Nous avons vu les insuffisances de cette conception « machinale » et la nécessité de placer cet usage sous le controle d'une compréhension et meme, dés que possible, d'un savoir rationnel. (2004, p. 291) [c]

In this quote, Brousseau proposes to prioritise the general concept of rational numbers over the decimal notation of numbers. This seems to be in line with the idea of TDI, which places the most general form of a concept at the beginning and then derives concrete instances of it (in various mathematical contexts, such as integers, rational or real numbers). In this way, particular instances of a concept will not contradict the general form of the knowledge originally established and will not act as a barrier to learning. Not taking the form of a global theory of the development of mathematical ideas, TDS must therefore (a posteriori) determine when such and such a situation would be 'best placed', and its project is often, in fact, to develop situations to meet the need for pre-established curricula. On the other hand, TDI does not allow for the development of situations without a deep reflection and a clear proposal on the whole curriculum.

The 'forces' that drive the student's learning

Davydov states that the 'abstract to concrete' approach to learning (which is about curriculum design rather than lesson design) contributes to the development of theoretical thinking in students. Let us now consider how this approach differs from TDS in its view of the role of the student, the task and the teacher in an educational context.

The student's internal forces

The concrete design of a didactical situation and its practical management in the classroom are central to TDS. Many of these didactical situations are designed as games that students have to win or problems that they have to solve. This implies that one of the necessary elements for successful learning is the natural disposition of students to want to win a game or solve a problem. In the case of the Race to Twenty, the desire to win is supposed to motivate students to seek winning strategies.

TDI, too, tends to find the main force of learning in the students themselves. But Davydov states that older preschool children show an increased desire for learning itself, a socially valued activity. The child would thus have an early desire to learn and nurturing the ability to learn is at the heart of TDI. The main objective of primary education is to transform the interest in learning into a conscious learning activity, which includes for example the conscious use of theoretical thinking. Ideally, by the end of primary school, students should still like to learn, and also know how to do it. Learning becomes their 'professional' activity, just as hairdressing is an activity of hairdressers, or selling is an activity of merchants. From this point of view, the Race to Twenty does not invite the students to focus on the learning part from the beginning, but rather tries to let them tame the game in a personal way.

TDS does not explain how and why students would develop their educational work skills so that they could eventually move into an adult learning mode (at university level for example). TDI, on the other hand, does not explain the value of the familiarisation stage with the context used to work on a concept. It can be said that on the TDS side, rules and contracts are the main didactical forces that feed the student's natural disposition to play or solve. According to TDI, play is only the initial form of learning that must then be developed into the conscious learning activity. Using Brousseau's terminology, we could say that Davydov is asking for the contract of play to be gradually replaced by a contract of conscious and responsible learning. Curiosity and desire to play or solve still remain, but they are now regulated by conscious learning. However, this difference can be seen as a complementarity. In the case of the Race to Twenty, it seems to me that both theories can be used simultaneously to describe and explain the gradual transition from play to conscious learning within the same activity. TDS describes well the didactical components, in particular the action-communication-validation-institutionalisation situations, which make the transformation of the didactical contract possible, and TDI explains the global direction of the didactical effort in order for the transformation to take place.

The structure of the task

The development of theoretical thinking within TDI is framed by the conscious learning activity. To enable the development of this activity in students, TDI offers a new form of task, Учебная задача (*uchebnaia zadacha* or learning problem), which students, accompanied by the teacher, usually solve in the classroom. This task may include a problem to solve or a question to answer—a core task. However, the real task that the students have to perform is the theoretical analysis of the problem and the creation of a new theory that will then enable the solving of a set of problems.

Working with a learning problem can in some ways be similar to working with a fundamental situation in TDS, which Brousseau describes as:

> les problèmes posés par une situation à la mise en œuvre d'un modèle (implicite ou explicite) préexistant, ou par une théorie à la prise d'une décision, provoquent l'évolution, la reprise ou le rejet et la formation des théories. (2004, p. 275) [d]

In fact, both Brousseau and Davydov propose to start with a problem and end with a (partial) theory. Brousseau's cycle of action-formulation-communication-validation-institutionalisation has the same objective as Davydov's call to create new understandings in a culturally shared theoretical form. So there do not seem to be any major differences from this point of view. This is probably what allows us to use the Race to Twenty as an example here. In the Race to Twenty, we find what both theories advocate: a core task (the game) and a part designed to theorise and generalise students' conceptions. But let us see what the teacher might do with the task in the classroom.

The role of the teacher

TDI identifies as a major problem in Russian schools the tendency of the majority of mathematics teachers to encourage empirical thinking in students at the expense of developing their theoretical thinking. Davydov explains that the mathematical concepts they are trying to teach lack foundation. For example, the origin of multiplication/division is to be found in the change of the unit of measurement of a quantity, while repeated subtraction is one of the multiple calculation methods for the division operation. Teaching based on repeated addition (or subtraction) creates a logical gap between the concept of multiplication/division and its origin in mathematical culture (the understanding of mathematicians), which would prevent a complete and deep understanding among learners [5]. According to Davydov:

> The main point, however, does not have to do with any particular deficiencies [of instruction]—but with the fact that in school teaching the divide between mathematical concepts and their origin leads to a complete lack of logic and order in the mathematics curriculum. (2008, p. 81)

For Davydov, teachers and students are thus victims of a poorly designed curriculum. And once a suitable curriculum for the development of theoretical reasoning is established, the teacher must take on the crucial role of guardian of the mathematical authenticity of the classroom activity. Davydov, therefore, sees the main problem not in the pedagogy, but in the curriculum.

On the contrary, Brousseau attributes students' difficulties in understanding mainly to the pedagogy employed by teachers (at that time, in France). Brousseau writes:

> Dans les situations de recherche, la pédagogie classique conduit le maitre à « exploiter » immédiatement ou presque la « bonne » déclaration. Il parle avec le premier ou un des premiers « qui trouvent ». Finalement. les échanges concernent 20% des enfants (les plus « vifs »). (2004, p. 269) [e]

According to Brousseau, this pedagogy does not support students' active participation and cognitive investment during classroom discussions, thus compromising the construction of their knowledge. TDS puts forward several theoretical concepts to explain what happens in the classroom and to enable the teacher to better manage students' learning.

One of the basic concepts of TDS is the *didactical contract*, which can be compared to the rules of a game. Some rules are explicitly communicated to the players, while others are implicit. Each participant in a didactical situation develops his or her own understanding of the didactical contract by using the explicit rules and reconstructing those that are implicit. For Brousseau, a didactical contract is what most affects the knowledge development process: "C'est à mon avis l'existence du contrat didactique qui assure le fonctionnement du processus, et non une quelconque loi de la genèse de la connaissance" (2004, p. 193) [f]. Thus, the success of learning with the Race to Twenty depends on the spoken and unspoken rules that the teacher establishes in class and not only on the design of the situations that make up the activity.

In TDS, the *milieu* supports and modifies the so-called traditional role of the teacher (*e.g.,* teaching explicitly) in the learning process. According to Brousseau, the milieu can be a physical environment (*e.g.,* a computer) or a logical environment (*e.g.,* a game) that reacts to the students' actions. The milieu helps the teacher to engage students in autonomous learning, which Brousseau calls devolution: "Dans la didactique moderne, l'enseignement est la dévolution à l'élève d'une situation a-didactique, correcte, l'apprentissage est une adaptation à cette situation" (p. 60) [g]. The Race to Twenty invites the student to apply a strategy, and the milieu—the rules of the game together with the strategies of the other players—determines whether the student wins or loses. The teacher and the milieu complement each other throughout the sequence (devolution-formation-validation-institutionalisation) in order for knowledge to emerge.

We can therefore see that in TDS, the teacher's role is to enable students to engage in autonomous discovery and in the construction of knowledge from the first interaction with the milieu (action situation). And Brousseau even explicitly prohibits the teacher from influencing the students' judgement during their first interactions with the milieu. Interaction with the teacher takes place in the communication and validation situations, and especially in the institutionalisation situation where the teacher plays the role of expert by providing culturally approved forms of expression for knowledge acquired.

For TDI, there is little detail on the role of the teacher (at least, in the texts I consulted). However, it can be understood that the teacher's role is to accompany students in solving a carefully selected learning problem, and to suggest, if necessary, a culturally developed tool to enable students to progress in their theoretical exploration of the problem (see, for example, Davydov & Kerr, 1995). My reading of Davydov suggests that, from the outset of a learning activity, the teacher explicitly invites students to analyse and model the problem in order to derive new knowledge by drawing on their theoretical thinking. In fact, the teacher is the custodian of mathematical culture within the mathematical activity that students undertake. This (newly created) knowledge is therefore formulated immediately in a culturally approved form.

Since the role of the teacher differs between the two theorists, the orientation of students during the learning process is also distinct. Although Davydov and Brousseau aim at the cognitive involvement of students during the learning process, for Davydov the initial orientation is towards theoretical thinking and the search for a general relationship is crucial for effective and efficient learning. For Brousseau, it is the student's free exploration in the milieu, followed by negotiation between peers that ensures this effectiveness. There is a real point of divergence here, as personal discovery is not a valued concept in TDI, which instead advocates the formation of knowledge in the student through theoretical reasoning and the use of tools available in the mathematical culture. It might be interesting to explore this point of divergence further.

Conclusion

I have analysed here some concepts of the two theories in order to find points of intersection and divergence. My reading of the two authors gives me the impression that their statements are often formally opposed. Nevertheless, I believe I have managed to coordinate the two readings in a sensible way, and to find important links.

In both theories, learning begins with a concrete activity to derive knowledge consistent with mathematical culture. The form of the specially designed activity plays a major role in learning as well as the teacher's interventions (or lack thereof). On the other hand, both authors often approach these commonalities from different perspectives: one from the point of view of didactical engineering and the other from the point of view of child development. TDS speaks more about the concrete didactical components to be used in the learning activity and TDI rather indicates the overall direction to be taken in the activity and in the curriculum. This suggests to me a complementarity of the two theories.

An important point of divergence, however, is identified in relation to the level of 'freedom' of the student in their personal exploration within a learning activity. TDI is positioned as a theory of child development in school learning, including the learning of mathematics. It seeks to explain the principles of transformation of a student into a competent learner. It therefore does not seek to describe the precise parameters of learning activities that may go so far as to make certain forms of thinking inevitable, but rather to ensure that the more general notions are acquired before the multiple concrete knowledge that is derived from them. The 'engineering' aspect is therefore less present in TDI than in TDS. TDS, on the other hand, values the student's free exploration in the milieu. Could this divergence (and especially the formulations found in the books) be the effect of different scientific cultures? Is it enough to question their possible complementarity? To me, it would seem not, but perhaps the adaptation of these theories in the classroom could teach us more. What do you think?

Notes

[1] The name *Theory of Developmental Instruction* was chosen by Vasily Davydov; it is a direct translation of the title of his last book: Davydov, V. V. (1996). Теория развивающего обучения. Moscow: INTOR. Formally, TDI was born in the field of psychology, because traditionally in Russia the field 'didactique des mathématiques' did not exist. However, Davydov and his colleagues were concerned with didactical issues. This allowed them, in addition to the creation of a theory, to design an original mathematics curriculum for the primary level, and very interesting didactical devices.

[2] Besides the cultural-historical theory of the development of the higher mental functions of L.S. Vygotsky, we can name A.N. Leontiev's activity theory, P. Galperin's theory of formation of mental acts, D.B. El'konin's periodisation of the mental development of children, and many others.

[3] I highlight the terms I will discuss later in the text.

[4] It is difficult to make a clear distinction between notion and concept by their use in the texts.

[5] For an interesting discussion of this topic see the article by Maffia and Mariotti in FLM **38**(3).

Translators' Notes

[a] Abstraction consists thus of identifying as an object of knowledge the "structure common to" diverse isomorphic games. Structure is the set of properties which, independently of the particularities of each example, govern them all. (Brousseau, 2002, p. 140)

[b] But if these theories are to have meaning for those using them, they "must" previously have functioned as solutions to problems given to each student in conditions which allow her either to find the solution by herself—or more exactly to construct it (possibly progressively)—or to borrow

it ready-made from among a number she could envisage, without any didactical intention or cultural pressure compelling her by substituting itself for her judgement. (Brousseau, 2002, p. 162)

[c] We have seen the inadequacies of this “mechanical” conception and the necessity of placing this use under the control of an understanding and even, as early as possible, of rational knowledge. (Brousseau, 2002, p. 223).

[d] Problems posed by a situation at the time of putting a pre-existing model (implicit or explicit) to work, or by a theory at the time of making of a decision provoke the evolution, the modification or the rejection and the formulation of theories. (Brousseau, 2002, p. 211)

[e] In situations of investigation, classical pedagogy leads the teacher to “exploit” a “good” statement almost immediately. She speaks to the first (or one of the first) children who “finds it”. In the end, the exchanges involve 20% of the children (the most “alert”). (Brousseau, 2002, p. 206)

[f] In my opinion the existence of the didactical contract is what assures the functioning of the process, and not an arbitrary law of the genesis of knowledge (Brousseau, 2002, p. 141).

[g] In modern *didactique*, teaching is the devolution to the student of an adidactical, appropriate situation; learning is the student’s adaptation to this situation” (Brousseau, 2002, p. 31).

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