Using History in Mathematics Education

JOHN FAUVEL

For decades if not centuries now, a few voices in each generation have urged the value and importance of using history in teaching mathematics—but so far without this insight taking firm and widespread root in the practice of teaching. This can be most starkly seen by comparing the content and guidance for the UK National Curriculum in mathematics, laid down for state schools by the British government in 1989, with some views held earlier in the century.

> The Historical aspect of Mathematics has never yet found its fitting place in teaching of the schools. [ ] Every boy ["(*Throughout the report the word BOY is to be taken as referring to pupils of either sex*)"] ought to know something of the more human and personal side of the subject he studies. [.] The history of mathematics will give us some help in framing our school syllabus. [.] Recommendation:] That portraits of the great mathematicians should be hung in the mathematical classrooms, and that reference to their lives and investigations should be frequently be made by the teacher in his lessons, some explanation being given of the effect of mathematical discoveries on the progress of civilization.
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> Mathematical Association Committee report, 1919
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> The teacher who knows little of the history of Mathematics is apt to teach techniques in isolation, unrelated either to the problems and ideas which generated them or to the further developments which grew out of them. [.] A knowledge of the arguments and dissensions between great mathematicians might induce healthy skepticism and discussion in the classroom and lead to a firmer grasp of principles [ ] One of the most valuable assets which the teacher can acquire from a knowledge of the history of his subject is an appreciation of the influence of current traditions. [ ] It is important to convey to the pupils the knowledge that much of what is taught today as a finished product was the result of centuries of groping or of spirited controversy [.] Mathematics can be properly taught only against a background of its own history.
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> Ministry of Education, 1958
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> In our view the mathematics teacher has the task [.] of helping each pupil to develop so far as is possible his appreciation and enjoyment of mathematics itself and his realization of the role which it has played and will continue to play both in the development of science and technology and of our civilization. [.] [High-attaining pupils] should be encouraged, too, to read books about mathematics and to learn something of the work of the great mathematicians of the past [.] Reference to the historical background of some of the topics which are being studied can both help to explain their importance and also add interest and depth to the A-level course.
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> Cockcroft Report, 1982

It is curious that no sign of these past concerns about using history in mathematics teaching is to be seen in the apotheosis of British educational aspirations, the National Curriculum. We find the historical perspective less noticeable at the beginning of the 1990s than in any official document about mathematics education for a century. How has this come about?

There is no lack of enthusiasm among many teachers, and it is not that people cannot have heard the message; there is no point in merely saying the same thing again, but louder. It is important, rather, to seek to understand why the value of using history is so hard to put across in the right quarters. What is getting in the way of this simple beneficial improvement? Then we begin to understand; for of course the change is far from simple, and may not be universally regarded as beneficial. What is fundamentally at issue may be a deep cleavage between different ways of looking at mathematics. A mathematics which consists of timeless truths and value-free facts may be fundamentally inconsistent with one which develops through human endeavour within social contexts. It is precisely the detached abstract unsullied purity of mathematics that has been its appeal to many students down the ages. We should not lose sight of the tenacity and depth—and to some extent validity, indeed—of this emotional aesthetic response.

But even a historically-conditioned mathematics may give rise to ideologies which subsequent generations find unacceptable, or at least unfashionable: one strong argument for the historical perspective in mathematics education, much in favour at the start of this century, was the biologistic one that mathematical development in the individual retraces the history of mathematics itself: an *ontogeny recapitulates phylogeny* kind of argument. This was the key feature of Herbert Spenser's influential views on education, back in the 1860s:

> The education of the child must accord both in mode and arrangement with the education of mankind as considered historically; or, in other words, the genesis of knowledge in the individual must follow the same course as the genesis of knowledge in the race

and reached its apogee in the work of Benchara Branford in the early 1900s. (See page 16, and diagram on next page.)

But historical experiences this century have made free discussion of the development of the race less welcome, and a little knowledge of mathematical history shows the course of its development to be, in any case, much less simple and linear than this analogy would require. The deep historical context of Branford's ideology is so clear to us now that this kind of analysis is hard to sustain with such innocence, even though at the time it was a powerful and effective model. The cause of invoking history in mathematics education may have suffered a setback through being tarred, however unfairly, with the brush of a

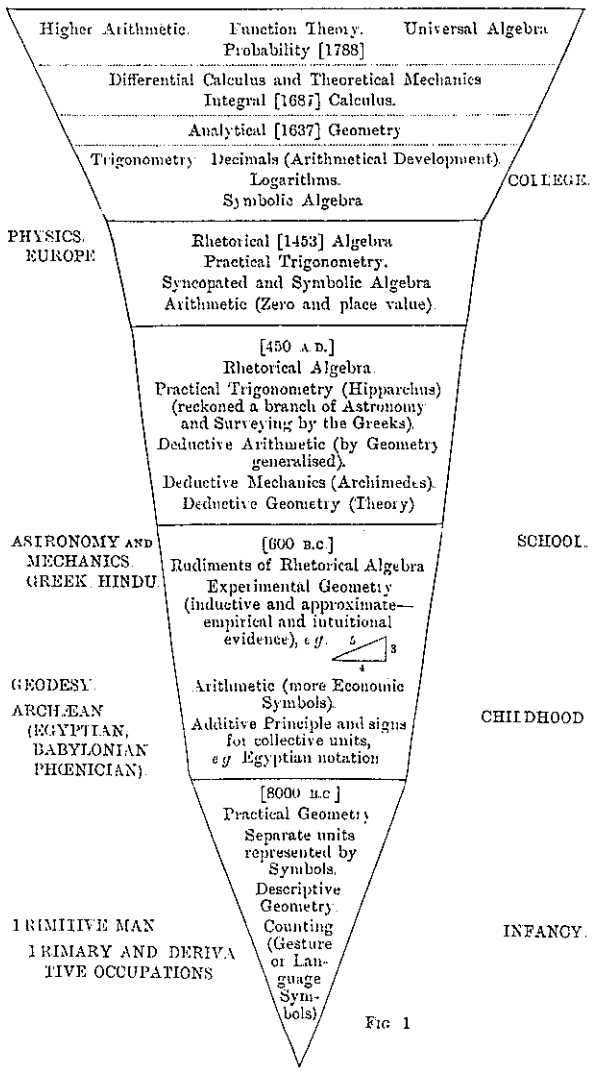

FIG 1

Miss Barwell's version of Benchara Branford's schema from *Mathematical Gazette* of 1913.

racist ideology (from which the history of mathematics itself has not been free).

> Pupils should develop their knowledge and understanding of the ways in which scientific ideas change through time and how the nature of these ideas and the uses to which they are put are affected by the social, moral, spiritual and cultural contexts in which they are developed
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> *Science in the National Curriculum*, 1989

Nor is this an abstract intellectual consideration. Notice something else which has not made it to the Mathematics National Curriculum; not only is any sense of historical perspectives and opportunities missing, but also the *social, moral, spiritual and cultural contexts* which have managed to gain a toehold in the Science Curriculum (further discussed in Pumfrey [1991]) There may still be something too dangerous about the combination of history and mathematics. The potential subversiveness of history is always a constant threat to authorities who need to control the past, and its conjunction to a mathematics which embodies quite other qualities may be an unsettling and provocative programme.

But there are other factors too behind the neglect of history in mathematics education. History is not just some kind of lubricant or additive that comes in a tube and can just be poured in at the right time, like fabric conditioner into one's washing machine. Making use of history is hard for pupils, whose historical framework and sense of the past can be very erratic, if it exists at all; and it is hard for teachers—who have usually learned little or no mathematical history during their training, let alone received training on how to use history with their pupils. Historians of mathematics, too, have not always found it easy to accommodate their sense of the complexity and subtlety of history to the somewhat cavalier way with historical accuracy and truth which may be a feature of some classroom presentations or even popular historical texts

Furthermore, the message that using history is a good idea is worthy, but incomplete. It is not hard to produce many good reasons for using history in maths teaching and to detail the benefits it can bring

Some reasons that have been advanced for using history in mathematics education

Helps to increase motivation for learning
Gives mathematics a human face
Historical development helps to order the presentation of topics in the curriculum
Showing pupils how concepts have developed helps their understanding
Changes pupils' perceptions of mathematics
Comparing ancient and modern establishes value of modern techniques
Helps to develop a multicultural approach
Provides opportunities for investigations
Past obstacles to development help to explain what today's pupils find hard
Pupils derive comfort from realizing that they are not the only ones with problems
Encourages quicker learners to look further
Helps to explain the role of mathematics in society
Makes mathematics less frightening
Exploring history helps to sustain your own interest and excitement in mathematics
Provides opportunity for cross-curricular work with other teachers or subjects

Many worthwhile and interesting papers have been written over the past decades drawing up lists such as this and exploring the arguments and illustrations which justify these claims—but there is a danger that this is seen as the end of the argument, and the changes will then miraculously happen, on account of their inherent persuasiveness. It is important to move beyond this stage, to take it as read, for the moment, that using history is a good thing—for numerous reasons—and to show *how* it might be incorporated into some classroom activities, *how* it might make the teaching of various specific things easier, *how* the extra work which may be needed at first has a long-term pay-off in improving the attainment of objectives within the mathematics syllabus, and so forth. Without a demonstrable

concrete payoff in the lives of teachers and pupils, it is all hot air.

Much valuable work is going on along these lines in France, for example, and an edition of papers by French teachers has recently been published by the Mathematical Association under the title *History in the mathematics classroom*. A great deal of effort by many people has gone into exploring how historical mathematical texts can be used in the classroom—not the only approach, but one with great potential. And other valuable work has been done abroad, notably in Israel where workers at the Weizmann Institute have developed historical packs for use with mathematics teachers; see for example the paper by Ofir elsewhere in this issue.

Once it can be seen from experience that using history does work, then there is a grounding for more persuasive argument, and the wealth of ways in which it can aid mathematics teachers and learners may be explored.

Some ways of using history in the mathematics classroom

Mention past mathematicians anecdotally
Provide historical introductions to concepts which are new to pupils
Encourage pupils to understand the historical problems to which the concepts they are learning are answers
Give "history of mathematics" lessons
Devise classroom or homework exercises using mathematical texts from the past
Direct dramatic activity which reflects mathematical interaction
Encourage the creation of poster displays or other projects with a historical theme
Setting projects about local mathematical activity in the past
Using critical examples from the past to illustrate techniques or methods
Explore past misconceptions/errors/alternative views to help in understanding and resolving difficulties for today's learners
Devise the pedagogical approach to a topic in sympathy with its historical development
Devise the ordering and structuring of topics within the syllabus on historically-informed grounds

There is an important distinction to be made between using the history of mathematics within the teaching of mathematics, and teaching the history of mathematics as a subject. To these may be added a third kind of historical involvement in mathematics education: giving a historical perspective to cultural studies of mathematics—discussions of mathematics in society, and the like. There are several reasons why it is important to keep these separate. *Firstly,* they are often confused; teachers may fear they are being urged to teach a subject they know little about and which is not on the syllabus, namely history of mathematics, when what they are actually being encouraged to do is to explore ways of helping their teaching of mathematics itself to become richer and more varied and effective in certain ways. All sorts of muddled discussions can arise through not being clear about this fundamental distinction.

History of mathematics ought to have an important role in general studies lessons, in history lessons and elsewhere; but that is a different role from its use within the mathematics classroom. G. Heppel explained this quite clearly a hundred years ago, in a 1893 talk to the Association for the Improvement of Geometrical Teaching, but the distinction and the reason for it have not always been seen so clearly in the intervening years.

The use of history in teaching mathematics

(1) The history of mathematics should be strictly auxiliary and subordinate to mathematical teaching.
(2) Only those portions should be dealt with which are of real assistance to the learner
(3) It is not to be made a subject of examination

Unless these conditions are observed, it is to be feared that the effect of the introduction of new matter for instruction would be injurious rather than beneficial. The ordinary schoolboy or schoolgirl now takes in hand quite as many subjects as he or she can satisfactorily study, and nobody wants the number to be increased.

G. Heppel, 1893

Secondly, the distinction between teaching history and using it is important not only to set teachers' minds at rest, but because of the potential use of history in teaching various components of the mathematical curriculum. Even though history itself receives no mention in the British National Curriculum for mathematics, this may not be a bar to its use in mathematics classrooms, since the intention is presumably for mathematics to be taught by the most effective means—which ought therefore, if this can be demonstrated, to include historical perspectives.

And *thirdly,* there is clearly pressure on teachers nowadays, which in many cases is welcomed, to contextualise mathematical studies in various directions. The Cockcroft report referred to the teacher's *"task [ ] of helping each pupil to develop so far as is possible his appreciation and enjoyment of mathematics itself and his realization of the role which it has played and will continue to play both in the development of science and technology and of our civilization"*. More recently, schools have become increasingly concerned about helping pupils from a range of cultural backgrounds to achieve their potential, as well as ensuring that all children can appreciate the diverse cultural roots of mathematics. Many teachers are also concerned about how to ensure that girls can be helped to continue to develop their fullest mathematical potential throughout the teenage years. All these issues are materially promoted by historical perspectives. This is another area in which there is a historical tradition of advocating the use of history. Benchara Branford in 1908 discussed the inadequate state of mathematical education for women in these terms:

> The questions of the development of the appropriate grouping and types of mathematics for the female sex [...] are in a condition still more obscure and immature than the corresponding questions for the male. This is not to be wondered at if we look back upon the history of education. Doubtless the solution of these problems for women can only come, and gradually, from women themselves, when more emancipated from their present day adherence to and imitation of male conventions and standards in education. Considerable research work in the history of mathematics, particularly in relation to the rise and growth of female arts and crafts, deliberate experiment with carefully-planned varieties of curricula, and

> woman's confidence in her own power to work out the best solutions, are undoubtedly all needed here [Branford, 1924; p 270]

Another dimension along which the uses of history vary is that of level or key stage, the stage of development which a pupil has reached. The possibilities and challenges open to a primary school teacher are very different from those of a *gymnasium* teacher, and only some experiences can be meaningfully shared. I make this rather obvious point because in talking about the uses of history there is an understandable temptation for people to assume, without recognizing this very explicitly, that the key stage or age or ability range of *their* pupils is the one under discussion; all sorts of unnecessary misunderstandings can arise from overlooking the breadth of varying possibilities, across a much wider spectrum than any single teacher is likely to meet. The other papers in this issue of *For the learning of mathematics* demonstrate something of the range of ages and levels for which an appropriately devised historical component can enrich classroom mathematics

No one pretends that making use of history in the classroom is easy. Until the day when all teacher education includes both history of mathematics and also training in its classroom uses, for different topics, appropriate ages and ability ranges, teachers will understandably come to this area a little warily, with a fear of not knowing enough and not having access to the right materials to make this approach easier or indeed possible for them. There is need for much discussion about what is needed in support, and how to help the classroom teacher with provision of courses, background reading, examples of classroom packs and other activities, and so on. It may be hoped that this time the movement for using history is more solidly based in the realities of what must be done to support teachers in the classroom, by example and by the provision of materials. The next few years will see the flourishing of a richer mathematical educational environment throughout Europe and the wider world, to which the papers in this issue of *For the learning of mathematics* hope to make a significant contribution.

References

Branford, Benchara *A study of mathematical education*. Oxford University Press, 1924 (new edition —original 1908)

Fauvel, John (editor) *History in the mathematics classroom: the IREM papers* The Mathematical Association, 1990

Pumfrey, Stephen "History of science in the National Science Curriculum: a critical review of resources and their aims" *British Journal for the History of Science* 24 [1991] 61-78