I Learn Mathematics from My Students —Multiculturalism in Action

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For twenty years I have taught a junior/senior level geometry course for mathematics majors and future teachers at Cornell University The core of the course emphasizes students learning geometry with reason, understanding and through their personal experiences of the meaning of geometry. To accomplish this I assign a series of inviting and challenging problems and encourage the students in which I listen to and comment on her/his thinking The students respond to my comments and I to their responses in a cycle that ends when both of us are satisfied The results of this dialogue are then used to stimulate the whole class discussions The text [Henderson, 1996] is based on the course and contains a discussion of its philosophy; in addition, the paper [Lo *et al.*, 1996] contains a fuller discussion of the course and the effects on the students.

What I have discovered is that through this process not only have the students learned, but also I have learned much about geometry from them. At first I was surprised—How could I, an expert in geometry, learn from students? But this learning has continued for twenty years and I now expect its occurrence. In fact, as I expect it more and more and learn to listen more effectively to them, I find that a larger portion of the students in the class are showing me something about geometry that I have never seen before

For the past six years I have kept a record of those students who have shown me geometry that I had not seen before I recorded each student who showed me mathematics that was new to me; if one student showed me more than one new piece of mathematics then I only counted that student once The results follow and cover the period 1988-1993 (I did not teach the course in 1994):

# of students	# who showed me new mathematics	% who showed me new mathematics
178	56	31%

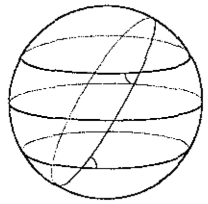
In this paper I give several examples of new mathematics (theorems and proofs) shown to me by my students. This will be followed by some reflections on the notion of proof which have helped me to make sense of why it is that I learn from my students This will lead to more data from my students and connect the discussion to issues of multiculturalism in mathematics and to our descriptions of what is mathematics

Mathematics that students have showed me

There follows a few examples of mathematics I have learned from my students. I picked these examples because they show particularly powerful and clear proofs. It is possible that some of these new pieces of mathematics are known by others, but at the time I had not seen them before. As far as I know, none of these proofs has previously appeared in print. For conciseness and clarity, these examples are written in my words and not in the students' words

Theorem 1: *The sum of the angles of a (geodesic) triangle on the sphere is more than a straight angle.*

Here is a proof that was discovered and shown to me by Mariah Magargee, a woman first year student who was taking a course for "students who did not yet feel comfortable with mathematics", which is taught in the same style and using some of the same problems as the geometry course. In class I stressed that the students should remember that latitude circles (except for the equator) are not geodesics and I urged them not to try to apply the notions of parallel to latitude circles Mariah ignored my urgings and came up with the following delightful proof:

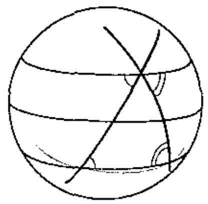


First note that:

> *Two latitude circles which are symmetric about the equator have the property that every (great circle) transversal has opposite interior angles congruent*

This follows because the two latitudes have half-turn symmetry about any point on the equator.

Now we can mimic the usual planar proof:



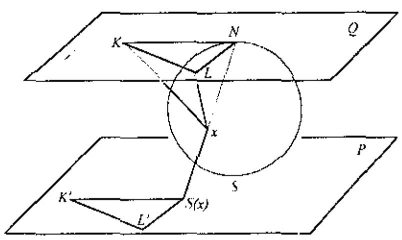
We see that the angles of the "triangle" in the figure sum to a straight angle. This is not a true spherical triangle because the base is a segment of a latitude circle instead of a (geodesic) great circle. If we replace this latitude segment by a great circle segment then the base angles will increase. Clearly, then, the angles of the resulting spherical triangle sum to more than a straight angle

You can check that any small spherical triangle can be derived in this manner.

Theorem 2: *Stereographic projection from a sphere to the plane is conformal* (i.e. preserves the size of angles.). [If the sphere is tangent to a plane P at its South Pole S then stereographic projection is the projection of the sphere from the North Pole N onto the plane P.]

The following proof was shown to me by Lucy Dladla, a Mosotho woman from South Africa. Instead of focusing on the plane P she focused on the plane Q tangent to the North Pole.

The angle between two great circles on the sphere is the same as the angle between the tangent vectors to these circles at the point of their intersection x. Let the tangents to these great circles at the point x intersect the plane Q in points L and K (if a tangent vector does not intersect the plane then use the opposite pointing vector). Let $S(x)$ be the projection of x on the plane P. Now connect the points K and L with the point N. $Kx \cong KN$ because they are two tangents to the sphere drawn from the same point; and $Lx \cong LN$ due to the same reason. $\Delta KLx \cong \Delta KLN$ by SSS, therefore $\angle KxL \cong \angle KNL$.



Now the circles are projected onto plane P as two curves emerging from the point $S(x)$, the angle between these curves being equal to that between their tangent vectors. These tangents $S(x)K'$ and $S(x)L'$ are projections of the tangents xK and xL and are therefore, intersections of the planes NKx and NLx with the projection plane P. But the planes NKx and NLx intersect the plane Q parallel to the plane P, along the straight lines NK and NL, therefore the straight lines $S(x)K'$ and $S(x)L'$ are parallel, respectively, to the lines NK and NL. Thus

$$\angle K'S(x)L' \cong \angle KNL \cong \angle KxL$$

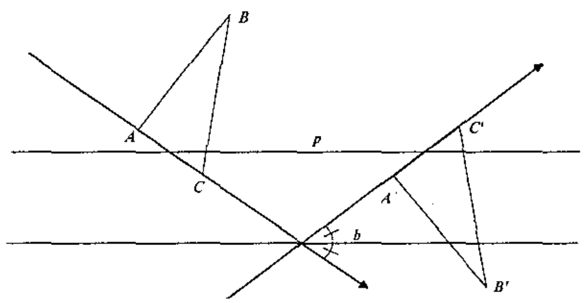
and the size of the angle is preserved.

Theorem 3: *Every isometry of the plane is the composition of one, two, or three reflections and is either the identity, a reflection, a rotation, a translation, or a glide reflection.*

The standard proof that I knew proceeded by using the Lemma *(Every isometry is determined by its effect on any (non-degenerate) triangle)* and then showing that the effect on a triangle can be accomplished by zero, one, two, or three reflections and proving that each of these cases corresponds to the identity, a reflection, a rotation, a translation, or a glide reflection. This proof is experienced by many students as being indirect and convoluted. In particular, Jun Kawashima, an Asian-American man, looked for a proof that would show that each isometry was the identity, a reflection, a rotation, a translation, or a glide reflection more directly.

The following is an outline of Jun's proof, which also uses the Lemma:

- If f is an isometry and ΔABC is any triangle, then the triangle is congruent to $\Delta A'B'C'$, where $f(A) = A'$, $f(B) = B'$, $f(C) = C'$. This congruence is either a direct congruence (no reflection needed) or not a direct congruence.
- If the congruence is direct and the lines AA', BB', CC' are parallel then the isometry is a translation along the direction of AA'.
- If the congruence is direct and the lines AA', BB', CC' are not parallel then the isometry is a rotation about the intersection of the perpendicular bisector of the lines AA', BB', CC'.
- If the congruence is not direct, then prolong AC and $A'C'$ to two directed lines and consider the bisector, b, of the angle between the two lines. Parallel transport this line (along one of the directed lines) to line, p, that is equidistant from A and A', as shown in the picture. Then we can see that the congruence (and thus by the Lemma also f) is equal to a glide reflection along p or a reflection through p

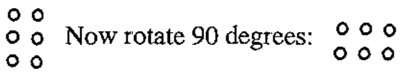


So the question emerges: Why is it that I am learning mathematics from my students? Clearly, a necessary condition is that I listen well to my students—if I did not listen there would be no chance for me to learn from them. But that does not answer why the students continue to come up with mathematics that is new to me. How can I make sense of this? I find that I can make some sense of this situation by reflecting on the notion of proof which is at the core of understanding, thinking, and listening effectively about mathematics.

Proof as a convincing communication that answers "Why?"

So, in this context, what is proof? A proof must do more than merely show *that* something is true. Here are some conclusions about proof that I have been led to by my experiences as a mathematician and teacher. Many of these understandings have come from listening to my students.

Why is $3 \times 2 = 2 \times 3$? To say "It follows from the Commutative Law", or "It can be proved from Peano's Axioms", does not answer the why-question. But most people will be convinced by: "I can count three 2's and then two 3's and see that they are both equal to the same, six". OK, now why is 2,657,873 × 92,564 = 92,564 × 2,657,873? We cannot count this—it is too large. But is there a way to see 3 × 2 = 2 × 3 without counting? Yes:

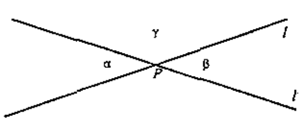


Most people will not have trouble extending this proof to include 2,657,873 × 92,564 = 92,564 × 2,657,873, or the more general $n \times m = m \times n$. Note that for the above to make sense I must have a meaning for $n \times m$ and a meaning for $m \times n$ *and these meanings must be different*. So naturally I have the question: "Why (or in what sense) are these things related?" A proof should help me experience relationships between the meanings—it is not just an argument to show THAT something is true. In my experience, to perform the formal mathematical induction proof starting from Peano's Axioms does not answer anyone's why-questions, unless it is such a question as: "Why does the Commutative Law follow from Peano's Axioms?" Most people (other than logicians) seem to have little interest in that question.

Conclusion 1: *In order for me to be satisfied by a proof, the proof must answer my why-question and relate my meanings of the concepts involved.*

As further evidence toward this conclusion, you have probably had the experience of reading a proof and following each step logically but still not being satisfied because the proof did not lead you to experience the answer to your why-question. In fact most proofs in the literature are not written out in such a way that it is possible to follow each step in a logical formal way. Even if they were so written most proofs would be too long and complicated for a person to check each step. Furthermore, even among mathematics researchers, a formal logical proof that they can follow step-by-step is not always satisfying. For example, my shortest research paper [Henderson, 1973] has a very concise simple proof that anyone who understands the terms involved can easily follow logically step-by-step. But, I have received more questions from other mathematicians about that paper than about any of my other research papers and most of the questions were of the sort: "Why is it true?", "Where did it come from?", "How did you see it?" They accepted the proof logically, they were convinced that it was true, but were not satisfied. To a large extent I was not able to answer these questions, and I find the same phenomenon in my students—*why-questions are hard to find and hard to articulate.*

Let us look at another example—the Vertical Angle Theorem: *If l and l′ are straight lines then the angle α is congruent to the angle β.*



The proof in most textbooks goes something like this:

> If $m(\alpha)$ denotes the measure of angle α, then $m(\alpha) + m(\gamma) = 180$ degrees $= m(\gamma) + m(\beta)$. Subtracting $m(\gamma)$ from both sides we conclude that $m(\alpha) = m(\beta)$ and thus that α is congruent to β.

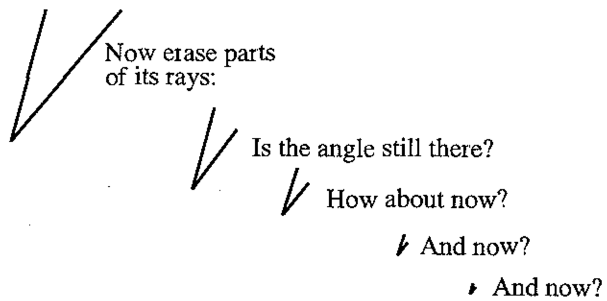
This proof is most satisfying to persons for whom the meaning of congruence of angles is in terms of measure and who see that the usual operations of arithmetic can be applied (with some care) to measures. I used to be satisfied. But several years ago a student in my geometry course objected to this proof because to her an angle is a geometric object that is congruent to another angle if it is possible to rigidly move one angle until it coincides with the other. She offered the following proof:

> Let h be a half-turn rotation about the point of intersection p. Since the straight lines have half-turn symmetry about p, $h(\alpha) = \beta$. Thus α is congruent to β.

My first reaction was that her argument could not possibly be a proof—it was too simple and seemed to leave out important parts of the standard proof. But she persisted patiently for several days and my understandings deepened. Now her proof is much more convincing to me than the standard proof. You may have had similar experiences with mathematics.

Conclusion 2: *A proof that satisfies someone else may not satisfy me because their meanings and why-questions are different from mine.*

You may ask: "But, at least in plane geometry, isn't an angle an angle? Don't we all agree on what an angle is?" Well, yes and no. Consider this acute angle:



The angle is somehow *at the corner*. It is difficult to express this formally. As evidence, I looked in most of the plane geometry books in the university library and found their definitions for "angle". I found nine different definitions! Each expressed a different meaning or aspect of "angle" and thus, potentially, each would lead to a different proof of the Vertical Angle Theorem. For example, if you see an angle as a pencil of rays, then reflection through the point p will directly take angle α to angle β. What would be the proof if you viewed an angle as a rotation (as many "modern" books do?)

Sometimes we have legitimate why-questions even with respect to statements traditionally accepted as axioms. The Commutative Law is one possible example. Another one is Side-Angle-Side (or SAS): *If two triangles have two sides and the included angle of one congruent to two sides and* *the included angle of the other, then the triangles are congruent.* You can find SAS listed in some geometry textbooks as an axiom to be assumed, in others it is listed as a theorem to be proved, and in still others as a definition of two triangles being congruent. But clearly, in any of these cases one can ask, "Why is SAS true in the plane?" This is especially pertinent since SAS is false for (geodesic) triangles on a sphere So one can naturally ask, "Why is SAS true on the plane but not on the sphere?" (Two sides and the included angle determine the endpoints of the third side but on the sphere there are, in general, two geodesic segments joining any two points)

Learning from students who differ from me

The above four examples of new mathematics are all from students who are women or persons of color (or both) Is this only a coincidence? I think not As I think back over the mathematics that I have learned from students, the most memorable mathematics is from students who differ from me (a White man) in gender or race The data also supports this in another way

	# of students	# who showed me new mathematics	% who showed me new mathematics
all students	178	56	31%
White men	85	25	29%
White women	58	21	36$
women of color*	22	5	23%
men of color*	13	5	38%
all Blacks	10	4	40%

*"Persons of color" is a term used in the USA to denote persons who are not considered White. In my class persons of color included: Asian, Asian-American, Black African, African-American, Native American, Hispanic, and Arab

Note that White women, men of color, and Blacks all have higher percentages who showed me new mathematics than the percentage of White men. This is significant because each of these groups is underrepresented in mathematics in the USA[2] I separated out the data for Blacks because it stands out, particularly given that there are so few Blacks participating in mathematics in the USA.

I have not found similar data in the literature, though this data is in line with [Fellows *et al.*, 1994, p 8] who use a somewhat similar teaching approach and observe that:

> In our experience . . once young women have become intrigued by a problem, they relate well to all aspects of mathematical creativity—the formal and systematic as well as the intuitive. For example, as often as not, the optimal solutions to the discrete math problems we have presented—and rigorous arguments to support these solutions—are offered first by girls

What is the explanation for this data? I do not have a clear answer to this, but I do have a partial answer It is possible that it is an effect of the self-selection process of the students in the course, but it also makes sense in terms of the conclusions above.

Conclusion 3: *Persons who differ the most from me (for example, in terms of cultural background and gender) are most likely to have different meanings and thus have different why-questions and different proofs.*

You should check this out in your own experience. Note that this conclusion implies that I must listen particularly carefully to the meanings and proofs expressed by persons of color and women because there is much which they see which I do not see. The table above shows that relatively fewer (percentage-wise) women of color showed me new mathematics. This may indicate that I need to listen more carefully to women of color

We should also be more critical of the standard histories of mathematics and mathematicians which have a decidedly Eurocentric emphasis I give two examples to illustrate the distortions in the histories of mathematics as it is usually presented The Pythagorean Theorem is attributed to the Greek Pythagoras, but clear statements, uses, and understandings of the result also exist in ancient Babylon, in ancient India, and probably in ancient China before the time of Pythagoras Most texts attribute the general solution of cubic equations to Cardano, a sixteenth century Italian However, a careful reading of Cardano's book shows that he thought that his technique did not work for all cubic equations and he refers the reader to the work of the Arabs. In fact, Omar al'Khayyam (tenth century in Persia) presented and proved a general (geometric) method which works for finding all solutions to all cubic equations See [Henderson, 1996], Chapter 14, for more discussion of Cardano and al'Khayyam For more examples and discussions of the Eurocentric biases in histories of mathematics see [Joseph, 1991].

Corollary: *I can learn much about mathematics by listening to persons whose cultural backgrounds or gender is different than mine Hearing someone else's proof may be difficult and may require considerable effort and patience on my part*

Perhaps it is true that women and persons of color are underrepresented in mathematics because they are not being well listened to by those of us already in mathematics

People understanding and communicating mathematics

Recently, a spirited debate surfaced in the pages of the *Bulletin of the AMS,* initiated in the July 1993 issue in an article by Arthur Jaffe and Frank Quinn [Jaffe/Quinn, 1993]. It was followed in the April 1994 issue by an article by William P Thurston [Thurston, 1994] and by a collection of "responses" and "responses to comments" by several different mathematicians [Atiyah, 1994] Thurston rejects the popular formal definition-theorem-proof model as an adequate description of mathematics and states that:

> If what we are doing is constructing better ways of thinking, then psychological and social dimensions are essential to a good model for mathematical progress . .
>
> . . The measure of our success is whether what we do enables *people* to understand and think more clearly and effectively about mathematics

I agree with this statement and offer this paper in that context I also offer this paper in the context of the widely acknowledged observation that there are too few women and persons of color participating fully in understanding and thinking about mathematics. Research in the social sciences suggests that successful learning takes place for many underrepresented minority and women students when instruction builds upon personal experience and provides for a diversity of ideas and perspectives (see [Belenky *et al*, 1986], [Cheek, 1984], [Valverde, 1984])

My experiences have shown me, and the data above suggests, that as we encourage and listen more effectively to students and, in particular, to women and persons of color, those of us already participating in mathematics will find that our own understandings and thinking about mathematics are being enriched. (See [Henderson, 1981].) The kind of listening that I am talking about here and the kind that I attempt in my classroom is described by Jere Confrey in the context of children's mathematics:

> Close listening involves an act of decentering by an adult or possibly a peer, in order to imagine what the view of the child might be like It includes repeated requests for a child to explain what the problem is that she is addressing, what she sees herself doing, and how she feels about her progress It requires one to ask for elaboration from the child about what, where, how, and why . A willingness to assume a rationality in the child's responses and to relinquish or modify one's own expected responses is a necessary part of the process [Confrey, 1993]

In addition, this is also in line with recent recommendations from various national bodies, for example, Mathematical Association of America [MAA, 1992], National Science Foundation [NSF, 1992], the Mathematical Sciences Education Board (MSEB) [MSEB, 1990 & MSEB, 1994]. For example the MSEB recommends in mathematics education

> . a style of instruction that rewards exploration, encourages experiments, respects conjectural approaches to solving problems . Students need to experience genuine problems—those whose solutions have yet to be developed by the students (or even perhaps by their teachers) Learning should be guided by the search to answer questions—first at an intuitive, empirical level; then by generalizing; and later by justifying (proving). [MSEB, 1990, p. 14]

My teaching background

My teaching is a product of Western Civilization. My known ancestors lived in England, Scotland, Ireland, Germany, and Luxembourg On my mother's side I am a descendant from a long line of academics stretching back (according to family traditions) to at least the seventeenth century On my father's side I am descended from working class people who placed high value on education My mode of teaching also has deep Western roots that reach back to the Socratic dialogues recorded by Plato in ancient Greece. More directly, my teaching has been influenced by my experiences in high school, college and graduate school In Ames, Iowa, my high school world literature teacher (Mary McNally) coaxed deep creative thinking out of us through her many writing assignments. At Swarthmore College in Pennsylvania, instead of attending classes I spent my last two years in student participation seminars and tutorials based on an old English academic model. At the University of Wisconsin my graduate mentor, R.H. Bing, taught without lectures or textbooks in a style which is often known as the Moore Method, named after Bing's own graduate mentor R.L. Moore at the University of Texas (See [Taylor, 1972], for more information on the Moore Method) R.L. Moore, around the turn of the century, was one of the first Americans to earn a Ph D in mathematics from an American university (University of Chicago)

My teaching of the geometry course is embedded in this background. I value diverse viewpoints in my classroom and I try to listen to all my students and to encourage each one to express their understandings and reasonings in their own words It was through this that I eventually discovered that I was learning from my students.

Multiculturalism and our views of mathematics

As I see it, all this is part of multiculturalism. People attach many different meanings onto "multiculturalism" and there seems to be no widely accepted definition For my purpose I define multiculturalism as listening to and learning from others who come from different experiences. Some persons paint a picture of multiculturalism as a force which is in opposition to Western Civilization and which contributes to what they perceive as a decline in Western Civilization. From my experiences, I would say instead that multiculturalism (in the sense I have defined) can be a natural part of Western or of any civilization and is a force which is not a cause of, but rather a significant part of the solution to, some of the current problems in Western Civilization. When we approach others with the attitude that we can listen to and learn from each other then we are all enriched.

In my own experience I gained new views of mathematics through listening to my students. And these views have also helped me to better hear the mathematics that my students are communicating Let me be clear that I am not advocating abandoning definitions, theorems, and proofs in mathematics or in the teaching of mathematics In fact, I advocate more attention to definitions and proofs at all levels of mathematics teaching. But I am also advocating looking at our assumptions, definitions, theorems, and proofs in less formal ways, and in ways that include social and individual aspects of experience David Hilbert, called by some the "father of formalism," wrote in the Preface to [Hilbert/Cohn-Vossen 1932]:

> In mathematics, as in any scientific research, we find two tendencies present. On the one hand, the tendency toward *abstraction* seeks to crystallize the *logical* relations inherent in the maze of material that is being studied, and the correlate the material in a systematic and orderly manner On the other hand, the tendency toward *intuitive understanding* fosters a more immediate grasp of the objects one studies, a live *rapport* with them, so to speak, which stresses the concrete meaning of their relations

As the students in my course gain a "live rapport" with the objects of study they construct many differing definitions and axioms Thus the definitions and axioms come from their experiences. According to the common formal views of mathematics we then "need" to settle on one consistent set of axioms and definitions. This is not the experience of the students in my course. Most semesters they come up with several different notions of "straightness", ten different notions of "angle", and seven versions of the parallel postulate. Rather than settling on one, it seems natural to hold and explore the complexity and interrelations among the differing notions It is not so much that the differing definitions and axioms contradict each other but, rather, that they enrich and supplement each other and point out differing points of view and aspects of our live rapport with "straightness", "angle", and "parallel". For more discussion of this issue refer to [Lo *et al.*, 1996] and the introduction to [Henderson, 1996] There is a common notion that it is important in mathematics to be consistent at the level of definitions and axioms. However, it is an empirically observable fact that neither mathematicians nor mathematical texts are consistent with one another For example, more than nine different definitions of angle can be found in plane geometry texts in the Cornell Library and there is no agreement across calculus texts as to whether the function $f(x) = 1/x$ is continuous or discontinuous. To shackle ourselves to an artificial consistency would be foolish and limiting

But: *What is the consistency of mathematics? Is it all arbitrary?* "Arbitrariness" is a dangerous notion There may be more than one possible starting point, but that does not mean that the starting points are arbitrary Differing contexts, differing points of view, and differing why-questions bring with them a demand for differing starting points. If the choice were arbitrary, then that does away with the need for any discussion. It is easy when discussing mathematics to slide from "if it is not absolute" to "then everything is completely relative/arbitrary" In most of these discussions there does not appear to be any middle ground In the geometry course the class holds onto the complexity of the multiple definitions and axioms But each student holds onto those notions which best relate to her/his why-questions The course demands that each student's definitions and axioms must be meaningful in the sense that they must lead to proofs that are convincingly communicated to others and answer — Why?

But listening effectively is not automatic. When I started using this teaching method twenty years ago I felt threatened when I could not understand a student's questions or explanations — after all, I was the expert in geometry. Gradually, after much persistence from the students, I began to realize that my old ways of understanding had blinded me from hearing alternatives Among the old ways that interfered with my effective listening were the common views of mathematics that are embedded in most of our textbooks. These views emphasize precise, consistent assumptions and definitions from which theorems are proved by applying certain logical rules to the assumptions and definitions. According to these views, the end results are mathematics that is certain. (See, for example, [Henley, 1991] and [Jaffe/Quinn, 1993] for arguments by mathematicians who support these types of views) I see no way within these views of mathematics to even start to account for the data given in this paper. I agree with Thurston [Thurston, 1994] that we must label these views as being inadequate descriptions of mathematics, especially mathematical experience and progress We must proceed in directions that include social and individual aspects of human mathematical experiences into our descriptions of mathematics [3] Unless we proceed in these directions, our very descriptions of mathematics will continue to be obstacles to progress and the full participation of all peoples in mathematics

Notes

[1] I am indebted to Kelly Gaddis for her assistance in data gathering for this paper and for some of the understandings expressed here which were sharpened through my conversations with her. In addition, Chris Breen, Jane-Jane Lo, Maria Terrell, John Volmink, and the members of Jere Confrey's mathematics education research group gave me valuable feedback on an early draft of this paper

[2] It is well known that Asians as a whole are not considered to be underrepresented in the United States, but it is not well known that Asian-Americans (native-born) are vastly underrepresented at least at the Ph D level (See [NRC 1977], Table G-WF-19.) In 1975, of the 15,569 persons in the USA with a Doctorate in mathematics 14,222 were White, 121 Black, 22 Native American, 72 Hispanic, 351 foreign-born Asian, and only 28 native-born Asian Since 1975 the Census Bureau has not distinguished between foreign born and native born.

[3] In addition to the discussions above see, for example, [Thurston, 1994], [MSEB, 1990, 1994], [Fellows *et al* , 1994], and the newsletter *Humanistic Mathematics* and in the mathematics education research literature, for example [Confrey, 1991]

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