

THIS IS NOT MATHEMATICS

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Many analyses of mathematics classroom communication distinguish mathematical discourse from non-mathematical discourse. For example, O'Halloran's (2005) lexicon of the mathematics register separated mathematical content and social organization. Similarly, Sierpiska (1997) differentiated the 'mathematical layer' from the 'didactic layer'. Setati (2005) had more nuance in her frame that she said "emerged from the data" (p. 449). She identified two 'mathematical discourses', namely conceptual and procedural discourses, and two 'non-mathematical discourses', namely regulatory and contextual discourses. Typically, this kind of distinction focuses on teacher talk which generally dominates classroom discourse.

We are not surprised at this common conceptual move to distinguish between mathematical and non-mathematical talk, because many researchers of mathematics education, like us, are former mathematics teachers. Recalling our experiences teaching mathematics, it was a distinction that seemed quite clear to us when we were trying to organize our classrooms and when we were trying to develop mathematics. When we thought students should be doing mathematics, we considered them 'off task' if they talked about anything else. We were responsible for developing mathematics, and thus quite resistant to other discourses.

This distinction between mathematical and non-mathematical communication for mathematics teachers is easily carried into analysis done by former mathematics teachers; we are predisposed to notice such apparently natural distinctions. Indeed, we have noticed ourselves gravitating toward the mathematical/non-mathematical distinction when we have analyzed classroom texts. For example, this was attempted in the first stages of analysis of instances of the word *just* in mathematics classroom interaction (Wagner & Herbel-Eisenmann, 2008) but abandoned because so many instances were inconclusive. When a teacher tells a student to "just use a calculator," for instance, it could be to avoid an argument among students or there could be a mathematical reason. The intention may be clear to the teacher, but it is unclear for students.

In this article, we challenge the distinction made by researchers between mathematical and non-mathematical talk and the corresponding distinction we have heard often (and used ourselves) in mathematics teaching between students being 'on task' or 'off task'. In our recent work in a research project exploring mathematics students' communication in the classroom, dialogue among French Immersion (FI) student participants provoked us to question our apparently natural inclination to judge communication as mathematical or not mathematical, and then to consider implications of favoring 'on task' mathematics.

We begin with some consideration of theory that addresses this distinction. Next, we illustrate the distinction using a short transcript from a group of Grade 10 FI mathematics students working on an investigation task. The

questions raised lead us into reflection on personal experience using mathematics with purpose. Finally, we close with discussion about the significance of this distinction in classrooms where students are learning mathematics in a language that is not their first language.

Theorizing mathematical and social contexts

At the root of our questioning the distinction between mathematical and non-mathematical talk we are really claiming that all language is social. This is not a new claim. Morgan (2006) drew on Halliday's theorization of language (Systemic Functional Linguistics) to argue that all instances of language have social effects:

Every instance of mathematical communication is thus conceived to involve not only signification of mathematical concepts and relationships but also interpersonal meanings, attitudes and beliefs. [...] Individuals do not speak or write simply to externalise their personal understandings but to achieve effects in their social world. (pp. 220-221)

A tradition of theorizing interpersonal positioning takes us further beyond the distinction between social and content effects. As illustrated in Figure 1, Herbel-Eisenmann *et al.* (2015) explained how communication acts have dynamic interaction with the storylines used to interpret and navigate interactions. The things people say bring to mind certain known stories in which the interlocutors imagine themselves inhabiting particular roles (each role with its particular rights and obligations). Reciprocally, the storylines chosen by the interlocutors to interpret their interaction are formative in the construction of communication acts. In other words, we decide what is happening in an interaction based on what is said and done, and we decide what to say based on our interpretation of what is happening in an interaction. The way we think of the interaction is key to our participation in the interaction.

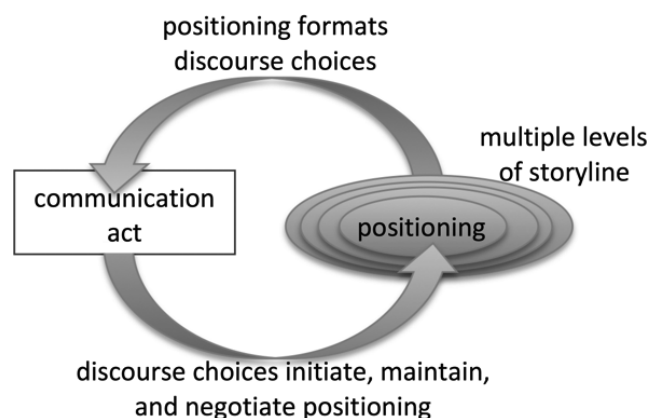


Figure 1. Multiple storylines in positioning, from Herbel-Eisenmann *et al.* (2015, p. 194)

Positioning theory describes how multiple storylines are used simultaneously and how different people may use different storylines to understand an interaction. Thus, while a teacher may be seeing an interaction as being mathematical, a student may see it as something else, or vice-versa. Similarly, a researcher may see an interaction in a certain way that may be significantly different from the ways the people involved in the interaction see it. Hence, any interaction may be considered mathematical while also fitting another storyline; ‘on task’ and ‘off task’ discourses may be interwoven in communication acts. Thus, it becomes increasingly difficult to judge what is and what is not mathematics.

Social and mathematical talk in group work

To illustrate the intersection of storylines, and the difficulty faced when trying to separate mathematical from non-mathematical discourse, we share here an example of a group of students working on a problem-solving task. In this case, the students were in a Grade 10 FI mathematics class. French Immersion is an optional school-based language-learning program in which students learn the target language not only in language arts courses but also via content instruction in other areas. These students often have limited or no contact with the target language outside the classroom setting (Baker, 2011). Such is likely the case for many of the participants in this study; they reside in an Anglo-dominant community.

Each group of three students was given a page with the following task and some images of cubes and cut up cubes (e.g., Figure 2): “A cube was painted red, and then cut into smaller cubes, $3 \times 3 \times 3$. How many of the small cubes have no red faces? How many have 1 red face? 2 red faces? 3 red faces? 4 red faces? 5? 6? How about a cube cut into $4 \times 4 \times 4$? Or $5 \times 5 \times 5$? Or $10 \times 10 \times 10$? Or $n \times n \times n$?” Each group was also given a set of twenty-seven solid white cubes snapped together in a $3 \times 3 \times 3$ configuration.

Here is a brief excerpt of group conversation among a group of three youths in the FI classroom. It is an example of the kind of interaction that prompted our reflection in this article. (All names are pseudonyms. English translation is in square brackets. Codeswitches, i.e., use of English, are underlined.)

Prior to where we begin this excerpt, Cal reads one of the posed questions and elaborates on it. He and Ben then argue

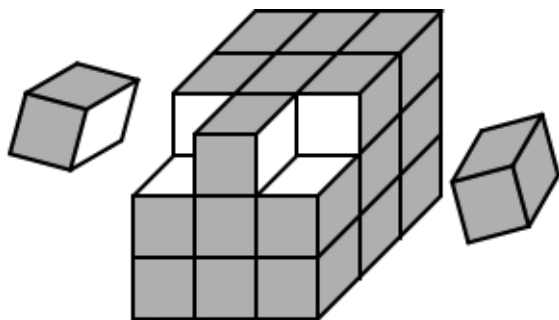


Figure 2. Cube Task

about the number of cubes without any red sides. Here, we show the continuation of the dialogue:

- 9 Cal On a fait ceci mal. [*We did this wrong.*]
- 10 Ben Ça c'est correct. C'est correct. C'est correct. Je promis que c'est correct. [*That's right. It's right. It's right. I promise that it's right.*]
- 11 Cal On a mis quatre et c'est huit parce que c'est ces quatre dans le milieu et c'est ses quatre là. [*We put four and it's eight because it's these four in the middle and these four there.*]
- 12 Ben [*takes cubes apart*] Un, deux, wait. [*One, two, wait.*]
- 13 Cal C'est quatre, c'est quatre. C'est huit, c'est huit. [*It's four, it's four. It's eight, it's eight.*]
- 14 Ben J'arrête. [*I'm stopping.*]
- 15 Cal Oui, regarde. Il y a un, deux, trois, quatre, un, deux, trois. Ha! [*Yes, look. There are one, two, three, four, one two three. Ha!*]
- 16 Ben Oh, oui. [*Oh, yes.*]
- 17 Cal Dix-sept cubes [*pause, calculating*] trente-six [*pause*] neuf fois six [*Seventeen cubes (pause, calculating) thirty-six (pause) nine times six*]
- 18 Ben [*Takes cubes apart, puts them back together, whistling*] Look at these. [*Meg smiles*]
- 19 Cal OK, j'ai ceci. [*OK, I have this.*] [*points to work*]
- 20 Ben Rubik's Cube action. [*Meg smiles*]
- 21 Ben Robert Smith [*pause*] Ol' Robbie. [*Meg smiles, laughs a little*]
- 22 Meg Qu'est-ce que c'est le nom de ton père? [*What is your father's name?*]
- 23 Ben Je ne sais pas. Je n'ai pas un père. [*I don't know. I don't have a father.*] Robert et Angela. [*Robert and Angela*] [*These are the names of Meg's parents*]
- 24 Meg Comment est-ce que tu sais? [*How do you know?*]

In this exchange, there seems to be a clear delineation between ‘on task’ (up to turn 20) and ‘off task’ dialogue (turns 21-24), indicated by a switch in topic. But here, we question the kind of analysis that would draw clear distinctions between ‘on task’ and ‘off task’ talk. The youths

brought storylines (made visible through the ‘off task’, non-mathematical talk) to the table that are seemingly unrelated to the task at hand—namely the discussion about their parents. However, we note that the apparently ‘off task’ interaction is evidence of a storyline that was at work already when the youths (at least Ben and Cal) were apparently ‘on task’. In other words, the part of the conversation that does not seem to be mathematical was also part of the interaction that seemed to be mathematical. There was little language that a teacher passing by would deem ‘off task’ in this group so it is tempting to assume little intersection between mathematical and non-mathematical storylines, and to see the short discussion about parenthood to be an abnormality. However, the smooth slide from one storyline to another indicates to us the significance of both storylines in the entire interaction.

The apparently clear separation of ‘on task’ and ‘off task’ talk is also marked by students’ use of their first language, English. There is little use of English in this exchange; based on longstanding theories and pedagogical practice, students in FI programs are expected to use only the target language in the classroom and use of any other language, including the first language (in this case English), is at the least greatly frowned upon or likely strictly forbidden (Turnbull & Dailley-O’Cain, 2009). Thus, we also question here whether the students would have discussed the ‘off task’ topic in greater detail had they felt permitted to do so in English (presumably, they did not). We wonder if these students lacked the linguistic repertoire required to engage in this ‘off task’ talk (we can only wonder, since lack of use does not strictly indicate lack of knowledge). For students learning mathematics in a second language, the storylines may be present but not able to be discussed.

What is clearer is that the storylines and languages came together to influence group dynamics and mathematics. The two male students, Ben and Cal, took charge of the mathematics and the communication. They were sitting on the same side of the table and interacting with each other, the worksheet, and the physical cubes. The female student, Meg, sat across mostly silently, said only three words within the first four minutes of group work and looked at the worksheet for the first time 12 minutes into the task (the entire task took 40 minutes). Storylines associated with the students’ parents contributed to the students’ communication choices, possibly affecting Meg’s choice to not participate and Ben’s choice to disclose his knowledge of Meg’s family. It could be that every communication act was affected as Meg’s father is a member of the school administration so his opinion of the boys might have an impact on them at some time. And we know there are other storylines at play. Gender, for example, may be at work as the boys dominated discussion and the girl remained silent. We suggest that all the possible storylines are intertwined. For example, gender may be a factor in the boys’ readiness to associate the girl with her family connections (just as it is clear that female politicians are subject to much more scrutiny than male politicians).

Instances of students’ drawing upon non-mathematical storylines have been reported in other research. Esmonde and Langer-Osuna (2012), for example, found that students working in heterogeneous groups drew on different actively

constructed ‘figured worlds’ (which are similar to storylines) during their interactions: one of mathematics learning and one of friendship and romance. The researchers showed how both of these figured worlds were gendered and racialized and affected the positioning, and thus the mathematics, within the groups. In an English as an additional language context, Barwell (2005) described how students used narratives to make sense of mathematical word problems, even when those narratives appeared to be what teachers would call ‘off task’. However, his analysis puts the narrative at the service of the mathematics, and not vice-versa. Chazan gave an account of students pasting their argument from the hallway into their mathematical interaction in class (Chazan & Ball, 1999), which turns the relationship around, the mathematics at the service of the students’ apparently unrelated narrative. These research accounts (narratives), along with our own (the one shown above and others from our research), cause us to reflect on the role of context in mathematics. Is it possible for mathematics to be purely ‘on task’ or is the notion some sort of denial of reality?

The value of ‘on task’ and ‘off task’ mathematics

Attention to the level of contextualization in mathematics learning guides us to question the value of decontextualized mathematics and to turn attention to the kinds of contextualization that go beyond typical views of mathematical contexts. In other words, in order to challenge the idea of a purely ‘on task’ mathematics we consider social contexts in which mathematics is used and ask how purely ‘on task’ mathematics may be useful in those situations. For this, we first reflect on our experiences with mathematics outside of school. We consider the way context worked in these experiences. We ask ourselves about the value of decontextualized ‘on task’ mathematics in these contexts and we encourage readers to think of their own examples.

For our first example that we share here, Karla once went on a birthday trip to New York City with three friends. One of the group of four often paid for a given activity (*e.g.*, a meal, museum entrance) and then they would figure out their separate bills at the end of each day. The calculations in this end-of-the-day figuring seemed straightforward, but the situations in the context were complicated—important to the friendship but not so important beyond that circle. The friends at the end of each day found themselves trying to remember the facts—the cost of what was eaten for dinner and factoring in who paid \$25 to go to the Guggenheim versus who took a \$3 subway ride downtown instead—but behind all this, there were more subjective questions that affect their relationship, such as “Should the birthday person pay the same as everyone else?” and “Should the more wealthy among us subsidize the relatively (perhaps temporarily) less wealthy?”

We ask what purpose a pure ‘on task’ mathematics served in the New York friends context. The pure, straightforward calculations based on ‘facts’ made it possible for the friends to embrace a storyline of fairness and equality in the friendship. However, underneath this communication within this storyline, there were subjective questions that may have influenced the selection of which ‘facts’ to include in the calculations. Furthermore, the group was using a purely ‘on

task' decontextualized mathematics for a social purpose (to underwrite the fairness storyline), which means that the decontextualization was in fact contextually contingent.

For our second shared example, David, along with a business partner, once ran a reforestation business. There were stipulations in their contracts that described how payment would be calculated based on the quality of a random sampling of their planted trees. Thus, the method of sampling and calculations made a huge impact on how much money they received for the contracts. The reforestation inspectors played with their sampling methods to achieve subjective purposes (e.g., sometimes threatening to do 'random' samples in the most difficult planting areas) while pointing to the perfection of their calculations and random sampling. Similarly, the business proprietors manipulated contexts to promote higher averages (e.g., David modeled the random sampling to predict the locations and ensure the quality of the planting in those places).

As with the friends in New York, a pure, straightforward calculation following the contract made it possible for the parties in the contract to embrace a storyline of non-partiality and fairness, but it also put some boundaries on the interaction to produce this fairness. Yet, there were subjective games played by both sides in the contracts, probably with awareness of this on both sides but unacknowledged in the communication. The people in this relationship were using purely 'on task' decontextualized mathematics for their social purposes (to underwrite the fairness storyline) which again means that the decontextualization was in fact contextually contingent.

For a larger in scope example of the value of 'on task', un-muddled mathematics, we draw attention to Statistics Canada, our national statistics agency. Its mandate is twofold, to "provide statistical information and analysis about Canada's economic and social structure to [...] develop and evaluate public policies and programs" and to "improve public and private decision-making for the benefit of all Canadians, [and also to] promote sound statistical standards and practices by [...] using common concepts and classifications to provide better quality data" (emphasis ours, Statistics Canada, 2016, n.p.). When our previous national-level government discontinued the backbone of the census data (the 'long-form census') communities and community groups raged their disapproval because of the importance of reliable statistics for fair planning and political dialogue.

Certain political interests desire to undermine access to reliable statistics for people without the means to do their own statistics. Statistics Canada exists because people manipulate mathematics and statistics for their ends. Yet, even the reliable statistics can be used in manipulative ways. To manipulate others with statistics, one needs the image of reliability and purity. Again, the apparently pure statistics (clean and fair) is used as if it is decontextualized to achieve a contextualized purpose.

The image of 'on task' mathematics

Our reflection on actual mathematics outside the classroom underscored the value of un-muddled (de-contextualized) calculations and communication that allows for the bracketing of emotion and relationship. Ironically, this reflection

also helped us see the impossibility of doing useful mathematics without impacting relationships. The positioning of the various players affects the way the mathematics is 'done' and interpreted. The true value of the mathematics and its use in practice resulted in other storylines coming into play. In our examples from our 'real life' experiences, we found that in order to make the mathematics, or the mathematical storyline, meaningful to us we needed to connect it to our other, non-mathematical, storylines. Thus, the mathematical object is not identical to its image.

These reflections remind us of a famous image. René Magritte's painting *The Treachery of Images* depicts a smoker's pipe with writing under it that says "Ceci n'est pas une pipe," which means "This is not a pipe." Of course, it is not a pipe; it is an image of a pipe. Magritte's work is a play on language that has been used by countless scholars to describe, and call into question, the semiotic relationship between signifiers (e.g., language, images) and the things they signify. For example, in the context of a mathematics teacher educator from the UK working with prospective teachers in Uganda, Bradford and Brown (2005) described a struggle with the concept of an ideal circle in relationship to the numerous circle-like objects in the interlocutors' experience: "This is not a circle." Which takes primacy? The ideal circle or lived experience with necessarily imperfect circles? Is the ideal circle but an image of a 'real' circle?

Magritte's painting problematizes the way we tend to interpret language and images, as standing unequivocally for the object (e.g., pipe, circle) they represent (Blakesley & Brooke, 2001). In turn, it helps us rethink our experiences with mathematics classroom communication. We are reflecting on the way teachers and researchers interpret communication in mathematics learning contexts with the urge to classify signs (language) as mathematics or not. With such classification, we miss the opportunity to consider how a sign is complex and can index multiple things.

When mathematics education researchers try to separate mathematical communication from other communication in the classroom, they are in effect saying "This is mathematics" or "This is not mathematics" (or, to borrow Magritte's language and structure, "Ce ne sont pas des mathématiques"). Similarly, when mathematics teachers try to keep their students 'on task', they are likewise judging students' utterances: "This is mathematics" or "This is not mathematics." Looking at students' classroom communication through Magritte's lens, we are prompted to question our view of what 'is' and what 'is not' mathematics, of what constitutes 'on task' and 'off task' mathematical talk. Our exploration motivates questions about the relationship between mathematics and other lines of student experience and intention.

'Off task' mathematics for the classroom

In short, we question the idea that there is a pure mathematics unconnected with human intention and experience, and we consider pedagogical implications of this tension. In particular, we focus this reflection on contexts where students do not learn in their first language, because these are the contexts that motivated this article. But we know that the same issues appear in any classroom, though they are exacerbated in multilingual contexts.

We question whether what students do in school is ‘real’ mathematics, or whether it is merely an image of mathematics. For instance, when students are ‘on task’, they are simply doing what their teacher has asked them to do. But when they are mixing their other wishes and storylines with their mathematics they are using mathematics for something else, which is potentially much more mathematical. This is why we are writing ‘on task’ in scare quotes throughout this article. Being ‘on task’ with mathematics tends to be blind rule following or, at best, following the teacher’s wishes (*c.f.*, Herbel-Eisenmann & Wagner, 2010)—it is an image of mathematics. By contrast, being ‘off task’ requires creative methods for intertwining mathematics with other storylines of interest. It positions mathematics in relation to human interests. Mathematics becomes a tool which can be used to mediate and interpret meaningful, ‘real life’ interactions. Thus, there is value not only in the ‘on task’ mathematical talk of curriculum outcomes and classroom tasks, but also in the ‘off task’ talk related to other storylines. At minimum, the influence of these ‘off task’ storylines on the mathematics of the classroom (such as how the mathematics unfolded in our classroom excerpt) should be recognized.

In conclusion, we have reflected that both ‘on task’ mathematics is necessary at times, and ‘off task’ mathematics is a reality requiring attention. Thus, we identify two needs.

First, we call for work that develops ways for engaging students in more complicated situations. We see connections in this to the Critical Language Awareness developed by Fairclough (1992). He and others found that students learn a new language better when they study the way the language is used for political purposes, as compared to classes where the focus is purely on language. We suggest that it could be the same for mathematics—better understanding of mathematics comes when taught in political contexts with, we suggest, the added benefit of developing language repertoire at the same time.

Fortunately, we have some strong examples of teachers engaging students in such ways. Stocker’s (2006a) book *Maththatmatters* is a powerful resource that can be used immediately by mathematics teachers and that can serve as a model for task design. His focus was on social justice contexts, but students may have other concerns. In a communication in FLM 26(2), Stocker (2006b) questioned Palm’s (2006) framework for developing ‘real-world’ mathematics contexts. Stocker (2006b) advocated for problems that are transformative with the purpose of making the world a better place. We claim that conflict among people who have particular ideas about making the world a better place and people who think that real-life contexts need not address one’s wishes for the world is in fact part of students’ experience, though this conflict is largely repressed and generally not supported (linguistically or otherwise) in mathematics classrooms.

Unfortunately, the examples we know of mathematics teachers engaging students in tensions do not draw attention to the role of linguistic diversity. Moreover, due to the paucity of ‘off task’ talk in our FI data, we wonder whether students in this educational context might need particular support for this kind of language. Given the oftentimes lim-

ited or in some cases non-existent contact these students have with the target language outside of the classroom walls, they may benefit from focused, explicit linguistic instruction in order to make connections to meaningful contexts and storylines. This, in turn, would contribute not only to meaningful mathematics but also to language use and thus language learning—a key aim of a language-learning program like immersion. How, then, can teachers in second language programs such as immersion, and in other linguistically diverse contexts, best support students’ development of language repertoires for a wider range of human experiences?

Thus, the second need we identify is the need for more research on the ways students connect various storylines to their mathematics in schoolwork, especially among groups with different language repertoires.

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