Grammatical Translation-Inhibitors in Two Classic Word Problem Sentences

MARTHA B. BURTON

The difficulty experienced by mathematics students in the meaningful use of their algebraic language is legendary. Translation difficulties between English and algebra arise in both directions, with the problems of the English-into-algebra translation task becoming apparent as soon as the student encounters "story problems" in elementary algebra. As we consider such translation problems, we should keep in mind not just the students and their mathematical capabilities but the characteristics of the languages themselves.

Although the translation task presented by a typical elementary-algebra word problem will involve summarizing several English sentences in one algebraic equation, it is as well to begin with simpler things. We will consider here only the translation task of one English sentence into one equivalent sentence in the algebraic language. We need to consider firstly the input and output sentences themselves, and secondly the means by which a person carries out the translation

English sentences and their algebraic equivalents

Not every English sentence has an algebraic translation, of course: "The puppy is chewing on your shoes again" certainly has none. The parts of speech in this sentence are different from those in sentences with algebraic equivalents: "The puppy has chewed five of your shoes already," "The puppy weighs 35 pounds now," "The proper dose is 15 micrograms," and "25 is ten more than the mystery number." In English sentences with algebraic equivalents, the subject may be a quantity, as in *dose* or *25* in the last two examples. Or the subject may be something that can be quantified, according to a measure function indicated by the English verb —as our example-puppy may be measured using the functions *number-chewed* or *pounds-weighed*

What becomes of the subject and verb of the English sentence, once it is translated into the algebraic language? An algebraic sentence describing real numbers must have for its verb some combination of the verbs *is* and *exceeds*; if the algebraic sentence is an equation rather than an inequality, only the verb *is* ("equals"), will belong in the algebraic sentence. The sentence will have the form

subject + *is* + subject-complement

Where the sentence's subject and subject-complement must be numbers or named quantities.

The subject of the algebraic sentence derives from the subject of the original English sentence If the English sentence already had a number or a named quantity as its subject, the English subject simply becomes the equational subject Thus, "The proper *dose* is 15 micrograms" becomes "*dose* = 15," while "*25* is ten more than the mystery number" becomes "*25* = 10 + number."

The other possibility is that the English subject represents a quantifiable entity rather than a quantity itself. In our examples the subject, *puppy*, does not itself become the subject of the corresponding algebraic sentences. Instead, the algebraic sentence takes as its subject the quantity resulting from the action on the English subject of the measure function indicated by the verb: "The *puppy* has chewed five of your shoes already" becomes "*number-chewed* = 5," while "The *puppy* weighs 35 pounds now" becomes "*puppy's-weight* = 35 "

How is a translation carried out?

Let us consider what must happen when a person attempts to rewrite an English sentence into the algebraic language The English sentence must first be parsed: read, understood, and stored. In considering this comprehension and storage, we must posit processes that we cannot describe explicitly, for the storage form of a sentence is not identical to the surface form in which the reader encountered it. In fact, as a reader processes a sentence word by word, its surface form disappears even as some more abstract version of the sentence is stored (The skeptical reader is invited to recall the verbatim, surface form of the previous sentence) [6, p. 111]

Taking the point of view that our knowledge of language is described by a transformational grammar, we can speculate that comprehension proceeds in something like the following form [6, p. 30] Each speaker of the language has available a set of *phrase-structure rules,* that is, sentence-describing rules of the form

sentence ⟶ noun-phrase + verb-phrase
noun-phrase ⟶ article + (adjective) + noun
verb-phrase ⟶ verb + noun-phrase

. .

These phrase-structure rules produce skeleton forms of just sentences that would be grammatically recognizable to the speaker of the language. Although it is obviously unclear exactly how these sentence-forms are stored in human cells and synapses, models for them are often taken to be *phrase-markers* — tree diagrams with the symbol "sentence" at the root. [6, p. 29] A sentence is recognized by a speaker of the language as grammatical as it is matched with a phrase-marker. Grammatical transformations can map certain phrase-markers into other phrase-markers, accounting for our ability to produce grammatical variations and paraphrases of a sentence from its abstract, stored, "base" form.

If it is accepted that some process even vaguely as outlined takes place when a human understands a sentence, then the translation of a sentence from its external English form into the algebraic language must begin with the human parsing the English sentence into its base form. This is done in accordance with the human's grammatical knowledge, so that the incorrect sentence "The of your shoes already" will be rejected. Only after a sentence has been parsed and stored can it be encoded into an equivalent algebraic sentence.

A momentary digression may illustrate these suggestions. The human translation ability that we cannot directly observe is mirrored in the humanly-constructed compilers that exchange sentences in one computer language for code segments in another. A compiler too must parse an incoming sentence, resolving it against a known grammar for the source language. An abstract version of the incoming sentence is then stored, typically as a tree diagram, in preparation for the production of code in the target language.

Whether we consider a compiler program — which we could comprehend fully, had we but world enough and time — or the more opaque human translator, some shared features should be emphasized. There are aspects of the syntactic understanding of sentences that are not surface phenomena. Parsing a source sentence must occur before the sentence is encoded into the new language. No correct encoding can be hoped for if there is an error in the stored syntactic form of the sentence. A translation failure may actually be a failure in the parsing of the incoming sentence.

What is stored in the sentence's base form?

Leaving aside the unknowable actual form of a stored sentence, we may ask what it is about the sentence that is stored. Consider the sentence "The puppy is chewing on your shoes again." Its subject, verb, and object (*puppy, chew,* and *shoes)* must be part of the stored information. Further, it is a reasonable guess that the same subject, verb and object will be stored even if the surface form of the sentence is passive: "Your shoes are being chewed again by the puppy." For the sentence to be understood it must be clear that the puppy is doing the chewing, and that the shoes are the targets of that action.

The early motivation for syntactic transformations on phrase-markers was as an explanation for the grammatical relationship between active-voice and passive-voice sentences just such as our example pair of the puppy and the shoes. [1, p. xxiv] Not everyone espouses a transformational theory of grammar, of course. An alternative "lexical-functional" theory suggests that information sufficient to account for active- and passive-voice sentence versions is stored in our mental lexicon entries for forms of the verb *chew:*

(1) chew: chew ‹ (subject) (object) ›
(2) chewed: chew ‹ (*by*-object) (subject) ›

The first-listed form of the verb *to chew* appears in an active-voice sentence, with the subject of the sentence *(puppy)* being the agent and the direct object *(shoes)* the recipient of the action. In the second-listed form of the verb the passive-voice sentence is indicated, with the agent of the action appearing in a prepositional phrase *(by the puppy)* and the recipient of the action appearing as the sentence's subject. Each listing of the verb has the same basic argument structure

chew ‹ agent patient ›.

[1, p. xxv] With both active and passive versions of the sentence indexed according to the verb form used, this scheme emphasizes the verb's grammatical importance.

Though one theory proposes a lexical rule and the other a phrase structure transformation to account for the change from active voice to passive voice, notice that the grammatical functions subject, verb and object retain their central importance. Adopting either point of view, we would have to conclude that these basic parts of speech are part of what the listener or reader stores in the base form of the sentence. Knowledge of a sentence's logical subject and verb resides at a much deeper level than that used in the old-fashioned exercised of diagramming sentences: recognizing and storing its subject and verb is central to understanding a sentence at all.

The grammar of two classic sentences

So far we have used as examples of English sentences with algebraic equivalents only a few, minimally simple, examples. We have been interested simply in illustrating what happens to the subject and verb of an English sentence as it is encoded algebraically. Now let us follow the same thread with some far more interesting example sentences.

In a series of investigations beginning with the 1981 paper of Clement, Lochhead and Monk on translation difficulties in learning mathematics, [5] a certain pair of short English example sentences has become almost symbolic of students' English-to-algebra translation difficulties:

> "There are six times as many students as professors at this university"

and

> "At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel."

These example sentences arrest the attention because they are so concise and clearly stated. Innocent of confusing excess verbiage, they are yet productive of widespread errors among students. The error rate among university students is not only high, but the type of error is predictable: a reversal of the placement in the algebraic translation of the variables for numbers of students and professors.

The suggestion to be made here is that there is some reason not to be astonished at this particular error: that the sentences, clear enough in English, represent translation tasks that are not entirely straightforward.

These sentences have an interesting similarity: their form makes it difficult to determine their subjects. We should begin by observing that there is no reason to fault the English form of the sentences, which seem to have been well enough understood by the students attempting to translate them. Still it is possible that not being able to identify the

English sentence's subject precluded the production of a subject for an algebraic sentence version — inhibiting the translation task

Each of these examples is an existential sentence, using the word *there* with a form of the verb *to be*· *"There are* six times as many students as professors at this university" and "At Mindy's restaurant, for every four people who ordered cheesecake, *there are* five people who ordered strudel "

In an existential sentence, the word *there* acts as a kind of dummy subject, with a form of *to be* as its corresponding verb It is a slot-filler, with the actual or logical subject appearing later in the clause. A there-sentence results from a grammatical transformation on a base form

> subject + (auxiliaries) + *be* + predication ⟶
> *there* + (auxiliaries) + *be* + predication [10, p 418]

For example, "There are six students in the room" is an existential there-sentence resulting from "Six students are in the room." The logical subject, the topic of the sentence, is *students,* while the nominal subject is the dummy word *there.*

When a human encounters a there-sentence, we conjecture that the there-phrase is deleted during parsing, and the logical subject and verb are stored instead. Since each of our present examples is a there-sentence, the parsing question in each case reduces to: what is the logical subject of the sentence?

Consider the first example, "There are six times as many students as professors at this university " The sentence's initial phrase "there are" acts as subject and verb in surface form only, and we conjecture that they are ignored upon parsing. The segment of the sentence " six times as many students as professors . " must include the actual subject and predicate, since the prepositional phrase "at this university" is certainly a noncontender. The obvious candidates for the role of subject would appear to be *students* and *professors* — indeed, students attempting the translation were offered letter names for numbers of students and professors. Yet neither of these candidate words seems to have the edge on the other, and it would seem that many readers of the sentence vacillate at this point

The subject of a sentence should tell us what the sentence is about, indicating the sentence's topic And this sentence is not about students and their doings, nor yet about professors and their activities. Rather, the sentence is about the *ratio* of students to faculty (a familiar ratio, in fact, one listed by many handbooks advising students on the choice of a college) The logical subject of the sentence is neither *students* nor *professors,* but the number *six* — acting as a noun rather than as an adjective.

If the subject is *six*, and the verb in the equation sentence must be *is* (a form of the verb *to be,* as indicated in the transformation rule for there-sentences) then what about the rest of the phrase " . . times as many students as professors . ."? Here the would-be translators of the sentence confront another difficulty in their mother tongue We will consider the phrase "times as many" later, but for now we should note that the base form of the sentence is

> six . is . . times-as-many students as professors

where the final phrase indicates the *ratio* S/P. Thus the algebraic translation of this sentence is 6 = S/P, which is equivalent to (for the algebraic language has its own grammatical transformations) the equation S = 6P

Now we turn to the second example, in which the patrons of Mindy's restaurant present us with an equally intriguing riddle. We begin as before, by inquiring as to the subject of the sentence Once more the prepositional phrase can be eliminated, leaving " . . for every four people who ordered cheesecake, there are five people who ordered strudel." We note again that the phrase "there are" fills a dummy grammatical slot, leaving us with ". . . for every four people who ordered cheesecake . . five people who ordered strudel " But the four cheesecake fans reside grammatically in a subordinate clause, so that the main clause of the sentence asserts that "five people . . ordered strudel."

The main clause has a simple enough translation. With an English subject *people* and verb *ordered,* the verb is accessed to indicate a measure of the subject and the algebraic version becomes *"strudel-orders* = 5 " But this is not quite right, for it translates only the main clause; and there is an attached clause ". . for every four people who ordered cheesecake " which will translate into *"cheesecake-orders* = 4." These are algebraic clauses, not sentences, and are related by the English phrase "for every " This tells us that, again, a ratio is being discussed, with the English phrase "for every" indicating — though not very clearly — the division operation Dividing one equation-clause by the other, and using letter variables S and C as the original authors suggested, we get

$$S/C = 5/4$$

or 4S = 5C, as hoped

English idiom inadequate for indicating division

A reader attempting to analyze the preferences of Mindy's customers might have eliminated the two-clause sentence form by first reworking the English sentence. In one possible rewriting " the strudel customers are 5/4 times as numerous as the cheesecake customers . . ." the operative phrase "for every" has been exchanged for the phrase "times as numerous." Both of these phrases indicate the division operation — but neither of them indicates it very forcefully

The phrase "times as (many, numerous,)" should be particularly noted, for upon algebraic translation it sometimes indicates multiplication and sometimes division

For example, consider Sarah, who is 30 years of age, and her five-year-old daughter Katie. We could describe the relationship in terms of Sarah's age (already a number): "Sarah's age is six times Katie's age " Or we could write an English sentence whose subject is Sarah herself — not a quantity, but quantifiable by age as indicated in the predicate: "Sarah is six times as old as Katie." As we have already seen, either of these sentences has the algebraic translation

> *sarah's-age* =6 * katie's-age.

We could also describe the relationship in an existential sentence: "There are six times as many candles on Sarah's cake as on Katie's" As before we observe that once the dummy subject is disregarded, the logical subject is the number six and the algebraic version is

$$six = \frac{\text{number-sarah's-candles}}{\text{number-katie's-candles}}$$

The ambiguity in the phrase "times as (many, much, old . .)" is a built-in characteristic of the English idiom. It does not appear to inhibit understanding of an English sentence, but is a source of difficulty once the sentence's translation into the algebraic language is attempted.

The overloaded use of the phrase "times as ." reflects a gap in our stock of relevant English phrases. Our mother tongue seems to lack conversational, non-arithmetical idiom to indicate the division operation. There is no such lack with respect to the other arithmetic operations: *more, fewer, leads* and *trails, twice as many, fifteen percent off* — example words come quickly to mind. But where are the phrases that nudge us to divide quantities?

One certainly cannot complain about perfectly clear and grammatical English sentences simply because they yield unexpected algebraic translations. But as we teach students to translate from one language into another — a skill they certainly need to be taught — it is only fair to alert them to the peculiarities of both of their languages.

References

1. Bresnan, Joan (ed.) *The mental representation of grammatical relations*. Cambridge, MA: MIT Press, 1982
2. Burton, Martha B. A Linguistic Basis for Student Difficulties with Algebra. *For the Learning of Mathematics* 8,1 (February 1988) 2-7
3. Burton, Martha B. Attempting Mathematics in a meaningless Language. In Sterrett, Andrew (ed.) *Using writing to teach mathematics (MAA Notes Number 16)* Washington, DC: Mathematical Association of America 1990
4. Clement, John. Algebra Word Problem Solutions: Thought Processes Underlying a Common Misconception. *Journal for Research in Mathematics Education* 13 (January 1982): 16-30
5. Clement, John; Lochhead, Jack; and Monk, George S. Translation Difficulties in Learning Mathematics. *American Mathematical Monthly* 88 (April 1981): 286-290
6. Foss, Donald J., and Hakes, David T. *Psycholinguistics. an introduction to the psychology of language*. Englewood Cliffs, NJ: Prentice-Hall 1978
7. Gray, Elizabeth. An Analysis of Syntactic Skills Used in Translating Natural Language Sentences into Algebraic Notation. Dissertation in preparation. Southeastern Louisiana University
8. Lochhead, Jack, and Mestre, Jose. From Words to Algebra: Mending Misconceptions. In Coxford, Arthur F. (ed.) *The ideas of Algebra, K-12 (1988 Yearbook)* Reston, VA: National Council of Teachers of Mathematics, 1988
9. Newby, Michael. *The structure of English: a handbook of English grammar*. Cambridge: Cambridge University Press, 1987
10. Quirk, Randolph; and Greenbaum, Sidney. *A concise Grammar of contemporary English*. New York: Harcourt Brace Jovanovich 1973

When I admit a new boy to my school I usually say to him: three eights? [ ] If he says twenty-four I ask How do you know? and he generally replies they told me at the other school Now you must tell a child that there are Three Persons in the Trinity because that is a dogma; but it is a sin against a child s mind to tell him what three eights are. when a boy says they told him three eights are twenty-four I usually reply Well here we say three eights are something else It is rare indeed that he raises any objection; he is usually quite prepared to believe that three eights is whatever the local head master chooses to make it

E.A. Greening Lamborn

---

men 1: have one s mind roused; hence both to love and to be mad (linked also by Shakespeare: The lunatic the lover of imagination all compact); also to think remember; show warn foretell and other extensions

Skr *mantra*: prayer counsel Vedic hymns *mantra* came to be used of a sacred passage or term for prayer or incantation supposedly embodying the god invoked and often repeated lengthily for its potent appeal Greatest of these is the term *OM* actually a triphone *a u m* a compression into one sound of three-times-three substances that comprise the universe: the three worlds earth atmosphere and heaven; the three (main) Hindu gods Brahma Vishnu Shiva; and the three sacred Vedic scriptures Tig Veda Yajur Veda Sama Veda Some add more personally another triad the god Vishnu (the sun) his wife Sri (good fortune) and oneself praying *OM* is discussed in various Upanishads among which the Mandukya is wholly devoted to its powers Representations of the lingam often bear the Sanskrit sign for *OM* and the sound is repeated and repeated in pious prayer

manas: mind *Ahriman*: hostile spirit *Ormuzd* shortened from *Ahura-mazda*: wise lord (*ahura*: lord) Via the Orient and Portuguese *mandarin*; the fruit *mandarine* from the golden robe of the Chinese official

Gk *mantis*: seer *mantic*: *praying mantis* *matos*: automatic *menthenein*: learn; *mathema*: thing learned; science E *mathesis*: mental discipline Metaphysical speculation is derided by Pope in *The Dunciad* (1742) iv:

> Mad mathesis alone was unconfined
> Too mad for mere material chains to bind

The suffix *math* was first applied to learning in general; hence *opsimath*: late learner (Gk *opse* late); *polymath* learned in many fields; *chrestomathy* (Gk *khrestos*: useful) Later the term *mathematical* became limited to the precise sciences: arithmetic geometry astronomy optics (In the quadrivium of medieval studies music took the place of optics; see *kuetuer*) In the 16th c on the analogy of the word *physics* *mathematics* with an s became the normal form requiring a singular verb [ ]

Joseph T Shipley